Pricing in a shipping market with waste shipments and empty container repositioning

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Abstract

In this paper, we study a shipping market with carriers providing service between two locations. Shipments are classified into two categories: goods and waste. Trade imbalance allows low-valued waste to be shipped at bargain rates. And if imbalance still exists, empty containers have to be repositioned from a surplus location to a shortage location. Carries decide prices, which will affect the demand. We build a monopoly and a duopoly model to find the optimal pricing strategy for carries. We also analyze how the profit of a carrier is affected by trade imbalance, price sensitivity, cost structures and competition intensity.

Keywords: waste shipments, empty container repositioning, price competition

1 Introduction

Maritime transportation is vital to the development of world economy since there are no other effective alternative to the long distance transportation of large amounts of freight. And container shipping which provides shipping service in standard units has become more and more important in international freight transportation since the mid-1960s. According to UNCTAD (2014), container ships carry an estimated 52 percent of global seaborne trade in terms of value. One main issue in the containerized transportation is the empty container repositioning (ECR) problem due to the imbalance of container flow. And it has become more conspicuous with the rapid growth of containerization and the regional difference in economic development. Table 1 summarized the annual containerized trade flows on three major trade routes for the years from 2009 to 2013. It can be seen that the volume from Asia to Europe was more than double of that in the opposite direction. And the similar situation exits on the transpacific route. Such huge imbalance indicates the scale of empty container movements since the empty containers have to be repositioned from a surplus area to a deficit area.

Table 1: Containerized trade flows on three major routes, 2009-2013 (Millions of TEUs)

Year	Europe Asia		Transpacific		Transatlantic	
	Asia - Eur	Eur - Asia	Asia - NA	NA - Asia	Eur - NA	NA - Eur
2009	11.5	5.5	10.6	6.1	2.8	2.5
2010	13.3	5.7	12.3	6.5	3.2	2.7
2011	14.1	6.2	12.4	6.6	3.4	2.8
2012	13.7	6.3	13.1	6.9	3.6	2.7
2013	14.1	6.4	13.8	7.4	3.8	2.8

Source: UNCTAD (2014)

Repositioning empty containers incurs extra cost but generates no profit. However, it is unavoidable due to the trade imbalance among different regions. For example, export-dominant ports like many ports in China are always in need of empty containers, while import-dominant ports always have surplus empty containers. Shipping waste and scrap instead of shipping empty containers has been adopted by practitioners to reduce operational cost.

Waste and scrap include paper waste, plastic scrap, metal scrap and some other recyclable items. The trade imbalance allows these low-valued products to be economically shipped in a long distance. Rather than having containers returned empty, carriers offer bargain rates for these shipments to offset ECR cost. Shipping a container full of metal scrap from Los Angeles to China is cheaper than sending it to Chicago by train (see Ford (2013)). Although shipping waste does not make money, it may help reduce cost of repositioning empty containers. Besides, compared with using new materials, manufacturing with recycled materials helps protect the environment and boost economic efficiency. All these benefits encourage the development of waste shipments. According to Morrison (2014), the U.S. sold $8.8 billion worth of waste and scrap to China in 2013, which is about 7.2% of total U.S. exports to China. And European Commission (2014) reported that the EU exported € 7.1 billion worth of waste and scrap to non-OECD Decision countries in 2013.

Although shipping waste has attracted great interest from industry analysts, it goes unnoticed by academics who study ECR problem. Actually, the purposes of shipping goods and waste for carriers are different: shipping good is to make money and shipping waste is to sustain the operations, just like repositioning empty containers. But the previous academic research does not differentiate these two, and just uses "laden containers" to differentiate from "empty containers". Besides, most work on ECR problem aims to find optimal repositioning strategy given the imbalanced flow of demand, and assumes that carriers can not affect demand by setting different prices. But in reality, carriers will charge more to the customer in the headhaul and charge less to the customer in the backhaul, so that the imbalance situation will be lessen.

We try to fill in this gap by studying the carrier's pricing decision in a shipping market with waste shipments and ECR. In this paper, the following questions will arise:

(1) What is the optimal pricing strategy for the carrier in a shipping market with waste shipments and ECR?

(2) Under what conditions should the carrier ship waste instead of repositioning empty containers? And compared to the no waste shipment case, how much improvement will be made?

(3) In case of competition, does the solution of Nash Equilibrium (NE) exists? If yes, what is the pricing strategy for the carriers?

(4) How the profit of carrier will be affected by different parameters like trade imbalance, cost structure, price sensitivity and competition intensity?

To address these problems, we first build a monopoly model to find and analyze the carrier's pricing strategy. The carrier's strategy falls into four categories: seeking the balance of goods; repositioning empty containers to achieve flow balance; shipping waste to achieve flow balance; shipping waste and repositioning empty containers to achieve flow balance. Compared with two traditional approaches, our approach generates more profit. Furthermore, we build a duopoly model to analyze competition between two symmetric carriers. We prove that there exists a unique NE in this game and solutions are provided. We also do some sensitivity analysis based on the results of these two models.

The main contribution of this paper is two-fold. First, this is the first paper to consider shipping waste in the container shipping market. Although shipping waste alone does not make money, it may help carrier offset ECR cost under some conditions. Second, most research on ECR problem assumes the demand of shipping service is given and studies the optimal repositioning strategy. Our work allows the demand to be affected by setting different prices, which further affects the number of empty containers to be shipped.

The rest of the paper is organized as follows. In Section 2, we provide a literature review on ECR problem and competition issue in the shipping market. Section 3 introduces the notations, assumptions and the model. We first analyze the pricing strategy in case of monopoly, which is presented in Section 4. In Section 5, we study the competition model with two symmetric carriers. Finally, conclusions are drawn in Section 6.

2 Literature review

ECR problem has drawn a lot of research attention since the beginning of containerization. And most of such work focus on the optimal repositioning strategy given the imbalanced flow of demand, that is the demand is not controllable and must be satisfied. Moon et al. (2010) considered the ECR problem with leasing and purchasing containers in a multi-port system. And mixed integer programming and genetic algorithms were adopted to solve this problem. Li et al. (2007) developed a (U,D) policy to allocate empty containers among multi-ports. For one port, empty containers are imported to U if the number of empty containers is less than U, or exported to D if the number of empty containers is more than D. And a heuristic algorithm was designed to compute the specific allocation of empty containers among ports. Ng et al. (2012) developed an optimal policy for empty container transfer problem between two adjacent ports/depots with backlogging and random demand and supply over a multi-period horizon. Stochastic dynamic programming was used to formulate this problem and structural characteristics of the optimal solution were presented. Song and Earl (2008) built an integrated model to solve empty vehicle repositioning and fleet-sizing problem in a two-depot system. A threshold control policy was presented as the optimal solution for the empty vehicle reposition problem. Long et al. (2012) proposed a two-stage stochastic programming model to minimize the ECR cost under random demand, supply and ship capacities. And the sample average approximation method was used to solve the model with large-scale case. Erera et al. (2009) developed a robust optimization approach to find a recoverable minimum cost repositioning plan. More research on ECR can be found in Song and Dong (2015), which provided a comprehensive literature review on

ECR problem mainly from the network scope.

All the above work assumes that the carrier can not affect demand. A recent work by Lu et al. (2015) considers coordinating pricing and ECR problem in a two-depot system. They used Markov decision process to formulate the problem and under mild assumptions they derived explicit structure of optimal strategies which is given by a price vector and two target inventory levels for inbound and outbound repositioning. The carrier in our model can also affect demand by setting different prices, which further affects the number of empty containers to be shipped. Besides, we consider shipping waste as an alternative approach to realize flow balance.

Our work is also related to the research on competition in the shipping market. Competition in the shipping industry is fierce and involves many different parties like shippers, carriers, terminal operators and ports. Álvarez-SanJaime et al. (2013) analyzed competition for freight transport between one road carrier and two liner shipping carriers by taking into account economics of scale in the liner shipping market. Besides, cooperative game model was adopted to analyze the horizontal integration between shipping lines. Ishii et al. (2013) proposed a game-theoretical model to study competition between two ports under demand uncertainty. Ports compete for customers by selecting port charges and a unique NE exists in this game. A case study of competition between the ports of Busan and Kobe was also presented. Lee et al. (2012) used a multi-level hierarchical approach to model the competition among oligopolistic ocean carriers, land carriers and terminal operators. Wang et al. (2014) studied the competition problem between two carriers in a new emerging liner shipping market. Each carrier decides optimal freight rate, service frequency and ship capacity to maximize his payoff. Three game-theoretical models (Nash game, Stackelberg game and deterrence game) were further investigated. They also considered a two-location market, but they assumed that two carriers only compete for customers in one direction, that is they implicitly assumed that the carriers have all containers returned empty.

Zhou and Lee (2009) is the most related work to ours. They built mathematical models to find the optimal pricing strategies for carriers in a shipping market with ECR. They also analyzed how market competition will affect carriers' profit. But they did not consider shipping waste and scrap, which is a useful tool for carriers to reduce operational cost. To the best of our knowledge, our paper is the first one to study pricing strategy in a shipping market with waste shipments.

3 Notations, assumptions and the model

Consider two liner shipping companies denoted by carrier 1 and 2 who compete on the shipping service between two ports denoted by A and B. Two kinds of shipping service are provided by both carriers: one is shipping goods which are usually of high value and the service is priced at high rates; the other one is shipping waste and scrap, which are low-valued products and the service is priced at low rates. There are potential demands for these two kinds of service in both directions. Laden containers (for goods and waste) and empty containers can be shipped in both directions. For convenience, we use shipment 1 and 2 to represent goods shipment from A to B and from B to A, respectively. And we use shipment 3 and 4 to represent waste shipment from A to B and from B to A, respectively. Since it is a two-location closed system, surplus containers need to be returned as empty containers, which are denoted by shipment 5 and 6.

The following notations are used in this paper and the superscript j denotes carrier $j(=1,2)$:

a_i^j	the potential demand of shipment $i(=1,2,3,4)$
b_i^j	the price sensitivity of shipment $i(=1,2,3,4)$
c_i^j	the cost of moving one container loaded with shipment $i(=1,2,3,4)$
c_5^j, c_6^j	the cost of moving one empty container from A to B and from B to A
d_i^j	the realized demand of shipment $i(=1,2,3,4)$
d_5^j, d_6^j	number of empty containers to be shipped from A to B and from B to A
p_i^j	the unit price of transporting shipment $i(=1,2,3,4)$
γ_i^j	the competition intensity of shipment $i(=1,2,3,4)$

Carriers compete for customers in price and each carrier seeks to maximize his profit by pricing the service. Define $x^+ = max(x,0)$. Therefore, the optimization problem for carrier j is:

$$max\ \Pi^j(p_1^j,p_2^j,p_3^j,p_4^j) = \sum_{i=1}^{4} d_i^j(p_i^j - c_i^j) - (d_2^j + d_4^j - d_1^j - d_3^j)^+ c_5^j - (d_1^j + d_3^j - d_2^j - d_4^j)^+ c_6^j \quad (1)$$

subject to:

$$d_i^j = a_i^j - b_i^j p_i^j + \gamma_i^j(p_i^k - p_i^j), i = 1,2,3,4 \quad (2)$$

$$d_i^j, p_i^j \geq 0, i = 1,2,3,4 \quad (3)$$

The first item in (1) indicates the profit that the carrier collects from the shipper by providing shipping service and the last two items indicate the ECR cost if there exists any imbalance. We assume that for each shipment i, the demand for one carrier's service depends linearly on his own and rival's price, which is indicated by (2). This says that when carrier j cuts the price of shipment i by one unit, the demand will increase by

$(b_i^j + \gamma_i^j)$: b_i^j are the increased demand stimulated by cheaper price and the remaining γ_i^j are demand attracted from the competitor. The larger γ_i^j means carrier j will grab more market share from his competitor by lowering the same amount of price, hence it can represent the intensity of competition. And the total demand for shipment i is $d_i(p_i^1, p_i^2) = (a_i^1 + a_i^2) - (b_i^1 + \gamma_i^1 - \gamma_i^2)p_i^1 - (b_i^2 + \gamma_i^2 - \gamma_i^1)p_i^2$. We assume that each carrier sets his price so that his own demand is nonnegative. We use this assumption in order to have a well-defined demand function for the other carrier if one decides not to provide service. In addition, we assume that all demand can be satisfied, i.e. there is no capacity constraint.

Note that all parameters are positive and we impose the following assumptions on these parameters:

- $c_5^j < min(c_1^j, c_3^j)$ and $c_6^j < min(c_2^j, c_4^j)$, that is the cost of shipping one empty container is less than the cost of shipping one laden container for each carrier. The relationship between these costs is understandable and can be observed in the industry.

- $a_1^j > b_1^j c_1^j$, $a_2^j > b_2^j c_2^j$, that is shipping goods is profitable if the price p_i^j is in the interval $(c_i^j, a_i^j / b_i^j)$. In practice, shipping high-valued goods is the main source of profits for the carrier.

- $a_3^j < b_3^j c_5^j$, $a_4^j < b_4^j c_6^j$, that is the pricing ceiling of shipping waste is below the corresponding empty container repositioning cost for each carrier. This assumption together with the first one indicate that transporting waste always loses money. But compared to repositioning empty containers, shipping waste may be a better way to achieve flow balance.

These assumptions facilitate our analysis and make the model more realistic.

4 Pricing strategy in a monopoly market

In this section, we first study the pricing strategy in case of monopoly. To study the market with one carrier will give us a better understanding of pricing strategy with the existence of waste shipment and ECR. And since shipping alliances are very popular in the liner shipping sector, the monopoly model is an approximation when one shipping alliance dominates some service route, using vessel-sharing agreement to offer integrated services and maximizing joint profit.

With only one carrier, we omit the superscript and the optimization problem is:

$$max\ \Pi(p_1,p_2,p_3,p_4) = \sum_{i=1}^{4} d_i(p_i - c_i) - (d_2 + d_4 - d_1 - d_3)^+ c_5 - (d_1 + d_3 - d_2 - d_4)^+ c_6 \quad (4)$$

subject to:

$$d_i = a_i - b_i p_i, i = 1,2,3,4 \quad (5)$$

$$d_i, p_i \geq 0, i = 1,2,3,4 \quad (6)$$

In case of monopoly, there is no competition between carriers, so the demand function reduces to (5). The following proposition gives the carrier's optimal pricing under different market conditions and cost structure.

Proposition 1. *In case of monopoly, the optimal pricing is as following:*
$(p_1^*, p_2^*, p_3^*, p_4^*) =$

$$\begin{cases} (\frac{a_1/b_1+c_1+T_0}{2}, \frac{a_2/b_2+c_2-T_0}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ max(-(c_3 - \frac{a_3}{b_3}), -c_5) < T_0 < min(c_4 - \frac{a_4}{b_4}, c_6) \\ (\frac{a_1/b_1+c_1+c_6}{2}, \frac{a_2/b_2+c_2-c_6}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ c_6 \leq T_0\ and\ c_6 < c_4 - \frac{a_4}{b_4} \\ (\frac{a_1/b_1+c_1-c_5}{2}, \frac{a_2/b_2+c_2+c_5}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ c_5 \leq -T_0\ and\ c_5 < c_3 - \frac{a_3}{b_3} \\ (\frac{a_1/b_1+c_1+T_1}{2}, \frac{a_2/b_2+c_2-T_1}{2}, \frac{a_3}{b_3}, \frac{a_4/b_4+c_4-T_1}{2}) & if\ T_1 \leq \frac{a_1}{b_1} - c_1\ and\ c_4 - \frac{a_4}{b_4} \leq T_1 < c_6 \\ (\frac{a_1/b_1+c_1-T_2}{2}, \frac{a_2/b_2+c_2+T_2}{2}, \frac{a_3/b_3+c_3-T_2}{2}, \frac{a_4}{b_4}) & if\ T_2 \leq \frac{a_2}{b_2} - c_2\ and\ c_3 - \frac{a_3}{b_3} \leq T_2 < c_5 \\ (\frac{a_1/b_1+c_1+c_6}{2}, \frac{a_2/b_2+c_2-c_6}{2}, \frac{a_3}{b_3}, \frac{a_4/b_4+c_4-c_6}{2}) & if\ c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1) \\ (\frac{a_1/b_1+c_1-c_5}{2}, \frac{a_2/b_2+c_2+c_5}{2}, \frac{a_3/b_3+c_3-c_5}{2}, \frac{a_4}{b_4}) & if\ c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2) \end{cases}$$

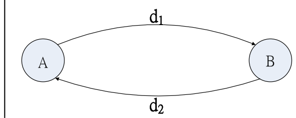
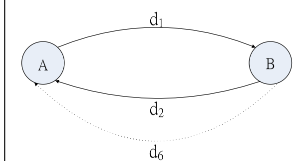
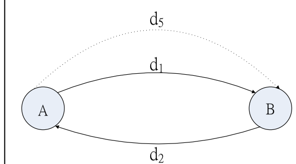
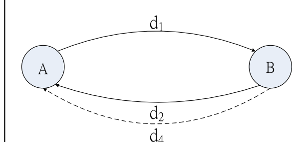
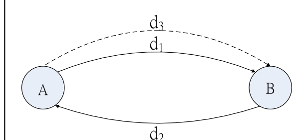
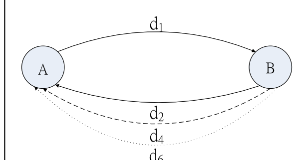
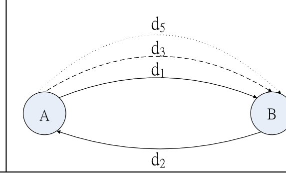
where $T_0 = \frac{(a_1-b_1c_1)-(a_2-b_2c_2)}{b_1+b_2}$, $T_1 = \frac{(a_1-b_1c_1)-(a_2-b_2c_2)-(a_4-b_4c_4)}{b_1+b_2+b_4}$, $T_2 = \frac{(a_2-b_2c_2)-(a_1-b_1c_1)-(a_3-b_3c_3)}{b_1+b_2+b_3}$.
And the corresponding demand and profit are summarized in Table 2.

We say that the realized demands of goods are balanced if $d_1 = d_2$, and imbalanced otherwise. Since it is a two-location closed system, the carrier should always achieve flow balance, i.e., $d_1 + d_3 + d_5 = d_2 + d_4 + d_6$. Then in case of imbalance, the carrier can ship waste or empty containers, or take both measures.

From Proposition 1, we find that the relationship between T_i, $(\frac{a_i}{b_i} - c_i)$ and corresponding ECR cost $c_5(c_6)$ is the key factor to determine the carrier's choice. Since $\frac{a_i}{b_i}$ is the price ceiling of shipment i, $(\frac{a_i}{b_i} - c_i)(i = 1,2)$ is the maximum possible profit of transporting one unit of goods. And $(c_i - \frac{a_i}{b_i})(i = 3,4)$ is the minimum possible loss of transporting one unit of waste. The meaning of T_i is a bit confusing. Actually T_0, T_1, T_2 satisfy the following three equations, respectively.

$$a_1 - b_1(c_1 + T_0) = a_2 - b_2(c_2 - T_0)$$
$$a_1 - b_1(c_1 + T_1) = a_2 - b_2(c_2 - T_1) + a_4 - b_4(c_4 - T_1)$$
$$a_2 - b_2(c_2 + T_2) = a_1 - b_1(c_1 - T_2) + a_3 - b_3(c_3 - T_2)$$

Table 2: Results of monopoly model

Condition	$(p_1^*, p_2^*, p_3^*, p_4^*)$ $(d_1^*, d_2^*, d_3^*, d_4^*)$ (d_5^*, d_6^*)	Π^*	Figure
Case 1: $T_0 < min(c_4 - \frac{a_4}{b_4}, c_6)$ $-T_0 < min(c_3 - \frac{a_3}{b_3}, c_5)$	$(\frac{a_1/b_1+c_1+T_0}{2}, \frac{a_2/b_2+c_2-T_0}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1+T_0)}{2}, \frac{a_2-b_2(c_2-T_0)}{2}, 0, 0)$ $(0, 0)$	$\frac{(a_1-b_1(c_1+T_0))^2}{4b_1} + \frac{(a_2-b_2(c_2-T_0))^2}{4b_2}$	
Case 2: $c_6 \leq T_0$, $c_6 < c_4 - \frac{a_4}{b_4}$	$(\frac{a_1/b_1+c_1+c_6}{2}, \frac{a_2/b_2+c_2-c_6}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1+c_6)}{2}, \frac{a_2-b_2(c_2-c_6)}{2}, 0, 0)$ $(0, \frac{(b_1+b_2)(T_0-c_6)}{2})$	$\frac{(a_1-b_1(c_1+c_6))^2}{4b_1} + \frac{(a_2-b_2(c_2-c_6))^2}{4b_2}$	
Case 3: $c_5 \leq -T_0$, $c_5 < c_3 - \frac{a_3}{b_3}$	$(\frac{a_1/b_1+c_1-c_5}{2}, \frac{a_2/b_2+c_2+c_5}{2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-c_5)}{2}, \frac{a_2-b_2(c_2+c_5)}{2}, 0, 0)$ $(\frac{(b_1+b_2)(-T_0-c_5)}{2}, 0)$	$\frac{(a_1-b_1(c_1-c_5))^2}{4b_1} + \frac{(a_2-b_2(c_2+c_5))^2}{4b_2}$	
Case 4: $T_1 \leq \frac{a_1}{b_1} - c_1$ $c_4 - \frac{a_4}{b_4} \leq T_1 < c_6$	$(\frac{a_1/b_1+c_1+T_1}{2}, \frac{a_2/b_2+c_2-T_1}{2}, \frac{a_3}{b_3}, \frac{a_4/b_4+c_4-T_1}{2})$ $(\frac{a_1-b_1(c_1+T_1)}{2}, \frac{a_2-b_2(c_2-T_1)}{2}, 0, \frac{a_4-b_4(c_4-T_1)}{2})$ $(0, 0)$	$\frac{(a_1-b_1(c_1+T_1))^2}{4b_1} + \frac{(a_2-b_2(c_2-T_1))^2}{4b_2} + \frac{(a_4-b_4(c_4-T_1))^2}{4b_4}$	
Case 5: $T_2 \leq \frac{a_2}{b_2} - c_2$ $c_3 - \frac{a_3}{b_3} \leq T_2 < c_5$	$(\frac{a_1/b_1+c_1-T_2}{2}, \frac{a_2/b_2+c_2+T_2}{2}, \frac{a_3/b_3+c_3-T_2}{2}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-T_2)}{2}, \frac{a_2-b_2(c_2+T_2)}{2}, \frac{a_3-b_3(c_3-T_2)}{2}, 0)$ $(0, 0)$	$\frac{(a_1-b_1(c_1-T_2))^2}{4b_1} + \frac{(a_2-b_2(c_2+T_2))^2}{4b_2} + \frac{(a_3-b_3(c_3-T_2))^2}{4b_3}$	
Case 6: $c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1)$	$(\frac{a_1/b_1+c_1+c_6}{2}, \frac{a_2/b_2+c_2-c_6}{2}, \frac{a_3}{b_3}, \frac{a_4/b_4+c_4-c_6}{2})$ $(\frac{a_1-b_1(c_1+c_6)}{2}, \frac{a_2-b_2(c_2-c_6)}{2}, 0, \frac{a_4-b_4(c_4-c_6)}{2})$ $(0, \frac{(b_1+b_2+b_4)(T_1-c_6)}{2})$	$\frac{(a_1-b_1(c_1+c_6))^2}{4b_1} + \frac{(a_2-b_2(c_2-c_6))^2}{4b_2} + \frac{(a_4-b_4(c_4-c_6))^2}{4b_4}$	
Case 7: $c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2)$	$(\frac{a_1/b_1+c_1-c_5}{2}, \frac{a_2/b_2+c_2+c_5}{2}, \frac{a_3/b_3+c_3-c_5}{2}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-c_5)}{2}, \frac{a_2-b_2(c_2+c_5)}{2}, \frac{a_3-b_3(c_3-c_5)}{2}, 0)$ $(\frac{(b_1+b_2+b_3)(T_2-c_5)}{2}, 0)$	$\frac{(a_1-b_1(c_1-c_5))^2}{4b_1} + \frac{(a_2-b_2(c_2+c_5))^2}{4b_2} + \frac{(a_3-b_3(c_3-c_5))^2}{4b_3}$	

That is, if the carrier charges shipment 1 and 2 at (c_1+T_0) and (c_2-T_0), respectively, he can achieve the balance of goods and get zero profit. Or if he charges shipment 1, 2 and 4 at (c_1+T_1), (c_2-T_1) and (c_4-T_1), respectively, he doesn't need to reposition empty containers and get zero profit. T_2 can be analyzed in a similar way. So we can regard T_0, T_1, T_2 as balancing cost. More specifically, T_0 is the balancing cost when carrier only transports shipment 1 and 2. And $T_1(T_2)$ is the balancing cost when carrier only transports shipment 1, 2 and 4 (3). Here balancing has two meanings: no empty containers to be shipped and break-even point. So if the balancing cost T_0 is the minimum compared to the loss of shipping waste and empty containers , the carrier should seek the balance of goods, just as the results of Case 1.

If the ECR cost c_5 (or c_6) is the minimum compared to the balancing cost and loss of shipping waste, the carrier should not seek the balance of goods but use empty containers to achieve flow balance. The results in Case 2 (Case 3 is its symmetric case) shows that the carrier should ignore the waste shipments and treat two directions of goods shipments independently. For shipment 1, the carrier solves the problem $max\ d_1(p_1-(c_1+c_6))$. And for shipment 2, the carrier solves the problem $max\ d_2(p_2-(c_2-c_6))$. The shipment of goods from B to A lessens the imbalance situation, so the carrier motivates customers in this direction with incentives (negative repositioning cost). And since the shipment of goods from A to B worsens the imbalance situation, the carrier charges more (positive repositioning cost) to the customers in this direction.

But when $c_3-\frac{a_3}{b_3}$ (or $c_4-\frac{a_4}{b_4}$) is minimum, the result is more complicated: the carrier may only ship waste to achieve flow balance (Case 4 and 5) or take both measures (Case 6 and 7). Unlike the empty container repositioning cost, which is fixed, the loss of shipping waste depends on the price charged by the carrier, which is in the interval $[0, \frac{a_i}{b_i}]$. As long as $c_3-p_3<c_5$ (or $c_4-p_4<c_6$), shipping waste is a cost saving way compared to shipping empty containers back. However, the amount of waste shipments is limited by the demand function, while the carrier can always ship all the surplus containers back if he wants. When the waste shipments are not enough to realize flow balance, the carrier should reposition surplus containers.

The condition in Case 4 (Case 5 is its symmetric case) implies $c_4-\frac{a_4}{b_4}\leq T_0$ and $T_1<c_6$, that is, the loss of shipping waste is less than the balancing cost T_0, so the carrier should ship waste. And balancing cost T_1 is less than c_6, so the carrier should not ship empty containers.

The condition in Case 6 (Case 7 is its symmetric case) implies $c_4-\frac{a_4}{b_4}\leq c_6$ and $c_6\leq T_1$. The ECR cost is less than the balancing cost T_1, so apart from shipping waste, the carrier should also reposition empty containers

Based on the above analysis, the carrier's pricing strategy can be summarized in the following proposition:

Proposition 2. *In case of monopoly,*

(1) If $max(-(c_3 - \frac{a_3}{b_3}), -c_5) < T_0 < min(c_4 - \frac{a_4}{b_4}, c_6)$*, the carrier should seek the balance of goods;*

(2) If $c_6 \leq T_0$ *and* $c_6 < c_4 - \frac{a_4}{b_4}$ *(or* $c_5 \leq -T_0$ *and* $c_5 < c_3 - \frac{a_3}{b_3}$*), the carrier should reposition empty containers to achieve flow balance;*

(3) If $T_1 \leq \frac{a_1}{b_1} - c_1$ *and* $c_4 - \frac{a_4}{b_4} \leq T_1 < c_6$ *(or* $T_2 \leq \frac{a_2}{b_2} - c_2$ *and* $c_3 - \frac{a_3}{b_3} \leq T_2 < c_5$*), the carrier should ship waste to achieve flow balance;*

(4) If $c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1)$ *(or* $c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2)$*), the carrier should ship waste and reposition empty containers to achieve flow balance.*

Next, we will analyze how the profit of a carrier is affected by different parameters.

The fundamental reason for ECR problem is the trade imbalance, so we first investigate how potential demand imbalance of normal goods will affect the carrier's profit. We fix the sum of potential demand of shipment 1 and 2, which equals to 100,000 TEUs and draw the figure of carrier's profit varying on potential imbalance, defined as $(a_1 - a_2)$. The value of other parameters are shown in the figure and the difference between Figure 1(a) and Figure 1(b) is the value of shipment 1's price sensitivity.

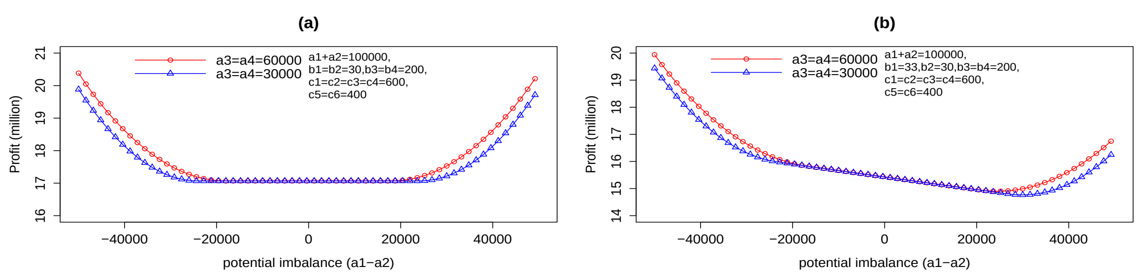


Figure 1: Profit varying on potential imbalance

The upper line in the Figure 1(a) shows that as the potential imbalance goes from -50,000 to 50,000, the carrier's profit first decreases, then remains unchanged, then increases. When the potential demand of goods in direction AB is significantly smaller than that in the opposite direction, the carrier first chooses to ship waste and reposition empty containers to realize the flow balance. Then, as the imbalance situation lessens, waste shipment is enough to realize flow balance. And when the imbalance lies in $[-18000, 18000]$, the optimal strategy for the carrier is to seek the balance of goods. When $a_3 = a_4 = 30,000$, shipping empty containers back is always a cost saving way to achieve flow balance compared to shipping waste, so the carrier either seeks the balance of goods or reposition empty containers to achieve flow balance. An interesting thing is that the demand imbalance will not damage the carrier's profit. On the contrary, when the potential imbalance is large enough, the carrier will make more profit as the imbalance situation intensifies.

Proposition 3. *In case of monopoly,*

(1) The profit of the carrier always decreases in the unit cost of shipping goods c_1 *and* c_2*;*

(2) The profit of the carrier decreases in the unit cost of shipping waste c_3 *(*c_4*) if parameters satisfy conditions in Case 5 and 7 (Case 4 and 6) and remains unchanged in other cases;*

(3) The profit of the carrier decreases in the unit ECR cost c_5 *(*c_6*) if parameters satisfy conditions in Case 3 and 7 (Case 2 and 6) and remains unchanged in other cases.*

It is intuitive to get Proposition 3 as the profit is decreasing in the cost. And under some conditions, the carrier will not ship waste or empty containers, so the profit may not be influenced by the cost of shipping waste or the cost of shipping empty containers. But when it goes to the duopoly model, results can be quite different (see Proposition 8).

Proposition 4. *In case of monopoly,*

(1) The profit of the carrier always decreases in the price sensitivity of goods b_1 *and* b_2*;*

(2) The profit of the carrier decreases in the price sensitivity of waste b_3 *(*b_4*) if parameters satisfy conditions in Case 5 and 7 (Case 4 and 6) and remains unchanged in other cases.*

The price sensitivity measures how the demand is affected by the price and a higher price sensitivity means that the carrier will lose more customers if he increases the price by the same amount. Therefore, the carrier can not capture more revenue from the shipper at a higher price sensitivity.

Now we will compare our pricing strategy with two traditional approaches. In current shipping industry, laden container transportation and ECR are usually considered separately, and the ECR decisions are made after the laden container transportation is planned. Then the decision maker for the laden containers will ignore shipping waste since it is always losing money. After the demand of goods is realized, surplus containers will be repositioned from a surplus location to a deficit location. Based on this idea, the optimization problem is: $max\ \Pi(p_1, p_2) = d_1(p_1 - c_1) + d_2(p_2 - c_2)$. Under this policy, the carrier essentially treats goods shipment in two directions independently. And the optimal solution is easy to find: $p_i^* = \frac{a_i/b_i+c_i}{2}, i = 1, 2$. The corresponding profit after deducting the repositioning cost is: $\Pi^* = \sum_{i=1}^{2} \frac{(a_i-b_ic_i)^2}{4b_i} - \frac{b_1+b_2}{2}(c_6T_0^+ + c_5T_0^-)$.

The second one is discussed by Zhou and Lee (2009). In their model, the carrier considers ECR cost when making decisions on laden containers, but the carrier only ships goods. Therefore, the objective function for the carrier is: $max\ \Pi(p_1, p_2) = d_1(p_1 - c_1) + d_2(p_2 - c_2) - c_5(d_2 - d_1)^+ - c_6(d_1 - d_2)^+$, and the optimal pricing is:

$$(p_1^*, p_2^*) = \begin{cases} (\frac{a_1/b_1+c_1+T_0}{2}, \frac{a_2/b_2+c_2-T_0}{2}) & \text{if } -c_5 < T_0 < c_6 \\ (\frac{a_1/b_1+c_1+c_6}{2}, \frac{a_2/b_2+c_2-c_6}{2}) & \text{if } T_0 \geq c_6 \\ (\frac{a_1/b_1+c_1-c_5}{2}, \frac{a_2/b_2+c_2+c_5}{2}) & \text{if } T_0 \leq -c_5 \end{cases}$$

Let Δ_1 and Δ_2 denote the differences in profit of our approach and two traditional approaches respectively, then we have the following results:

Table 3: Differences in profit of our approach and two traditional approaches

Cases	Δ_1	Δ_2
Case 1	$\frac{b_1+b_2}{4}[(2c_6 - T_0^+)T_0^+ + (2c_5 - T_0^-)T_0^-]$	0
Case 2	$\frac{(b_1+b_2)c_6^2}{4}$	0
Case 3	$\frac{(b_1+b_2)c_5^2}{4}$	0
Case 4	$\frac{(a_4-b_4(c_4-T_1))^2}{4b_4} + \frac{(b_1+b_2)(T_1^2+2T_0(c_6-T_1))}{4}$	if $T_0 < c_6$: $\frac{(a_4-b_4(c_4-T_1))^2}{4b_4} + \frac{(b_1+b_2)(T_0-T_1)^2}{4}$ if $T_0 \geq c_6$: $\frac{(a_4-b_4(c_4-T_1))^2}{4b_4} + \frac{(b_1+b_2)(c_6-T_1)(2T_0-T_1-c_6)}{4}$
Case 5	$\frac{(a_3-b_3(c_3-T_2))^2}{4b_3} + \frac{(b_1+b_2)(T_2^2-2T_0(c_5-T_2))}{4}$	if $-T_0 < c_5$: $\frac{(a_3-b_3(c_3-T_2))^2}{4b_3} + \frac{(b_1+b_2)(-T_0-T_2)^2}{4}$ if $-T_0 \geq c_5$: $\frac{(a_3-b_3(c_3-T_2))^2}{4b_3} + \frac{(b_1+b_2)(c_5-T_2)(-2T_0-T_2-c_5)}{4}$
Case 6	$\frac{(a_4-b_4(c_4-c_6))^2}{4b_4} + \frac{(b_1+b_2)c_6^2}{4}$	$\frac{(a_4-b_4(c_4-c_6))^2}{4b_4}$
Case 7	$\frac{(a_3-b_3(c_3-c_5))^2}{4b_3} + \frac{(b_1+b_2)c_5^2}{4}$	$\frac{(a_3-b_3(c_3-c_5))^2}{4b_3}$

Note that $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ for all cases. Compared with the first one, our approach always generates more profit(unless $a_1 - b_1c_1 = a_2 - b_2c_2$, then $\Delta_1 = 0$). Rather than treating customers in two directions equally, we charge less to the customers who lessen the imbalance situation, and charge more to the customers who worsen the imbalance situation. The cost saving on repositioning empty containers is more than the decrease in profit of shipping goods. This emphasizes the importance of considering ECR problem when pricing shipping service.

Compared with Zhou and Lee's, our approach also has better results. When the parameters satisfy first three conditions, both pricing schemes do not transport waste shipments, so the profits are the same. But in the last four cases, shipping waste is a cost-saving strategy compared to repositioning empty containers, so our approach generates more profit. Even though shipping waste alone is always losing money, it should be considered as an alternative option for achieving flow balance sometimes.

5 Pricing strategy in a duopoly market

The first model only considers the market with one carrier or without competition. Now we consider the situation where two carriers choose prices simultaneously and independently to maximize their own profit. Here we only consider the symmetric carriers, so we assume $a_i^1 = a_i^2 = a_i, b_i^1 = b_i^2 = b_i, \gamma_i^1 = \gamma_i^2 = \gamma_i, c_i^1 = c_i^2 = c_i, c_5^1 = c_5^2 = c_5, c_6^1 = c_6^2 = c_6$. Having symmetric carriers facilitates our analysis while still allowing us to obtain interesting insights relating to the presence of competition. Then the optimization problem for carrier $j(=1,2)$ is:

$$max\ \Pi^j(p_1^j, p_2^j, p_3^j, p_4^j) = \sum_{i=1}^{4} d_i^j(p_i^j - c_i) - (d_2^j + d_4^j - d_1^j - d_3^j)^+ c_5 - (d_1^j + d_3^j - d_2^j - d_4^j)^+ c_6 \quad (8)$$

subject to:

$$d_i^j = a_i - b_i p_i^j + \gamma_i(p_i^k - p_i^j), i = 1,2,3,4 \quad (9)$$

$$d_i^j, p_i^j \geq 0, i = 1,2,3,4 \quad (10)$$

Theorem 1. *There exists a unique symmetric pure strategy NE in this game.*

This theorem means that we can make a unique prediction of carriers' pricing decisions and no carrier has the incentive to deviate from this NE. Besides, the price decisions of two carriers are the same, which are given in the next proposition.

Proposition 5. *In case of duopoly with symmetric carriers, the NE prices are as following:*
$(p_1^{j*}, p_2^{j*}, p_3^{j*}, p_4^{j*}) =$

$$\begin{cases} (\frac{a_1/(b_1+\gamma_1)+c_1+T_0^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-T_0^D}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ max(-(c_3 - \frac{a_3}{b_3}), -c_5) < T_0^D < min(c_4 - \frac{a_4}{b_4}, c_6) \\ (\frac{a_1/(b_1+\gamma_1)+c_1+c_6}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-c_6}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ c_6 \leq T_0^D, c_6 < c_4 - \frac{a_4}{b_4} \\ (\frac{a_1/(b_1+\gamma_1)+c_1-c_5}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+c_5}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}) & if\ c_5 \leq -T_0^D, c_5 < c_3 - \frac{a_3}{b_3} \\ (\frac{a_1/(b_1+\gamma_1)+c_1+T_1^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-T_1^D}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4/(b_4+\gamma_4)+c_4-T_1^D}{2-\rho_4}) & if\ T_1^D \leq \frac{a_1}{b_1} - c_1, c_4 - \frac{a_4}{b_4} \leq T_1^D < c_6 \\ (\frac{a_1/(b_1+\gamma_1)+c_1-T_2^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+T_2^D}{2-\rho_2}, \frac{a_3/(b_3+\gamma_3)+c_3-T_2^D}{2-\rho_3}, \frac{a_4}{b_4}) & if\ T_2^D \leq \frac{a_2}{b_2} - c_2, c_3 - \frac{a_3}{b_3} \leq T_2^D < c_5 \\ (\frac{a_1/(b_1+\gamma_1)+c_1+c_6}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-c_6}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4/(b_4+\gamma_4)+c_4-c_6}{2-\rho_4}) & if\ c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1^D) \\ (\frac{a_1/(b_1+\gamma_1)+c_1-c_5}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+c_5}{2-\rho_2}, \frac{a_3/(b_3+\gamma_3)+c_3-c_5}{2-\rho_3}, \frac{a_4}{b_4}) & if\ c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2^D) \end{cases}$$

where $T_0^D = \frac{\frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_2-b_2c_2}{2-\rho_2}}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2}}$, $T_1^D = \frac{\frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_2-b_2c_2}{2-\rho_2} - \frac{a_4-b_4c_4}{2-\rho_4}}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_4}{2-\rho_4}}$, $T_2^D = \frac{\frac{a_2-b_2c_2}{2-\rho_2} - \frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_3-b_3c_3}{2-\rho_3}}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3}}$, $\rho_i = \frac{\gamma_i}{b_i+\gamma_i}$. *And the corresponding demand and profit are summarized in Table 4.*

By conducting a similar analysis used in the monopoly model, each carrier's pricing strategy can be summarized in the following proposition:

Proposition 6. *In case of duopoly with symmetric carriers,* *(1) If $max(-(c_3 - \frac{a_3}{b_3}), -c_5) < T_0^D < min(c_4 - \frac{a_4}{b_4}, c_6)$, each carrier should seek the balance of goods;*

(2) If $c_6 \leq T_0^D$ and $c_6 < c_4 - \frac{a_4}{b_4}$ (or $c_5 \leq -T_0^D$ and $c_5 < c_3 - \frac{a_3}{b_3}$), each carrier should reposition empty containers to achieve flow balance;

(3) If $T_1^D \leq \frac{a_1}{b_1} - c_1$ and $c_4 - \frac{a_4}{b_4} \leq T_1^D < c_6$ (or $T_2^D \leq \frac{a_2}{b_2} - c_2$ and $c_3 - \frac{a_3}{b_3} \leq T_2^D < c_5$), each carrier should ship waste to achieve flow balance;

(4) If $c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1^D)$ (or $c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2^D)$), each carrier should ship waste and reposition empty containers to achieve flow balance.

Compared to Proposition 2, the only difference is that T_i changes into T_i^D. And if $\rho_1 = \rho_2 = \rho_3 = \rho_4$, we have T_i=T_i^D, which means the conditions under which the carrier should seek the balance of goods or use empty containers and waste shipments to achieve flow balance for monopoly case and duopoly case are the same, although the prices and realized demands are different, so are the profits.

Now we will compare the results of duopoly model with those of monopoly model. If the competition intensity $\gamma_i (= 1, 2, 3, 4)$ equals to zero, two carriers can be treated as providing service in two separate markets, but only having half of the potential demand. Then compared with the case of monopoly, each carrier sets the same price but the demand is halved, so the profit is also halved. And the total profit of two carriers are the same as that of the monopolist. When the competition exists, carrier will cut his own price to attract demand from the competitor. Although each carrier's demand is more than half of the monopolist's, the total profit of two carriers is decreasing. That is how strategic alliance will benefit carriers apart from reducing cost via economies of scale and increasing the utilization of space. The impact of competition intensity will be further investigated in the next proposition.

Proposition 7. *In case of duopoly with symmetric carriers,*

(1) The profit of each carrier always decreases in the competition intensity of goods γ_1 and γ_2;

(2) The profit of each carrier decreases in the competition intensity of waste γ_3 (γ_4) if parameters satisfy conditions in Case 5 and 7 (Case 4 and 6) and remains unchanged in other cases.

As γ_i increases, carrier has more incentives to lower his own price so that he can attract more demand from his rival. But the increase in demand can not compensate for the decrease in unit profit. Finally, the price war becomes a lose-lose game. And the profit will not be affected by the competition intensity γ_3 (γ_4) sometimes, because carrier will not transport shipment 3 (4) at all under some circumstances.

Table 4: Results of duopoly model

Condition	$(p_1^{j*}, p_2^{j*}, p_3^{j*}, p_4^{j*})$ $(d_1^{j*}, d_2^{j*}, d_3^{j*}, d_4^{j*})$ (d_5^{j*}, d_6^{j*})	Π^{j*}
Case 1: $T_0^D < min(c_4 - \frac{a_4}{b_4}, c_6)$ $-T_0^D < min(c_3 - \frac{a_3}{b_3}, c_5)$	$(\frac{a_1/(b_1+\gamma_1)+c_1+T_0^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-T_0^D}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1+T_0^D)}{2-\rho_1}, \frac{a_2-b_2(c_2-T_0^D)}{2-\rho_2}, 0, 0)$ $(0,0)$	$\frac{(a_1-b_1(c_1+T_0^D))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2-T_0^D))^2}{(2-\rho_2)^2(b_2+\gamma_2)}$
Case 2: $c_6 \leq T_0^D$, $c_6 < c_4 - \frac{a_4}{b_4}$	$(\frac{a_1/(b_1+\gamma_1)+c_1+c_6}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-c_6}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1+c_6)}{2-\rho_1}, \frac{a_2-b_2(c_2-c_6)}{2-\rho_2}, 0, 0)$ $(0, (\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2})(T_0^D - c_6))$	$\frac{(a_1-b_1(c_1+c_6))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2-c_6))^2}{(2-\rho_2)^2(b_2+\gamma_2)}$
Case 3: $c_5 \leq -T_0^D, c_5 < c_3 - \frac{a_3}{b_3}$	$(\frac{a_1/(b_1+\gamma_1)+c_1-c_5}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+c_5}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-c_5)}{2-\rho_1}, \frac{a_2-b_2(c_2+c_5)}{2-\rho_2}, 0, 0)$ $((\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2})(-T_0^D - c_5), 0)$	$\frac{(a_1-b_1(c_1-c_5))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2+c_5))^2}{(2-\rho_2)^2(b_2+\gamma_2)}$
Case 4: $T_1^D \leq \frac{a_1}{b_1} - c_1$ $c_4 - \frac{a_4}{b_4} \leq T_1^D < c_6$	$(\frac{a_1/(b_1+\gamma_1)+c_1+T_1^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-T_1^D}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4/(b_4+\gamma_4)+c_4-T_1^D}{2-\rho_4})$ $(\frac{a_1-b_1(c_1+T_1^D)}{2-\rho_1}, \frac{a_2-b_2(c_2-T_1^D)}{2-\rho_2}, 0, \frac{a_4-b_4(c_4-T_1^D)}{2-\rho_4})$ $(0,0)$	$\frac{(a_1-b_1(c_1+T_1^D))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2-T_1^D))^2}{(2-\rho_2)^2(b_2+\gamma_2)} + \frac{(a_4-b_4(c_4-T_1^D))^2}{(2-\rho_4)^2(b_4+\gamma_4)}$
Case 5: $T_2^D \leq \frac{a_2}{b_2} - c_2$ $c_3 - \frac{a_3}{b_3} \leq T_2^D < c_5$	$(\frac{a_1/(b_1+\gamma_1)+c_1-T_2^D}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+T_2^D}{2-\rho_2}, \frac{a_3/(b_3+\gamma_3)+c_3-T_2^D}{2-\rho_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-T_2^D)}{2-\rho_1}, \frac{a_2-b_2(c_2+T_2^D)}{2-\rho_2}, \frac{a_3-b_3(c_3-T_2^D)}{2-\rho_3}, 0)$ $(0,0)$	$\frac{(a_1-b_1(c_1-T_2^D))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2+T_2^D))^2}{(2-\rho_2)^2(b_2+\gamma_2)} + \frac{(a_3-b_3(c_3-T_2^D))^2}{(2-\rho_3)^2(b_3+\gamma_3)}$
Case 6: $c_4 - \frac{a_4}{b_4} \leq c_6 \leq min(\frac{a_1}{b_1} - c_1, T_1^D)$	$(\frac{a_1/(b_1+\gamma_1)+c_1+c_6}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2-c_6}{2-\rho_2}, \frac{a_3}{b_3}, \frac{a_4/(b_4+\gamma_4)+c_4-c_6}{2-\rho_4})$ $(\frac{a_1-b_1(c_1+c_6)}{2-\rho_1}, \frac{a_2-b_2(c_2-c_6)}{2-\rho_2}, 0, \frac{a_4-b_4(c_4-c_6)}{2-\rho_4})$ $(0, (\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_4}{2-\rho_4})(T_1^D - c_6))$	$\frac{(a_1-b_1(c_1+c_6))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2-c_6))^2}{(2-\rho_2)^2(b_2+\gamma_2)} + \frac{(a_4-b_4(c_4-c_6))^2}{(2-\rho_4)^2(b_4+\gamma_4)}$
Case 7: $c_3 - \frac{a_3}{b_3} \leq c_5 \leq min(\frac{a_2}{b_2} - c_2, T_2^D)$	$(\frac{a_1/(b_1+\gamma_1)+c_1-c_5}{2-\rho_1}, \frac{a_2/(b_2+\gamma_2)+c_2+c_5}{2-\rho_2}, \frac{a_3/(b_3+\gamma_3)+c_3-c_5}{2-\rho_3}, \frac{a_4}{b_4})$ $(\frac{a_1-b_1(c_1-c_5)}{2-\rho_1}, \frac{a_2-b_2(c_2+c_5)}{2-\rho_2}, \frac{a_3-b_3(c_3-c_5)}{2-\rho_3}, 0)$ $((\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3})(T_2^D - c_5), 0)$	$\frac{(a_1-b_1(c_1-c_5))^2}{(2-\rho_1)^2(b_1+\gamma_1)} + \frac{(a_2-b_2(c_2+c_5))^2}{(2-\rho_2)^2(b_2+\gamma_2)} + \frac{(a_3-b_3(c_3-c_5))^2}{(2-\rho_3)^2(b_3+\gamma_3)}$

Proposition 8. *In case of duopoly with symmetric carriers,*

(1) If parameters satisfy conditions in Case 3, the profit of each carrier is decreasing (unchanging, increasing) in the ECR cost c_5 *if* $c_5 < (=, >) \frac{\frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)} - \frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)}}$ *;*

(2) If parameters satisfy conditions in Case 2, the profit of each carrier is decreasing (unchanging, increasing) in the ECR cost c_6 *if* $c_6 < (=, >) \frac{\frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)} - \frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)}}$*;*

(3) If parameters satisfy conditions in Case 7, the profit of each carrier is decreasing (unchanging, increasing) in the ECR cost c_5 *if* $c_5 < (=, >) \frac{\frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)} - \frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)} - \frac{a_3-b_3c_3}{(2-\rho_3)^2/(1-\rho_3)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)} + \frac{b_3}{(2-\rho_3)^2/(1-\rho_3)}}$*;*

(4) If parameters satisfy conditions in Case 6, the profit of each carrier is decreasing (unchanging, increasing) in the ECR cost c_6 *if* $c_6 < (=, >) \frac{\frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)} - \frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)} - \frac{a_4-b_4c_4}{(2-\rho_4)^2/(1-\rho_4)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)} + \frac{b_4}{(2-\rho_4)^2/(1-\rho_4)}}$*;*

(5) If parameters satisfy conditions in Case 1, 4 and 5, the profit of each carrier remains unchanged in the ECR cost c_5 *and* c_6*.*

It is unusual to see that the profit may increase in the ECR cost sometimes. As the ECR cost increases, the carrier will try to reduce the trade imbalance. Under Case 3, the increase in c_5 leads to: (1) decrease in equilibrium price and increase in equilibrium demand of shipment 1; (2) increase in equilibrium price and decrease in equilibrium demand of shipment 2; (3) decrease in the number of empty containers to be shipped from A to B. The final effect on the profit depends on whether the positive effect or the negative effect will dominate. And only when c_5 increases to $\frac{\frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)} - \frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)}}$, the positive effect will outweigh the negative effect; otherwise, the profit is decreasing in the c_5 (see Figure 2(a)). Note that $c_5 \leq -T_0^D$, and $\frac{\frac{a_2-b_2c_2}{(2-\rho_2)^2/(1-\rho_2)} - \frac{a_1-b_1c_1}{(2-\rho_1)^2/(1-\rho_1)}}{\frac{b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{b_2}{(2-\rho_2)^2/(1-\rho_2)}} < -T_0^D$ can be possible only when $\rho_1 < \rho_2$. Therefore, the increase in ECR cost leading to the increase in the profit will not happen in the monopoly market ($\rho_1 = \rho_2 = 0$). The effect of c_5 on the profit under Case 7 can be analyzed in a similar way and the numerical result is in Figure 2(b).

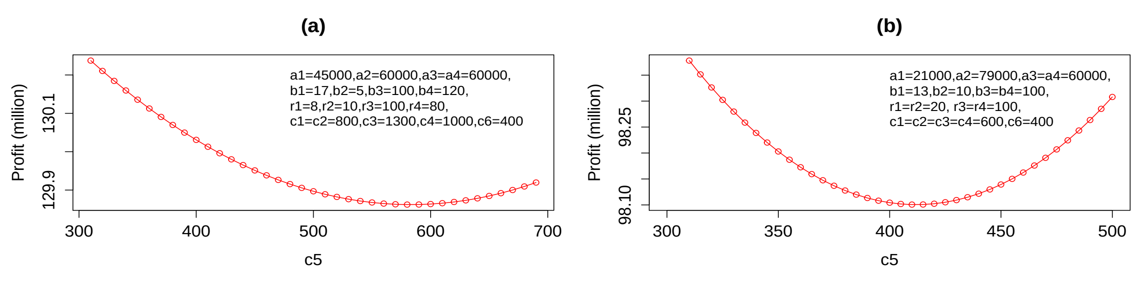


Figure 2: Profit varying on ECR cost c_5

Waste shipment 3 plays a similar role under Case 5, so the profit may increase in the unit cost of shipment 3 sometimes (see Figure 3).

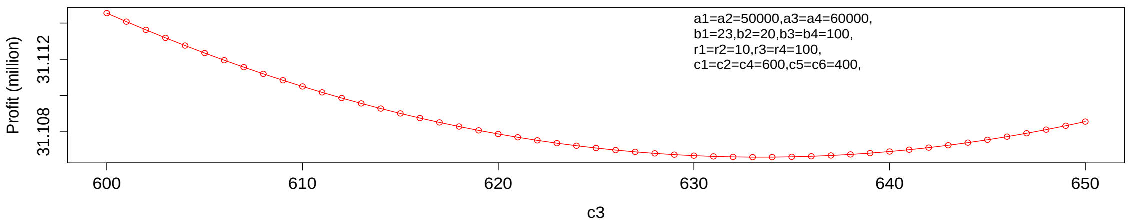


Figure 3: Profit varying on c_3

Since the profit may increase in the ECR cost and waste shipment cost, how about pricing as if the carrier has higher cost? As a result, the carrier will be worse off if he unilaterally deviates from the NE prices. This can be seen from the definition of NE. Only when two carriers act in the same way, this strategy will generate more profits.

6 Conclusions

In spite of much research on ECR, little has considered shipping waste and scrap, which is an environmentally-friendly and cost-effective alternative approach to achieve flow balance. In this paper, we study optimal pricing strategies for carriers in a monopoly and duopoly market with waste shipment and ECR. Depending on different market conditions and cost structure, the carrier's strategy falls into four groups: seeking the balance of goods; repositioning empty containers to achieve flow balance; shipping waste to achieve flow balance; shipping waste and repositioning empty containers to achieve flow balance. Compared with work without waste shipment, our model generates better results. We also investigate how the profit is affected by trade imbalance, price sensitivity, cost structures and competition intensity.

One limitation of our work is that we only consider a two-port closed system, which is a simplification of complicated shipping network in the real life. One may study a multi-port system with routing decision, but the analysis can he far more challenging. Besides, we use a deterministic linear model to describe the demand function which fails to capture the uncertainty in the shipping market. Some stochastic models like additive model, multiplicative model and logit model can be used in the further research.

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Appendix

A.1 Proof of proposition 1

Define two new problems:

$$\overline{P^M}: \qquad max\ \Pi(p_1,p_2,p_3,p_4)=\sum_{i=1}^{4} d_i(p_i-c_i)-(d_2+d_4-d_1-d_3)c_5$$

subject to:

$$d_2+d_4-d_1-d_3\geq 0$$
$$d_i=a_i-b_ip_i, i=1,2,3,4$$
$$d_i,p_i\geq 0, i=1,2,3,4$$

$$\underline{P^M}: \qquad max\ \Pi(p_1,p_2,p_3,p_4)=\sum_{i=1}^{4} d_i(p_i-c_i)-(d_1+d_3-d_2-d_4)c_6$$

subject to:

$$d_1+d_3-d_2-d_4\geq 0$$
$$d_i=a_i-b_ip_i, i=1,2,3,4$$
$$d_i,p_i\geq 0, i=1,2,3,4$$

Let $\mathbf{p}^*$,$\overline{\mathbf{p}}$ and $\underline{\mathbf{p}}$ denote the optimal solutions of original problem denoted by P^M and two new problems $\overline{P^M}$ $\underline{P^M}$, respectively. And the corresponding demand and optimal value of these three problems are $\mathbf{d}^*$, $\overline{\mathbf{d}}$, $\underline{\mathbf{d}}$ and Π^*, $\overline{\Pi}$, $\underline{\Pi}$.Then we must have $\Pi^*=max(\overline{\Pi},\underline{\Pi})$. (Since $\overline{\mathbf{p}}$ is feasible to P^M, we have $\Pi^*\geq\sum_{i=1}^4\overline{d_i}(\overline{p_i}-c_i)-(\overline{d_2}+\overline{d_4}-\overline{d_1}-\overline{d_3})^+c_5-(\overline{d_1}+\overline{d_3}-\overline{d_2}-\overline{d_4})^+c_6=\sum_{i=1}^4\overline{d_i}(\overline{p_i}-c_i)-(\overline{d_2}+\overline{d_4}-\overline{d_1}-\overline{d_3})c_5=\overline{\Pi}$. Similarly, we have $\Pi^*\geq\underline{\Pi}$, then $\Pi^*\geq max(\overline{\Pi},\underline{\Pi})$. Besides, if $d_2^*+d_4^*-d_1^*-d_3^*\geq 0$, then $\mathbf{p}^*$ is feasible to $\overline{P^M}$, so we have $\overline{\Pi}\geq\sum_{i=1}^4 d_i^*(p_i^*-c_i)-(d_2^*+d_4^*-d_1^*-d_3^*)c_5=\Pi^*$. And if $d_2^*+d_4^*-d_1^*-d_3^*<0$, then $\mathbf{p}^*$ is feasible to $\underline{P^M}$, so we have $\underline{\Pi}\geq\sum_{i=1}^4 d_i^*(p_i^*-c_i)-(d_1^*+d_3^*-d_2^*-d_4^*)c_6=\Pi^*$. Therefore, $\Pi^*\leq max(\overline{\Pi},\underline{\Pi})$.)

$\overline{P^M}$ is a convex programming problem, and we can use Karush-Kuhn-Tucker conditions to solve. The optimal pricing for $\overline{P^M}$ is given by:

$$(\overline{p_1},\overline{p_2},\overline{p_3},\overline{p_4})=\begin{cases}(\frac{a_1/b_1+c_1+T_0}{2},\frac{a_2/b_2+c_2-T_0}{2},\frac{a_3}{b_3},\frac{a_4}{b_4}) & \text{if } max(-c_5,\frac{a_3}{b_3}-c_3))<T_0<c_4-\frac{a_4}{b_4}\\ (\frac{a_1/b_1+c_1-c_5}{2},\frac{a_2/b_2+c_2+c_5}{2},\frac{a_3}{b_3},\frac{a_4}{b_4}) & \text{if } c_5\leq -T_0 \text{ and } c_5<c_3-\frac{a_3}{b_3}\\ (\frac{a_1/b_1+c_1+T_1}{2},\frac{a_2/b_2+c_2-T_1}{2},\frac{a_3}{b_3},\frac{a_4/b_4+c_4-T_1}{2}) & \text{if } c_4-\frac{a_4}{b_4}\leq T_1\leq\frac{a_1}{b_1}-c_1\\ (\frac{a_1/b_1+c_1-T_2}{2},\frac{a_2/b_2+c_2+T_2}{2},\frac{a_3/b_3+c_3-T_2}{2},\frac{a_4}{b_4}) & \text{if } T_2<c_5 \text{ and } c_3-\frac{a_3}{b_3}\leq T_2\leq\frac{a_2}{b_2}-c_2\\ (\frac{a_1/b_1+c_1-c_5}{2},\frac{a_2/b_2+c_2+c_5}{2},\frac{a_3/b_3+c_3-c_5}{2},\frac{a_4}{b_4}) & \text{if } c_3-\frac{a_3}{b_3}\leq c_5\leq min(\frac{a_2}{b_2}-c_2,T_2)\end{cases}$$

$\underline{P^M}$ is quite similar to $\overline{P^M}$ and the optimal pricing can be obtained in the same way. Then we only need to compare the profits in these two problems and find the optimal solution for the original problem P^M. The comparing process is easy but tedious for writing, so we omit this part.

Then we can calculate the corresponding demand and profit by $d_i^* = a_i - b_i p_i^*$ and $\Pi^* = \sum_{i=1}^4 d_i^*(p_i^* - c_i) - (d_2^* + d_4^* - d_1^* - d_3^*)^+ c_5 - (d_1^* + d_3^* - d_2^* - d_4^*)^+ c_6$, which is summarized in Table 2. □

A.2 Proof of proposition 2

This proposition follows the results in Table 2. □

A.3 Proof of proposition 3

We only analyze the impact of c_1, c_3 and c_5, and the impact of c_2, c_4 and c_6 can be analyzed in a similar way.

(1)

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{b_2}{b_1+b_2}\frac{a_1 - b_1(c_1+T_0)}{2} - \frac{b_1}{b_1+b_2}\frac{a_2 - b_2(c_2 - T_0)}{2} = -d_1^* < 0 \quad (Case\ 1)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{a_1 - b_1(c_1+c_6)}{2} = -d_1^* < 0 \quad (Case\ 2)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{a_1 - b_1(c_1-c_5)}{2} = -d_1^* < 0 \quad (Case\ 3)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{b_2+b_4}{b_1+b_2+b_4}\frac{a_1 - b_1(c_1+T_1)}{2} - \frac{b_1}{b_1+b_2+b_4}\frac{a_2 - b_2(c_2-T_1)}{2} - \frac{b_1}{b_1+b_2+b_4}\frac{a_4 - b_4(c_4-T_1)}{2}$$
$$= -d_1^* < 0 \quad (Case\ 4)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{b_2+b_3}{b_1+b_2+b_3}\frac{a_1 - b_1(c_1-T_2)}{2} - \frac{b_1}{b_1+b_2+b_3}\frac{a_2 - b_2(c_2+T_2)}{2} + \frac{b_1}{b_1+b_2+b_4}\frac{a_3 - b_3(c_3-T_2)}{2}$$
$$= -d_1^* < 0 \quad (Case\ 5)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{a_1 - b_1(c_1+c_6)}{2} = -d_1^* < 0 \quad (Case\ 6)$$

$$\frac{\partial \Pi^*}{\partial c_1} = -\frac{a_1 - b_1(c_1-c_5)}{2} = -d_1^* < 0 \quad (Case\ 7)$$

(2)

$$\frac{\partial \Pi^*}{\partial c_3} = \frac{b_3}{b_1+b_2+b_3}\frac{a_1 - b_1(c_1-T_2)}{2} - \frac{b_3}{b_1+b_2+b_3}\frac{a_2 - b_2(c_2+T_2)}{2} - \frac{b_1+b_2}{b_1+b_2+b_4}\frac{a_3 - b_3(c_3-T_2)}{2}$$
$$= -d_3^* < 0 \quad (Case\ 5)$$

$$\frac{\partial \Pi^*}{\partial c_3} = -\frac{a_3 - b_3(c_3-c_5)}{2} = -d_3^* < 0 \quad (Case\ 7)$$

$$\frac{\partial \Pi^*}{\partial c_3} = 0 \quad (Case\ 1\ 2\ 3\ 4\ and\ 6)$$

(3)

$$\frac{\partial \Pi^*}{\partial c_5} = \frac{a_1 - b_1(c_1 - c_5)}{2} - \frac{a_2 - b_2(c_2 + c_5)}{2} = -d_5^* < 0 \quad (Case\ 3)$$
$$\frac{\partial \Pi^*}{\partial c_5} = \frac{a_1 - b_1(c_1 - c_5)}{2} - \frac{a_2 - b_2(c_2 + c_5)}{2} + \frac{a_3 - b_3(c_3 - c_5)}{2} = -d_5^* < 0 \quad (Case\ 7)$$
$$\frac{\partial \Pi^*}{\partial c_5} = 0 \quad (Case\ 1\ 2\ 4\ 5\ and\ 6)$$

□

A.4 Proof of proposition 4

We only analyze the impact of b_1 and b_3, and the impact of b_2 and b_4 can be analyzed in a similar way.

(1)

$$\begin{aligned}\frac{\partial \Pi^*}{\partial b_1} &= \frac{(a_1 - b_1(c_1 + T_0))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 + T_0)}{2} + \frac{c_1 + T_0}{b_1 + b_2}(\frac{a_1 - b_1(c_1 + T_0)}{2} - \frac{a_2 - b_2(c_2 - T_0)}{2}) \\ &= \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 1)\end{aligned}$$
$$\frac{\partial \Pi^*}{\partial b_1} = \frac{(a_1 - b_1(c_1 + c_6))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 + c_6)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 2)$$
$$\frac{\partial \Pi^*}{\partial b_1} = \frac{(a_1 - b_1(c_1 - c_5))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 - c_5)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 3)$$
$$\begin{aligned}\frac{\partial \Pi^*}{\partial b_1} &= \frac{c_1 + T_1}{b_1 + b_2 + b_4}(\frac{a_1 - b_1(c_1 + T_1)}{2} - \frac{a_2 - b_2(c_2 - T_1)}{2} - \frac{a_4 - b_4(c_4 - T_1)}{2}) \\ &+ \frac{(a_1 - b_1(c_1 + T_1))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 + T_1)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 4)\end{aligned}$$
$$\begin{aligned}\frac{\partial \Pi^*}{\partial b_1} &= \frac{c_1 - T_2}{b_1 + b_2 + b_3}(\frac{a_1 - b_1(c_1 - T_2)}{2} - \frac{a_2 - b_2(c_2 + T_2)}{2} + \frac{a_3 - b_3(c_3 - T_2)}{2}) \\ &+ \frac{(a_1 - b_1(c_1 - T_2))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 - T_2)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 5)\end{aligned}$$
$$\frac{\partial \Pi^*}{\partial b_1} = \frac{(a_1 - b_1(c_1 + c_6))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 + c_6)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 6)$$
$$\frac{\partial \Pi^*}{\partial b_1} = \frac{(a_1 - b_1(c_1 - c_5))^2}{4b_1^2} - \frac{a_1}{b_1^2}\frac{a_1 - b_1(c_1 - c_5)}{2} = \frac{d_1^*(d_1^* - a_1)}{b_1^2} = -\frac{d_1^* p_1^*}{b_1} < 0 \quad (Case\ 7)$$

(2)

$$\begin{aligned}\frac{\partial \Pi^*}{\partial b_3} &= \frac{c_3 - T_2}{b_1 + b_2 + b_3}(\frac{a_1 - b_1(c_1 - T_2)}{2} - \frac{a_2 - b_2(c_2 + T_2)}{2} + \frac{a_3 - b_3(c_3 - T_2)}{2}) \\ &+ \frac{(a_3 - b_3(c_3 - T_2))^2}{4b_3^2} - \frac{a_3}{b_3^2}\frac{a_3 - b_3(c_3 - T_2)}{2} = \frac{d_3^*(d_3^* - a_3)}{b_3^2} = -\frac{d_3^* p_3^*}{b_3} < 0 \quad (Case\ 5)\end{aligned}$$
$$\frac{\partial \Pi^*}{\partial b_3} = \frac{(a_3 - b_3(c_3 - c_5))^2}{4b_3^2} - \frac{a_3}{b_3^2}\frac{a_3 - b_3(c_3 - c_5)}{2} = \frac{d_3^*(d_3^* - a_3)}{b_3^2} = -\frac{d_3^* p_3^*}{b_3} < 0 \quad (Case\ 7)$$
$$\frac{\partial \Pi^*}{\partial b_3} = 0 \quad (Case\ 1\ 2\ 3\ 4\ and\ \ 6)$$

□

A.5 Proof of theorem 1

The result is proven in three steps:

(1) Existence of NE

Debreu (1952), Glicksberg (1952) and Fan (1952) suggest that "Game $G = (S_i, \pi_i)_{i=1}^n$ has a pure strategy NE if the strategy space for each player S_i is convex and compact; the payoff function π_i is quasi_concave in s_i; the payoff function π_i is a continuous function in $s = (s_1, s_2, ..., s_n)$."

For carrier j, the decision variable p_i^j is in the interval $[0, p_{i\ max}]$. And $p_{i\ max} = a_i/b_i$, which solves $d_i^1(p_i^1, p_i^2) = 0$ and $d_i^2(p_i^1, p_i^2) = 0$, so the strategy space is convex and compact. The payoff function is concave since $\sum_{i=1}^4 d_i^j(p_i^j - c_i)$, $-(d_2^j + d_4^j - d_1^j - d_3^j)^+$ and $-(d_1^j + d_3^j - d_2^j - d_4^j)^+$ are concave in $(p_1^j, p_2^j, p_3^j, p_4^i)$. Π^j is a nonnegative weighted sum of three concave functions, which is also concave. And obviously, the payoff function is continuous. Therefore, we can conclude that there exists at least one pure strategy NE in our problem.

(2) Symmetric NE

Cachon and Netessine (2004) suggests that for a symmetric game satisfying the three conditions listed in the existence of NE argument mentioned above, there exists at least one symmetric pure strategy NE.

(3) Uniqueness of NE

Cachon and Netessine (2004) points out that "If the best response mapping is a contraction on the entire strategy space, there is a unique NE in the game."

Given carrier 2's price decision $(p_1^2, p_2^2, p_3^2, p_4^2)$, carrier 1's best response function can be derived from Proposition 1, which is:
$(p_1^1, p_2^1, p_3^1, p_4^1) = f_1(p_1^2, p_2^2, p_3^2, p_4^2) =$

$$\begin{cases}
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1+T_0^2}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2-T_0^2}{2}, \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}\right) & if\ max\left(\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} - c_3, -c_5\right) < T_0^2 < min\left(c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}, c_6\right) \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1+c_6}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2-c_6}{2}, \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}\right) & if\ c_6 \le T_0^2, c_6 < c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1-c_5}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2+c_5}{2}, \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}\right) & if\ c_5 \le -T_0^2, c_5 < c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1+T_1^2}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2-T_1^2}{2}, \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, \frac{\frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}+c_4-T_1^2}{2}\right) & if\ T_1^2 < c_6, c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} \le T_1^2 \le \frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} - c_1 \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1-T_2^2}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2+T_2^2}{2}, \frac{\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}+c_3-T_2^2}{2}, \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}\right) & if\ T_2^2 < c_5, c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} \le T_2^2 \le \frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} - c_2 \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1+c_6}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2-c_6}{2}, \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, \frac{\frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}+c_4-c_6}{2}\right) & if\ c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} \le c_6 \le min\left(\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} - c_1, T_1^2\right) \\
\left(\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1}+c_1-c_5}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2}+c_2+c_5}{2}, \frac{\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}+c_3-c_5}{2}, \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}\right) & if\ c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} \le c_5 \le min\left(\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} - c_2, T_2^2\right)
\end{cases}$$

where $T_0^2 = \frac{(a_1+\gamma_1 p_1^2-(b_1+\gamma_1)c_1)-(a_2+\gamma_2 p_2^2-(b_2+\gamma_2)c_2)}{b_1+\gamma_1+b_2+\gamma_2}$, $T_1^2 = \frac{(a_1+\gamma_1 p_1^2-(b_1+\gamma_1)c_1)-(a_2+\gamma_2 p_2^2-(b_2+\gamma_2)c_2)-(a_4+\gamma_4 p_4^2-(b_4+\gamma_4)c_4)}{b_1+\gamma_1+b_2+\gamma_2+b_4+\gamma_4}$, $T_2^2 = \frac{(a_2+\gamma_2 p_2^2-(b_2+\gamma_2)c_2)-(a_1+\gamma_1 p_1^2-(b_1+\gamma_1)c_1)-(a_3+\gamma_3 p_3^2-(b_3+\gamma_3)c_3)}{b_1+\gamma_1+b_2+\gamma_2+b_3+\gamma_3}$.

The best response function for carrier 1 is a piece-wise linear function. And carrier 2 also has such piece-wise linear function $(p_1^2, p_2^2, p_3^2, p_4^2) = f_2(p_1^1, p_2^1, p_3^1, p_4^1)$. Then the best response mapping can be written as:

$$f(p_1^1, p_2^1, p_3^1, p_4^1, p_1^2, p_2^2, p_3^2, p_4^2) = (f_1(p_1^2, p_2^2, p_3^2, p_4^2), f_2(p_1^1, p_2^1, p_3^1, p_4^1)) \tag{11}$$

Now we need to prove that this mapping is a contraction mapping.

Let $f(p_1^1, p_2^1, p_3^1, p_4^1, p_1^2, p_2^2, p_3^2, p_4^2) = (g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8)$ and define the following matrix H, where the entry h_{ij} is:

$$h_{ij} = \begin{cases} \frac{\partial g_i}{\partial p_j^1} & \text{if } j \leq 4 \\ \frac{\partial g_i}{\partial p_{j-4}^2} & \text{if } j > 4 \end{cases}$$

To show the mapping (11) is a contraction mapping, it suffices to show $\|H\| < 1$ for any one matrix norm. The best response mapping (11) is a piece-wise linear function with 49 segments, and we only show that $\|H\|_\infty < 1$ in one segment.

Let $T_2^1 = \frac{(a_2+\gamma_2 p_2^1-(b_2+\gamma_2)c_2)-(a_1+\gamma_1 p_1^1-(b_1+\gamma_1)c_1)-(a_3+\gamma_3 p_3^1-(b_3+\gamma_3)c_3)}{b_1+\gamma_1+b_2+\gamma_2+b_3+\gamma_3}$.

If $max(\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} - c_3, -c_5) < T_0^2 < min(c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}, c_6)$ and $c_3 - \frac{a_3+\gamma_3 p_3^1}{b_3+\gamma_3} \leq c_5 \leq min(\frac{a_2+\gamma_2 p_2^1}{b_2+\gamma_2} - c_2, T_2^1)$, then

$$(g_1, g_2, g_3, g_4) = (\frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 + T_0^2}{2}, \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 - T_0^2}{2}, \frac{a_3 + \gamma_3 p_3^2}{b_3 + \gamma_3}, \frac{a_4 + \gamma_4 p_4^2}{b_4 + \gamma_4})$$

$$(g_5, g_6, g_7, g_8) = (\frac{\frac{a_1+\gamma_1 p_1^1}{b_1+\gamma_1} + c_1 - c_5}{2}, \frac{\frac{a_2+\gamma_2 p_2^1}{b_2+\gamma_2} + c_2 + c_5}{2}, \frac{\frac{a_3+\gamma_3 p_3^1}{b_3+\gamma_3} + c_3 - c_5}{2}, \frac{a_4 + \gamma_4 p_4^1}{b_4 + \gamma_4})$$

Therefore,

$$\begin{aligned} \|H\|_\infty =& max(\frac{\gamma_1}{2(b_1+\gamma_1)} + \frac{\gamma_1+\gamma_2}{2(b_1+\gamma_1+b_2+\gamma_2)}, \frac{\gamma_2}{2(b_2+\gamma_2)} + \frac{\gamma_1+\gamma_2}{2(b_1+\gamma_1+b_2+\gamma_2)}, \\ & \frac{\gamma_3}{b_3+\gamma_3}, \frac{\gamma_4}{b_4+\gamma_4}, \frac{\gamma_1}{2(b_1+\gamma_1)}, \frac{\gamma_2}{2(b_2+\gamma_2)}, \frac{\gamma_3}{2(b_3+\gamma_3)}, \frac{\gamma_4}{b_4+\gamma_4}) < 1 \end{aligned}$$

And $\|H\|_\infty < 1$ in other segments can be verified in a similar way. Therefore, the uniqueness of NE can be guaranteed. □

A.6 Proof of proposition 5

According to the best response function calculated in the proof of Theorem 1, if
(1) $max(\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} - c_3, -c_5) < T_0^2 < min(c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}, c_6)$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 + T_0^2}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 - T_0^2}{2}, p_3^1 = \frac{a_3 + \gamma_3 p_3^2}{b_3 + \gamma_3}, p_4^1 = \frac{a_4 + \gamma_4 p_4^2}{b_4 + \gamma_4}.$$

Since $p_i^1 = p_i^2$ for $i = 1, 2, 3, 4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 + T_0^D}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 - T_0^D}{2-\rho_2}, p_3^j = \frac{a_3}{b_3}, p_4^j = \frac{a_4}{b_4}.$$

where $\rho_i = \frac{\gamma_i}{b_i+\gamma_i}, T_0^D = (\frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_2-b_2c_2}{2-\rho_2})/(\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2})$.
To satisfy the constraint on T_0^2, we have $max(\frac{a_3}{b_3} - c_3, -c_5) < T_0^D < min(c_4 - \frac{a_4}{b_4}, c_6)$.

(2) $c_6 \leq T_0^2, c_6 < c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 + c_6}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 - c_6}{2}, p_3^1 = \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, p_4^1 = \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}.$$

Since $p_i^1 = p_i^2$ for $i = 1, 2, 3, 4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 + c_6}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 - c_6}{2-\rho_2}, p_3^j = \frac{a_3}{b_3}, p_4^j = \frac{a_4}{b_4}.$$

To satisfy the constraint on T_0^2, we have $c_6 \leq T_0^D, c_6 < c_4 - \frac{a_4}{b_4}$.

(3) $c_5 \leq -T_0^2, c_5 < c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 - c_5}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 + c_5}{2}, p_3^1 = \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, p_4^1 = \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}.$$

Since $p_i^1 = p_i^2$ for $i = 1, 2, 3, 4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 - c_5}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 + c_5}{2-\rho_2}, p_3^j = \frac{a_3}{b_3}, p_4^j = \frac{a_4}{b_4}.$$

To satisfy the constraint on T_0^2, we have $c_5 \leq -T_0^D, c_5 < c_3 - \frac{a_3}{b_3}$.

(4) $T_1^2 < c_6, c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} \leq T_1^2 \leq \frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} - c_1$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 + T_1^2}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 - T_1^2}{2}, p_3^1 = \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, p_4^1 = \frac{\frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} + c_4 - T_1^2}{2}.$$

Since $p_i^1 = p_i^2$ for $i = 1, 2, 3, 4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 + T_1^D}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 - T_1^D}{2-\rho_2}, p_3^j = \frac{a_3}{b_3}, p_4^j = \frac{\frac{a_4}{b_4+\gamma_4} + c_4 - T_1^D}{2-\rho_4}.$$

where $T_1^D = (\frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_2-b_2c_2}{2-\rho_2} - \frac{a_4-b_4c_4}{2-\rho_4})/(\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_4}{2-\rho_4})$.
To satisfy the constraint on T_1^2, we have $T_1^D \leq \frac{a_1}{b_1} - c_1, c_4 - \frac{a_4}{b_4} \leq T_1^D < c_6$.

(5) $T_2^2 < c_5, c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} \leq T_2^2 \leq \frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} - c_2$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 - T_2^2}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 + T_2^2}{2}, p_3^1 = \frac{\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} + c_3 - T_2^2}{2}, p_4^1 = \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}.$$

Since $p_i^1 = p_i^2$ for $i = 1,2,3,4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 - T_2^D}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 + T_2^D}{2-\rho_2}, p_3^j = \frac{\frac{a_3}{b_3+\gamma_3} + c_3 - T_2^D}{2-\rho_3}, p_4^j = \frac{a_4}{b_4}.$$

where $T_2^D = (\frac{a_2-b_2c_2}{2-\rho_2} - \frac{a_1-b_1c_1}{2-\rho_1} - \frac{a_3-b_3c_3}{2-\rho_3})/(\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3})$.
To satisfy the constraint on T_2^2, we have $T_2^D \le \frac{a_2}{b_2} - c_2, c_3 - \frac{a_3}{b_3} \le T_2^D < c_5$.

(6) $c_4 - \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} \le c_6 \le min(\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} - c_1, T_1^2)$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 + c_6}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 - c_6}{2}, p_3^1 = \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3}, p_4^1 = \frac{\frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4} + c_4 - c_6}{2}.$$

Since $p_i^1 = p_i^2$ for $i = 1,2,3,4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 + c_6}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 - c_6}{2-\rho_2}, p_3^j = \frac{a_3}{b_3}, p_4^j = \frac{\frac{a_4}{b_4+\gamma_4} + c_4 - c_6}{2-\rho_4}.$$

To satisfy the constraint on T_1^2, we have $c_4 - \frac{a_4}{b_4} \le c_6 \le min(\frac{a_1}{b_1} - c_1, T_1^D)$.

(7) $c_3 - \frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} \le c_5 \le min(\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} - c_2, T_2^2)$

$$p_1^1 = \frac{\frac{a_1+\gamma_1 p_1^2}{b_1+\gamma_1} + c_1 - c_5}{2}, p_2^1 = \frac{\frac{a_2+\gamma_2 p_2^2}{b_2+\gamma_2} + c_2 + c_5}{2}, p_3^1 = \frac{\frac{a_3+\gamma_3 p_3^2}{b_3+\gamma_3} + c_3 - c_5}{2}, p_4^1 = \frac{a_4+\gamma_4 p_4^2}{b_4+\gamma_4}.$$

Since $p_i^1 = p_i^2$ for $i = 1,2,3,4$, by solving the equations, we have

$$p_1^j = \frac{\frac{a_1}{b_1+\gamma_1} + c_1 - c_5}{2-\rho_1}, p_2^j = \frac{\frac{a_2}{b_2+\gamma_2} + c_2 + c_5}{2-\rho_2}, p_3^j = \frac{\frac{a_3}{b_3+\gamma_3} + c_3 - c_5}{2-\rho_3}, p_4^j = \frac{a_4}{b_4}.$$

To satisfy the constraint on T_2^2, we have $c_3 - \frac{a_3}{b_3} \le c_5 \le min(\frac{a_2}{b_2} - c_2, T_2^D)$.

Then we can calculate the corresponding demand and profit by $d_i^j = a_i - b_i p_i^j + \gamma_i(p_i^k - p_i^j)$,and $\Pi^j(p_1^j, p_2^j, p_3^j, p_4^j) = \sum_{i=1}^4 d_i^j(p_i^j - c_i^j) - (d_2^j + d_4^j - d_1^j - d_3^j)^+ c_5^j - (d_1^j + d_3^j - d_2^j - d_4^j)^+ c_6^j$, which is summarized in Table 4. □

A.7 Proof of proposition 6

This proposition follows the results in Table 4. □

A.8 Proof of proposition 7

We only analyze the impact of γ_1 and γ_3 , and the impact of γ_2 and γ_4 can be analyzed in a similar way. Let $f_i = \frac{1}{(2-\rho_i)^2(b_i+\gamma_i)} = \frac{b_i+\gamma_i}{(2b_i+\gamma_i)^2}$, then $\frac{\partial f_i}{\partial \gamma_i} = -\frac{\gamma_i}{(2b_i+\gamma_i)^3} < 0$. Therefore,

$\frac{\partial \Pi^{j*}}{\partial \gamma_1} < 0$ (Case 2 3 6 7) and $\frac{\partial \Pi^{j*}}{\partial \gamma_3} < 0$ (Case 7) can be easily checked. Besides,

$$\frac{\partial \Pi^{j*}}{\partial \gamma_1} = -\frac{d_1^{j*}}{(2b_1+\gamma_1)(b_1+\gamma_1)} \frac{d_1^{j*}\rho_1 \frac{b_2}{2-\rho_2} + b_1 d_2^{j*} \frac{\rho_2}{2-\rho_2}}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2}} < 0 \quad (Case\ 1)$$

$$\frac{\partial \Pi^{j*}}{\partial \gamma_1} = -\frac{d_1^{j*}}{(2b_1+\gamma_1)(b_1+\gamma_1)} \frac{d_1^{j*}\rho_1 (\frac{b_2}{2-\rho_2} + \frac{b_4}{2-\rho_4}) + b_1 (d_2^{j*} \frac{\rho_2}{2-\rho_2} + d_4^{j*} \frac{\rho_4}{2-\rho_4})}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_4}{2-\rho_4}} < 0 \quad (Case\ 4)$$

$$\frac{\partial \Pi^{j*}}{\partial \gamma_1} = -\frac{d_1^{j*}}{(2b_1+\gamma_1)(b_1+\gamma_1)} \frac{d_1^{j*}\rho_1 (\frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3}) + b_1 (d_2^{j*} \frac{\rho_2}{2-\rho_2} - d_3^{j*} \frac{\rho_3}{2-\rho_3})}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3}} < 0 \quad (Case\ 5)$$

$$\frac{\partial \Pi^{j*}}{\partial \gamma_3} = -\frac{d_3^{j*}}{(2b_3+\gamma_3)(b_3+\gamma_3)} \frac{d_3^{j*}\rho_3 (\frac{b_2}{2-\rho_2} + \frac{b_1}{2-\rho_1}) + b_3 (d_2^{j*} \frac{\rho_2}{2-\rho_2} - d_1^{j*} \frac{\rho_1}{2-\rho_1})}{\frac{b_1}{2-\rho_1} + \frac{b_2}{2-\rho_2} + \frac{b_3}{2-\rho_3}} < 0 \quad (Case\ 5)$$

Then Proposition 7 follows these results. □

A.9 Proof of proposition 8

(1) In case 3:

$$\begin{aligned}\frac{\partial \Pi^{j*}}{\partial c_5} &= \frac{2(a_1-b_1(c_1-c_5))}{(2-\rho_1)^2(b_1+\gamma_1)} b_1 + \frac{2(a_2-b_2(c_2+c_5))}{(2-\rho_2)^2(b_2+\gamma_2)}(-b_2) \\ &= \frac{2(a_1-b_1c_1)}{(2-\rho_1)^2/(1-\rho_1)} - \frac{2(a_2-b_2c_2)}{(2-\rho_2)^2/(1-\rho_2)} + (\frac{2b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2b_2}{(2-\rho_2)^2/(1-\rho_2)})c_5\end{aligned}$$

(2) In case 2:

$$\begin{aligned}\frac{\partial \Pi^{j*}}{\partial c_6} &= \frac{2(a_1-b_1(c_1+c_6))}{(2-\rho_1)^2(b_1+\gamma_1)}(-b_1) + \frac{2(a_2-b_2(c_2-c_6))}{(2-\rho_2)^2(b_2+\gamma_2)} b_2 \\ &= -\frac{2(a_1-b_1c_1)}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2(a_2-b_2c_2)}{(2-\rho_2)^2/(1-\rho_2)} + (\frac{2b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2b_2}{(2-\rho_2)^2/(1-\rho_2)})c_6\end{aligned}$$

(3) In case 7:

$$\begin{aligned}\frac{\partial \Pi^{j*}}{\partial c_5} &= \frac{2(a_1-b_1(c_1-c_5))}{(2-\rho_1)^2(b_1+\gamma_1)} b_1 + \frac{2(a_2-b_2(c_2+c_5))}{(2-\rho_2)^2(b_2+\gamma_2)}(-b_2) + \frac{2(a_3-b_3(c_3-c_5))}{(2-\rho_3)^2(b_3+\gamma_3)} b_3 \\ &= \frac{2(a_1-b_1c_1)}{(2-\rho_1)^2/(1-\rho_1)} - \frac{2(a_2-b_2c_2)}{(2-\rho_2)^2/(1-\rho_2)} + \frac{2(a_3-b_3c_3)}{(2-\rho_3)^2/(1-\rho_3)} \\ &+ (\frac{2b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2b_2}{(2-\rho_2)^2/(1-\rho_2)} + \frac{2b_3}{(2-\rho_3)^2/(1-\rho_3)})c_5\end{aligned}$$

(4) In case 6:

$$\begin{aligned}\frac{\partial \Pi^{j*}}{\partial c_6} &= \frac{2(a_1-b_1(c_1+c_6))}{(2-\rho_1)^2(b_1+\gamma_1)}(-b_1) + \frac{2(a_2-b_2(c_2-c_6))}{(2-\rho_2)^2(b_2+\gamma_2)} b_2 + \frac{2(a_4-b_4(c_4-c_6))}{(2-\rho_4)^2(b_4+\gamma_4)} b_4 \\ &= -\frac{2(a_1-b_1c_1)}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2(a_2-b_2c_2)}{(2-\rho_2)^2/(1-\rho_2)} + \frac{2(a_4-b_4c_4)}{(2-\rho_4)^2/(1-\rho_4)} \\ &+ (\frac{2b_1}{(2-\rho_1)^2/(1-\rho_1)} + \frac{2b_2}{(2-\rho_2)^2/(1-\rho_2)} + \frac{2b_4}{(2-\rho_4)^2/(1-\rho_4)})c_6\end{aligned}$$

Then Proposition 8 follows these results. □