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Is Three a Crowd?

Small Group Provision of a Public Good

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Abstract

Suppose a group of individuals within a large community trust one another sufficiently to be able to co-ordinate their contributions to a public good. The alternative is to accept a Nash equilibrium. We show that under a wide range of reasonably plausible circumstances a relatively small group size (in many cases just three) suffices for this to be beneficial to group members.

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It seems generally accepted that when trust exists life is more pleasant than when it is absent or in short supply. Even within the realm of economic theory this idea is not new. It was central to the thinking of Adam Smith and David Hume who both thought the spread of trust throughout a community an essential part of the development process (Bruni and Sugden (2000)). More recently it has been linked to the success of institutions (La Porta et al (1997) and financial development (Guisio et al. (2004)). An extensive experimental literature (see Sapienza et al (2013) and the references therein) suggests there exists a significant level of trust among individuals. Within the realm of public economics it is possible that trust can explain in some way why laboratory experiments and econometric studies consistently show that agents voluntarily donate more to public goods than would be predicted by the standard model set out in Bergstrom, Blume and Varian (BBV) (1986). This classic paper, the foundation for a large body of subsequent work, takes the final provision of a public good to be the outcome of a Nash equilibrium. To explain the anomaly two theoretical strategies have been developed. Firstly, we have the ‘warm glow’ (Andreoni (1990)) which takes individuals to gain utility from the very act of donating. This preserves the idea of a Nash equilibrium, but changes individual motivation. Secondly, donations might stem from some social convention based on reciprocity (Sugden (1984), Benchekroun and Van Long (2008)). Here individuals contribute to the public good under the expectation that others will do the same, the expectation itself reflecting some form of social capital. This is trust at a societal level, as individuals feel under some obligation to conform to social norms. It takes us away from simple Nash equilibrium.

The origin of trust in this sense has been debated for a long time, going back at least to 18th century Italian and Scottish writers (Bruno and Sugden (2000)). Levati (2006) and Woersdorfer (2010) provide differing modern treatments of this question, and make extensive references to the modern literature. In this paper, by contrast, we consider a form of reciprocity stemming from trust at the individual, not societal, level. That is, individuals trust individuals known to them, but not others. This form of trust is surely one feature of "social capital", and may indeed be a vital part of any story in which trust emerges spontaneously within society. However, for the purpose of the exercise to follow, trust is taken as given. Individual motivation lies solely in the provision of the public good (no warm glow). Trust is just an instrument for achieving a desirable provision of the same. Hence, reciprocity is assumed to operate only within a most likely small sub-group of the economy, not as a general social norm. Starting with the foundation model (BBV), recall that with a Nash equilibrium agents either cannot communicate with one another or cannot credibly pre-commit to any particular action. Contrast this with the standard definition of trust given by Sapienza et al. quoting Gambetta (2000): "When we say we trust someone or that someone is trustworthy we implicitly mean that the probability that he will perform an action that is beneficial [to us] is high enough for us to consider engaging in some form of co-operation with him." Clearly, then, in the foundation model there is no trust. This raises an obvious question: how much trust do we need in order for total donations to rise above the Nash level and for the foundation model to be undermined? This is the key question we try to answer.

Our first problem is that of measuring trust. We adopt a simple approach. Note that the quotation above has two sides: I am trusting in that I believe someone will perform some action when she says she will; and that person is trustworthy if she does in fact perform an action when she says (promises) that she will. In consequence we have a minimum (non-zero) level of “trust” in an economy if exactly two individuals exist who both trust one another and who are both trustworthy. If we add a third trusted and trustworthy person to this group aggregate trust increases (by one unit if we like). Trust similarly increases if we add a fourth individual to the group, and so on. We assume just one group of mutually trusting people within society. We assume individuals to be identical in all respects, that members of this group agree that each will contribute a given amount to the public good, and finally that all contribute what was promised. Hence the group would be unanimous in deciding on its overall contribution, and our question becomes this: how large would such a group have to be in order for total donations to the public good to rise above the Nash level?

There are two papers which relate closely to our argument. The first, Buchholz, Haslbeck and Sandler (BHS) (1998), posed the same question. Assuming a fixed but small group of individuals who can co-ordinate their donations (who in our terms trust one another), they found two conditions under which such a group might gain from so doing: firstly, agents outside the potential group must have implausibly high marginal propensities to consume the public good; and, secondly, the size of the trusting group has to be implausibly large. They conclude that partial co-operation in the sense assumed here is unlikely. By contrast, in this paper, with qualifications, we reach conclusions that are almost exactly the reverse. The second paper, Buchholz, Cornes and Ruebbelke (BCR) (2014), undertakes a very thorough analysis of what might happen if there is a *given* single co-operating group within a community. Here co-operation, or in our terms trust, is characterised by ‘matching’, by which each co-operating agent provides a quantity of the public good to match the base donation made by each co-operator. The consequence of this is to alter (usually lower) the perceived price of the public good to each co-operating agent. There are three possible types of equilibrium: (a) the co-operating group and the outside group of non-co-operators both donate to the public good; (b) only the co-operating group donates; (c) only outside agents donate. The last of these comes about through the co-operating group deciding to cut their donations to zero and so gain from free-riding on the resulting increased donations of the outside group. In general there is no reason to suppose that the Nash outcome (zero matching) is what the trusting group will choose. However, two questions are left somewhat open in their analysis: (a) what size is a viable co-operating group likely to be; (b) what are the circumstances under which we might expect one type of equilibrium rather than another to emerge? In what follows we attempt to provide partial answers.

We set out our main argument diagrammatically in the next section. Formal analysis is presented in section 2, and extended in section 3. Comments and conclusions are given in the final section.

1. The basic argument

The basis of our argument can be found by looking at Figure 1, derived from Cornes and Sandler (1989). Assume an economy of individuals who are in all respects identical. Andreoni (1988) shows that with a sufficiently large number of individuals only those of a particular type can be expected to donate to the public good. This type would typically be individuals who are wealthier and/or have a stronger taste for the public good. If we bear in mind that we are dealing only with this type of person our assumption that agents are homogenous is not as restrictive as it might appear. There is a private good x and a public good G. We assume a constant cost economy with units of both goods chosen such that the price of each is unity. The individual income expansion path is drawn on Figure 1 and intersects the vertical line drawn where $x = M$ at point L, M being individual income. In this model, under the conditions of BBV, a unique equilibrium exists and agents all donate the same amount to the public good. Diagrammatically, equilibrium for any n is found when the symmetry line MC ($G = n(M - x)$) intersects the income expansion path at point A (Ignore the indifference curve for the moment). Clearly, as n increases, the line MC pivots clockwise, and G and x both increase as does individual utility. The equilibrium point approaches L as n goes to infinity. In what follows we assume a large economy, and take individual utility without co-ordination to be that at point L. Again, this is not as restrictive as it appears: the pivot of line MC on Figure 1 is greatest when n is small which suggests that convergence to point L is likely to be fast.[1]

Now imagine that there is a group of m mutually trusting individuals, able to co-ordinate fully their donations to the public good, as described earlier. They all agree amongst themselves to donate a given (equal) amount to G. If so, under the assumption that the rest of community donates nothing, the implicit budget constraint each person faces will be $M = x + m^{-1}G$. Replacing n by m this now appears on Figure 1 as the line MC. As shown, our group would be just willing to co-ordinate their contributions. From the diagram, it is clear that they will only gain if the group is large enough, i.e. MC is steep enough. If so, the provision of public good *must* increase above that at point L. Agents outside the group will donate nothing[2]. Thus the effective price of the public good to a group member does indeed become m^{-1}, the Lindahl price within the group. Recall that the price of provision is unity. Hence, the level of public good provision will be determined by standard individual utility maximisation, with the equation $M = x + m^{-1}G$, described by line MC, being each group member's budget constraint. Clearly, as m increases the price of the public good to an individual group member falls, and provision of G by the group rises. The path taken would be traced out by the price expansion curve on the diagram (not drawn). The critical question we address is this: at what level of m does the utility of donating within such a group exceed the limit utility at point L?

The critical point on Figure 1 is B.[3] To find the minimum size of group we find the (slope of that) symmetry line which is tangential to the limit indifference curve, and then take the smallest integer at least as large as this value, m^*. Our diagram, of course, cannot tell us how large or small m^* has to be. It can, though, suggest determining factors. Firstly, the location of L seems relevant, the lower it is the lower m^* is likely to be. To see this, just take any indifference curve, and shift it downwards vertically on the diagram. This would typically raise the importance of x in individual consumption. If so, with a given marginal willingness to pay, the limit provision of G would be lower, and fewer trusting individuals would be required to achieve a higher provision. Thus, contrary to BHS, who deal with marginal changes in public good contributions, we can expect the critical size of group to vary *inversely* with respect to the weight of private consumption in individual preferences. We call the impact of a lower limit point "Effect One". Secondly, suppose that point L and the *MRS* there remain unchanged, but that the curvature of the indifference curve diminishes. The tangency point will now be on a symmetry line with a gentler slope (above L) with a lower m and possibly m^*. We call the impact of a gentler slope for the limit indifference curve "Effect Two"[4]. One way to explain would be to say that, locally, agents now place a higher relative value on the public good. Finally, it is worth recalling that when m increases we get a greater pivot (greater marginal benefit from cost sharing) from the first individuals to join the group, and that this marginal benefit decreases rapidly with m. This suggests that the required number of co-operators might actually be rather small.

Note that the key comparison here is between points B and L, which do not lie in the same neighbourhood. To proceed we must therefore specify an individual utility function. The constant elasticity of substitution (CES) utility function seems a natural choice. It incorporates the Cobb-Douglas as a special case and provides a convenient way to examine the impact of increased substitutability between x and G. However, as we shall find in the next section, which investigates how well our conjectures stand up to formal analysis, there are subtleties to be dealt with in the way in which changing the elasticity of substitution (σ) changes the slope of the indifference curves.

2. The CES Utility Function

We assume each individual maximises:

$$U = (ax^{-\rho} + (1-a)G^{-\rho})^{-\frac{1}{\rho}}$$

Here $a \in (0, 1)$ captures the importance of x in individual preferences, and $\sigma = \frac{1}{\rho+1}$ is the elasticity of substitution. From the conjectures given in section 1 these are the parameters of particular interest. Suppose the individual faces the standard budget constraint $M = x + pG$ (just for the moment treat G as a private good and suspend the unit normalisation). By following standard procedures the demand functions for x and G are given by:

$$G = \frac{M(1-a)^{\sigma}p^{-\sigma}}{a^{\sigma} + (1-a)^{\sigma}p^{1-\sigma}} \qquad x = \frac{Ma^{\sigma}}{a^{\sigma} + (1-a)^{\sigma}p^{1-\sigma}}$$

Given that the marginal rate of substitution ($MRS_{Gx} = -\frac{dx}{dG}$) is equal to $\frac{1-a}{a}\left(\frac{x}{G}\right)^{\frac{1}{\sigma}} = p$, at the limit point L on Figure 1 consumption is given by the equations:

$$x_L = M \qquad\qquad G_L = M\left(\frac{1-a}{pa}\right)^{\sigma} \qquad (1)$$

Hence limit utility is:

$$U_L = M(a + (1-a)^{\sigma}(pa)^{1-\sigma})^{\frac{\sigma}{\sigma-1}}. \qquad (2)$$

The derivative of the marginal rate of substitution with respect to σ at any consumption point is given by:

$$\frac{\partial MRS_{Gx}}{\partial \sigma} = -\frac{1-a}{a}\left(\frac{x}{G}\right)^{\frac{1}{\sigma}}\frac{1}{\sigma^2}\ln\left(\frac{x}{G}\right)$$

Taking the sign of $-\ln\left(\frac{x}{G}\right)$, this is negative if and only if $x > G$. Hence, below the 45° line from the origin on Figure 1, a rise in the elasticity of substitution at any point would cause the new indifference curve to pivot clockwise. Above the line the reverse is the case, so overall indifference curves flatten using the point where *x* equals *G* as a fulcrum. Note too that with the normalisation restored if $a > ½$ $G_L < M$. Hence if σ rises G_L will fall and the indifference curve at the original limit point will become steeper. The two effects isolated in section 1 work against one another. For $a < ½$ the two effects again work against one another but with roles reversed. These properties help us understand the propositions that follow.

With the supply price (p) of the public good fixed at unity we now find the utility for members of a mutually trusting group size of size *m*. Recall from section 1 that if this group co-operates in order to provide the public good donations by the rest of the community will be zero. Within the group people, being identical, will be unanimous about the quantity of *G* to be provided. Given their trust in one another each takes the price of the public good to be m^{-1} and the consequent utility maximising quantity of *G* will be the agreed provision by the group. Thus to find the level of *G* and *x* consumed simply substitute this into the demand equations:

$$G = \frac{M(1-a)^{\sigma}\left(\frac{1}{m}\right)^{-\sigma}}{a^{\sigma} + (1-a)^{\sigma}\left(\frac{1}{m}\right)^{1-\sigma}} \qquad x = \frac{Ma^{\sigma}}{a^{\sigma} + (1-a)^{\sigma}\left(\frac{1}{m}\right)^{1-\sigma}}$$

Substitute these expressions into the utility function:

$$U(m) = M\left\{a^{\sigma} + (1-a)^{\sigma}\left(\frac{1}{m}\right)^{1-\sigma}\right\}^{\frac{1}{\sigma-1}} \tag{3}$$

From Figure 1 we find the minimum size of group by equating the utility at point L (U_L) with utility at point B ($U(m)$). Solving for *m* yields:

$$m = \left[\frac{\left(a+(1-a)^{\sigma}(a)^{1-\sigma}\right)^{\sigma} - a^{\sigma}}{(1-a)^{\sigma}}\right]^{\frac{1}{\sigma-1}} \tag{4}$$

If *m* is not an integer simply take the lowest integer greater than *m* to find *m**. This gives us the minimum size of co-operative group that would be willing to form in a large economy, given that the alternative is simple Nash equilibrium. To proceed we first establish some key properties of the *m* function. We then illustrate for various parameter values the number of members a trusting group would require if members are to benefit by co-ordinating donations.

Property 1

$$\frac{\partial m}{\partial a} < 0$$

Proof

Given that $m = \left[\frac{(a+(1-a)^{\sigma}(a)^{1-\sigma})^{\sigma}-a^{\sigma}}{(1-a)^{\sigma}}\right]^{\frac{1}{\sigma-1}}$, define the term in square brackets as y, so that $m = y^{\frac{1}{\sigma-1}}$. Also define $z = \frac{a}{1-a}$ so that $y = [(z + z^{1-\sigma})^{\sigma} - z^{\sigma}]$.

As $\frac{dz}{da} > 0$, we can evaluate the derivative of *m* with respect to *a* using *z*. Given that:

$$\frac{dm}{dz} = \frac{1}{\sigma-1} y^{\frac{2-\sigma}{\sigma-1}} \frac{dy}{dz}$$

The sign of the derivative depends on whether σ is greater or less than unity together with the sign of $\frac{dy}{dz}$.

$$\frac{dy}{dz} = \sigma(z + z^{1-\sigma})^{\sigma-1}(1 + (1-\sigma)z^{-\sigma}) - \sigma z^{\sigma-1}$$

$$= \sigma z^{\sigma-1}[(1 + z^{-\sigma})^{\sigma-1}(1 + (1-\sigma)z^{-\sigma}) - 1]$$

Hence:

$$\frac{dy}{dz} \gtrless 0 \text{ as } \frac{(1+(1-\sigma)z^{-\sigma})}{(1+z^{-\sigma})^{1-\sigma}} \gtrless 1$$

Now employing a second order Taylor expansion on the function $f(z^{-\sigma}) = (1 + z^{-\sigma})^{1-\sigma}$ we have:

$$(1 + z^{-\sigma})^{1-\sigma} = 1 + (1-\sigma)z^{-\sigma} - \sigma(1-\sigma)\tfrac{1}{2}(1 + \widetilde{z^{-\sigma}})^{-(\sigma+1)}z^{-2\sigma}$$

where $\widetilde{z^{-\sigma}} \in (0, z^{-\sigma})$, and suitably valued. It follows from this that:

$$\frac{(1+(1-\sigma)z^{-\sigma})}{(1+z^{-\sigma})^{1-\sigma}} > 1 \text{ if } \sigma < 1 \text{ with } \frac{(1+(1-\sigma)z^{-\sigma})}{(1+z^{-\sigma})^{1-\sigma}} < 1 \text{ if } \sigma > 1.$$

In both cases it follows that $\frac{dm}{dz}$ (or $\frac{\partial m}{\partial a}$) < 0. *Q.E.D.*

A rise in *a* will always lower G_L. The diagrammatic explanation is that a rise in *a* causes the MRS_{Gx} to fall (indifference curves to become steeper on Figure 1). Given normality the new limit point at which $MRS_{Gx} = 1$ must therefore lie below the old one. This confirms our earlier conjecture that the more tastes are directed towards the private good the lower the number of people needed to form a co-operative coalition to provide the public good. Given the smaller provision of *G* in the Nash equilibrium fewer mutually trusting people are needed to co-ordinate their donations so as to increase their utility. The role of σ is more complicated, but is partially dealt with by the next properties.

Property 2

As σ goes to zero required group size m goes to infinity.

From equation (1), as $\sigma \rightarrow 0, G_L \rightarrow M$. (Remember $p = 1$.) $\sigma = 0$ is the case of perfect complements, so here point B will approach point L and the slope of the line MC will become a vertical line. Hence *m* goes to infinity. The economic explanation for this result seems to lie in the fact that with perfect complements the Nash equilibrium is Pareto optimal. With a low σ we are already close to Pareto optimality, and the potential gains from co-ordination are correspondingly limited. A viable trusting group would have to be very large.

Property 3

As σ goes to infinity required group membership goes to:

(a) *Infinity if* $a < 0.5$

(b) 2 *if* $a = 0.5$

(a) From eq. (1) as $\sigma \rightarrow \infty$ $G_L \rightarrow \infty$. At the limit the goods are perfect substitutes. Now consider an artificial value of *m*, m'', determined by using the slope of the tangent to the indifference curve at G_L (equals unity for any σ), rather than the indifference curve itself. m'' is found as the slope of the line linking point M to the point at which the tangent hits the G axis. Clearly, $m'' \leq m$ for all values of σ. As G_L goes to infinity so too will m'' and therefore m and m^*. Clearly, if $G_L \rightarrow \infty$ Effect One must ultimately dominate Effect Two even when the latter works to diminish m, as it must for $G_L > M$.

(b) From (1) $a = 0.5$ implies $G_L = M$ for all σ. The limit indifference curve going through point L will be a straight line with slope minus one (perfect substitutes). This will hit the vertical axis at $G = 2M$, which implies a limit value of m of 2. Here only Effect Two operates and as it will do so only in the area of Figure 1 above the 45^{o} line it must cause m to fall.

The empirically interesting case of $1 > a > 0.5$ is not so straightforward. The limit indifference map, of course, is that of perfect substitutes, straight line indifference curves with $MRS_{Gx} = a^{-1}(1 - a)$. In this case $m = \frac{a}{1-a}$ (the slope of corresponding indifference curves we would draw on Figure 1) which will be high for high values of a. However, as against this G_L is converging to zero as σ increases, and the MRS_{Gx} at the L naturally remains at unity along this path which may result in large rises in m only occurring at very high values of σ. Effects One and Two work against one another, so we have to resort to computation to find out which is the stronger, particularly for the more plausible values of the parameters. We now turn to this task.

On Figure 2 we draw the surface of the m function for values of the elasticity of substitution up to 2. The picture illustrates our formal analysis of the m equation. For $a < 0.5$ we see that, as expected, m falls and then rises as σ rises. The diagram also shows an inverse relationship between m and σ when $a > 0.5$, although the change in m is not great once the elasticity of substitution reaches about 0.5. On Figure 3 we extend the range of σ. The change in scale suppresses some of the detail at the edge of the diagram (compare the vertical scale for m in both diagrams). The main feature to observe is the lack of feature in the bulk of the diagram (also apparent on Figure 2). Clearly in the 'flat' portion of the diagram, which encompasses reasonably plausible values of the parameters, m^* does not vary by very much. In the case of the high values of σ illustrated, Effect One offsets Effect Two. It would appear therefore that for the impact of these effects to be reversed σ would have to be very high[5].

To see what size of group would be needed to deliver what in fact will be a Pareto improving quantity of the public good consult Figure 4. Here we see the minimum value for a consistent with individuals being willing to form groups of sizes 2, 3, 5, 8, 10. All of these might be thought of as relatively 'small'. Given that $\frac{\partial m}{\partial a} < 0$ any coalition with a size above those indicated on the diagram would willingly provide the public good. It is unlikely that people to want to spend a large fraction of their income on the public good, almost certainly well below 50% ($a > 0.5$). As the diagram shows, except for very low values of σ we can expect a trusting group of size 10 to form.

If there is a moderate degree of substitutability (σ = 2), supposing $a = 0.9$ a group of 3 suffices. The flat area of Figures 2 and 3 in fact corresponds roughly to points where m^* equals 3. If σ is sufficiently (but not excessively) high it is possible that only two people (minimum trust) is necessary. Broadly speaking our simulation matches the conjectures we made in section 1. The preliminary conclusion is that for a wide range of plausible values for the key parameters of the model, a group of just three individuals would be willing to provide the public good. In this sense three, or possibly even two, is a ‘crowd’![6]

3. Two Extensions

We now consider two extensions to the analysis of section 2.

(a) Non-homothetic utility functions

As explained in section 1, the CES utility function is a natural one to use when analysing the question of trust, given the likely role of the elasticity of substitution in determining the minimum viable size of co-operating group. Nevertheless, in spite of its advantages, it is somewhat restrictive. To see what difference non-homotheticity makes to our results we now consider a wider class of utility functions.

First, take the simple case of quasi-linear utility: $U = v(x) + G$. Here the whole of the income effect goes to G, with the income expansion path in Figure 1 being vertical (we assume M is sufficiently large to avoid corner solutions). By adapting our earlier diagrammatic exercise it is easy to see that as community size increases without limit G goes to infinity. As n increases G_L also increases without limit. Hence, the minimum size of group (m) likewise goes to infinity as n increases. Thus for the large economy we assume, the numbers required for a co-operating group to want to form become very large. The Nash equilibrium is now robust. The same conclusion holds whenever there is no finite limit to G. That is, whenever the income expansion path on Figure 1 fails to cross the vertical line from point M we will have the same result. While a clear counter-example to the conclusion of the previous section this is not empirically plausible. It assumes a zero or negligible income effect on the private good and therefore a low weight for x at high incomes.

Our second example makes the opposite assumption: $U = x + v(G)$. Now there is a zero income effect on G. Although again rather special, this type of utility function is more plausible and is used quite often in public economics. The income expansion path on Figure 1 is now a horizontal line and the earlier analysis applies. Consider the function $U = x + \beta G^{\alpha}$, $0 < \alpha < 1$. By following the same procedure, equating the limit utility to the "Lindahl" utility when there are m co-operators and solving for m, it can be shown[7] that the formula for the minimum size of group is $m = (1-\alpha)^{-\frac{1-\alpha}{\alpha}}$. This translates to integer values for m of 2 or 3 with 2 being for the higher values of α. Our earlier conclusions continue to hold.

Now consider a generalisation of the CES utility function: $U = [a(x-\beta_1)^{-\rho} + (1-a)(G-\beta_2)^{-\rho}]^{-\frac{1}{\rho}}$. A special case of this is the Stone-Geary function, widely used in empirical work, where $\rho = 0$ or $\sigma = 1$. Unfortunately, this example is more complicated. Following the familiar procedures, the solution for m is given (in implicit functional form) by the equation:

$$f = \left(M - \beta_1 - \frac{\beta_2}{m}\right)\left\{a^{\sigma} + (1-a)^{\sigma}\left(\frac{1}{m}\right)^{1-\sigma}\right\}^{\frac{1}{\sigma-1}} - (M-\beta_1)[a + (1-a)^{\sigma}a^{1-\sigma}]^{\frac{\sigma}{\sigma-1}}$$

$$= 0$$

There is no general analytic solution for m. However, one or two comments can be made before presenting some simulations.

- The derivative of the first expression of f with respect to m is positive, so for any set of parameters there is a unique value for m.

- If $\beta_2 = 0$ there remains just one *m* in the equation. In this case the equation can be solved. It is not difficult to show that the solution is the same as in section 2, so all properties found there continue to hold. With $\beta_2 > 0$ *M* has a direct effect on *m*, in contrast to previous examples.
- Given that the derivative of *f* with respect to β_2 is negative it follows that $\frac{\partial m}{\partial \beta_2} > 0$. This relates to an earlier argument. Raising β_2 shifts the indifference curves up vertically on Figure 1, making *G* in some sense more important in individual consumption (Effect One), and therefore raising the minimal size of trusting group. This means that we must expect group size to be higher than in Section 2.
- The final comment concerns *a*, which as before indicates the weighting of the private good in (non-subsistence) consumption. Now, in contrast to the original CES case, we find that *m* does not decline monotonically when *a* rises. For *a* sufficiently high *m* rises with *a* and in fact approaches infinity as *a* approaches unity. The explanation for this lies in the fact that now the limit value of *G* approaches β_2, not zero. Indifference curves become vertical above $G = \beta_2$. This means we approach a situation akin to perfect complements, which we know from Section 2 causes the critical size of the trusting group to approach infinity. There is, though, a slight element of implausibility in the situation described. The quantity β_2 of the public good is in some sense vital to individuals, and yet production of *G* beyond this point delivers very little extra benefit.

The significance of the last point depends on how high *a* has to be for *m* to be excessively high. To answer this question we need examples. This is our next task.

In doing this we note that in the above equation it is not so much M that is of importance but the difference between M and the β terms. Recall that from Andreoni's findings the individuals we are dealing with are likely to be among the wealthier members of society perhaps with a strong preference for the public good. Accepting this, it is reasonable to suppose that the β terms are comparatively small in relation to M. For this reason our favoured simulations assume: $M = 100$; $\beta_1 = 10, 20, 30$; $\beta_2 = 30, 20, 10$. The β's are small compared to M, but not too small, thus avoiding similarity to findings of Section 2. Figures 5, 6, and 7 show how the minimum size of group changes with a for four possible values of σ. Outcomes where group size is much above 20 have been suppressed in order to show the basic pattern which is much the same across the diagrams. The functions are now U-shaped, this being more pronounced the higher is σ. The large "flat" area for m has shrunk noticeably for high values of σ, and in this case is restricted to values of a between 0.45 and 0.95. In some cases 2 or 3 individuals suffice for a viable group, but generally the numbers required are between 5 and 7, still reasonably low. a has to be quite high for m to be really large except for when σ is high.

To see the impact of a higher value for β_2 consider Figure 8, which is drawn for $\beta_2 = 70$, $\beta_1 = 30$, $M = 100$ and values for σ of 0.5, 1, 2, and 10. A comparison of Figures 7 and 8 shows that the general difference between the corresponding functions is not great. The functions are again U-shaped, being flatter the lower is σ, with minima between $a = 0.5$ and $a = 0.8$. As with the C.E.S. function the schedules take high values for a low (not illustrated), although this has to be very low with low σ. With a high σ (= 10) a itself has to be high (0.45) before m reaches a realistic level. Is three still a crowd? The answer generally is no. As expected, with a higher value for β_2 m rises and with σ = 0.5 is fairly constant at 10. For high values of σ m can be significantly lower even reaching three for σ = 10 around about the value a = 0.6. If we (arbitrarily) take 10 to be a maximum number for a viable co-operating group then for most cases illustrated we could expect such a group to form as long as $a < 0.95$. Even though this is not so favourable to undermining the Nash equilibrium as in Section 2 this is still very likely to happen if we bear in mind that the individuals we study are among the wealthier and/or more motivated members of the whole community.

(b) **The Commons**

Our second extension of the analysis of section 2 deals with a commons type of public good. This is particularly significant in the environmental context. Here individuals' private consumption damages some facility providing a public benefit, as for example with noise pollution. We assume that the only way to alleviate the problem is to restrict individual consumption. This question has been examined by Vicary (2011) in such a way as to draw comparison with the standard "subscription" type of public good that has, thus far, been the focus of our attention. The main general conclusion is that commons public good models have a sort of "skew-symmetric" relationship with the key subscription model. We illustrate this here with a simple model related to Vicary (2009) and illustrate another such feature.

Assume a "Garden of Eden" level of public good provision, $\bar{G}$. Assume two private goods, x which is environmentally neutral, and y for which each unit of consumption lowers the provision of G by an amount β. Each individual i is interested in their overall consumption $z_i = ax_i + by_i$, and G. Our identical individuals all solve the same utility maximisation problem:

Max $U_i = U_i(z_i, G)$

subject to $M_i = x_i + py_i$

Both x and y are delivered at constant cost (unity and p respectively). To ensure that individuals will want to consume good y we also assume $b > ap$. Take G_{-i} to be the initial provision of G, before i makes his/her consumption decision. Hence the equation $G = G_{-i} - \beta y_i$ tells i the impact of his/her consumption of y on G. By using the budget constraint to eliminate x from the z equation and then using the equation for G to eliminate y we can recast the individual utility maximisation problem as:

Max $U_i = U_i(z_i, G)$

Subject to $M_i + \left(\frac{b-ap}{\beta a}\right) G_{-i} = \left(\frac{1}{a}\right) z_i + \left(\frac{b-ap}{\beta a}\right) G$

The budget equation here appears as the line aMA on Figure 9. Note the minimum level of z is consumed when $z = ax = aM$, and the maximum when $z = bMp^{-1}$. As G_{-i} falls the budget line shifts downwards and the Income Expansion Path (IEP) traces out the locus of tangencies with the individual's indifference curves (not drawn).

Now assume all agents are identical in all respects. This means that they will all make the same (unco-ordinated) decisions. Hence they all consume the same amount of x and y and therefore of z. It can be shown that if this is so the line of symmetry is given by:

$$G = \left(\frac{\beta n a}{b-ap}\right) M + \bar{G} - \left(\frac{\beta n}{b-ap}\right) z$$

This is the line *a*MB that appears on Figure 9. Equilibrium is therefore found at point Q. Note that if $z = aM$ (no y consumption) this equation reduces to $G = \bar{G}$ so that increasing n pivots the line MB clockwise around the point $(aM, \bar{G})$. The consequence is that the limit point as n goes to infinity is L, with G falling, not rising, with n. Also note that we could imagine, following Andreoni, that the economy is composed of individuals of differing types, each having their own income expansion path and limit point. Suppose a finite number of types with individuals being added one by one with type according to underlying probabilities. As n, the number of individuals in the economy, goes to infinity the provision of G will with probability one converge to the limit point of the type with the least (not most) interest in the public good.

In a large economy, then, what are the chances of partial co-operation achieving a (Pareto) improvement? The answer is simple: very little. Close to the limit agents either consume x only and do no environmental damage, or, if members are of one particular type, do very little damage individually but a large amount collectively. Were a few of these people to agree together to cut y consumption they would achieve little. Partly this is because a small group would have been doing little damage in the first place. However, even this would probably largely be offset by the reaction of the rest of the community. If G does rise as a result, the level of G_{-i} will rise for other agents, who would then react by increasing their y consumption. More precisely, suppose agents are identical in all relevant respects, and suppose there is a small group, m, of mutually trusting individuals. Suppose the economy starts at Q, taking n

to be large. For the small group the individualised price of the public good is now lower and, assuming zero reaction on the part of the rest of the economy, each individual member will face a budget line starting at Q and moving up and to the left steeper than the initial budget line. Suppose this group decides to consume only *x* (choosing the end point on the vertical line down from *aM*). Final (now Stackelberg) equilibrium, allowing for the reaction of the rest of the economy, will be found as before only with a symmetry line pivoted anti-clockwise from *aMB*. If *n* is large and *m* is small the pivot will be minimal. The new consumption point of group members would involve a (most likely small) rise in *G* as a result of their cut in *z*. Suppose now our group just lowers *y* consumption. This would lower the effective value of $\bar{G}$ for the rest of the community in comparison to the previous example. The modified symmetry line will have the same slope as in the previous case but with a lower intercept. Provision of *G* falls below that in which the co-operating group chooses $y = 0$, but is still above provision at Q. Trusting individuals suffer a (lower) loss of *z* and in return enjoy a smaller gain in *G*. By varying their *z* consumption and accounting for the response by non-group members we could construct a budget line for the group. The group is viable if this goes above the initial indifference curve. However, there is no need to pursue the exercise further as the argument is effectively the same as BHS (1998) (see their Figure 1)[8]. Their pessimistic conclusions follow. To summarise, with a commons the conditions for a viable trusting group are difficult to achieve. In contrast, even allowing for the slightly less favourable findings of the previous sub-section, in many plausible circumstances we should not be surprised to find co-operative groups forming when the public good is of a subscription nature. This is another instance of the two cases being skew-symmetric with respect to one another.

4. Conclusion

The main overall conclusion to take from our analysis is that the properties of the foundation model are not robust with respect to the degree of trust within an economy. We supplement this statement with three observations.

Why are our findings so much at variance with those of BHS? The answer lies in the fact that they kept to the assumption of an internal solution for donations by individuals outside the group. Once it is possible for individuals in a co-operating group to achieve a utility above that at point L on Figure 1 donations by the rest of the community *must* be forced down to zero. At this point it would be impossible for the rest of the community to offset the extra donations by the co-operating group. Bear in mind too that when a, (measuring the importance of the private good in individual consumption) is high, G_L, is low. Indeed for $a > 0.5$ total community expenditure on the public good in the BBV model is less than individual income, possibly very substantially so. In the light of this observation, together with the substantial cost saving from having just a few individuals co-ordinating their donations, the fact that a small group of agents might gain from co-ordinating their efforts comes as less of a surprise.

Secondly, note that BCR do not use the limit point L. Our argument still applies, though, as can easily be seen from further inspection of Figure 1. With a finite economy the initial Nash equilibrium lies on the income expansion path below and to the left of L. If point B is available, it will be preferable for the trusting group. Indeed, in *any* final equilibrium in which outside agents donate to the public good, G *must* be less than at L with the utility of the co-operators being lower than at B. It follows that once B or anything better is available the *only* type of final equilibrium we would observe is one in which outsiders do not donate. For this, a group of three usually suffices when agents have a CES utility function. A higher, but still relatively small, number would be required for many of the cases examined in section 3.

Finally, consider the international context that both BHS and BCR had in mind. Numbers are likely to be fewer than assumed here and the degree of trust may be lower[9]. More to the point, many international public goods, such as the question of global warming, are better described as commons public goods. While both types of good are subject to prisoner dilemma problems the characteristics of equilibrium when action is not co-ordinated differ as between the two cases. Agents can always increase the damage they do, rather than cut their donations. This means that the assumption of an internal utility maximisation solution for individuals outside the co-operating group, as used by BHS, is now plausible. Hence with a commons, the scope for partial co-operation is much less, and their conclusions continue to hold.

Footnotes

*I am very grateful to the referees for valuable comments on earlier versions of this paper.

1. The focus on point L distinguishes our analysis from that of Buchholz, Cornes and Ruebbelke (2014). This is not a matter of great importance, as explained in the conclusion.

2. We are therefore trying find conditions under which outside agents contribute nothing. Taking the co-operating group to be one agent, this outcome could be thought of as either a Nash or a Stackelberg equilibrium.

3. Up to this point the argument is simply a graphical account of Proposition 7 in Buchholz et al (2014).

4. I am very grateful to a referee for suggesting this distinction.

5. The packages used for computing (Excel and Matlab) were not able to find a case of a rise in σ causing a rise in m when a was more than ½. This seems to have been caused by the fact that certain values required in the calculations came too close to zero.

6. In Benchekroun and Van Long's dynamic model one possible solution is for the economy to be stuck at the static Nash equilibrium indefinitely, with 'social capital' being zero (2008; page 244). Although they do not deal with direct co-ordination between agents, our argument suggests that if we suppose the personal trust we have examined here translates into some positive amount of social capital (people might simply observe *G*, not individual behaviour) this solution to their model might in practice be unstable. Social capital would subsequently grow to a higher level through the mechanism they analyse.

7. Derivations for Section 3 are given in an appendix available at http://eprint.ncl.ac.uk/pub_details2.aspx?pub_id=231695

8. One difference is that the number of outsiders causing damage may not be fixed: other types might be induced to increase their *y* consumption above zero if co-operators lower their own *y*. This simply strengthens their original conclusions.

9. David Hume, who provided us with one of the earliest recognisably modern treatments of trust, indeed suggested (*Treatise of Human Nature*, Book 3 section 11) that conventional rules of morality have less force in an international context.

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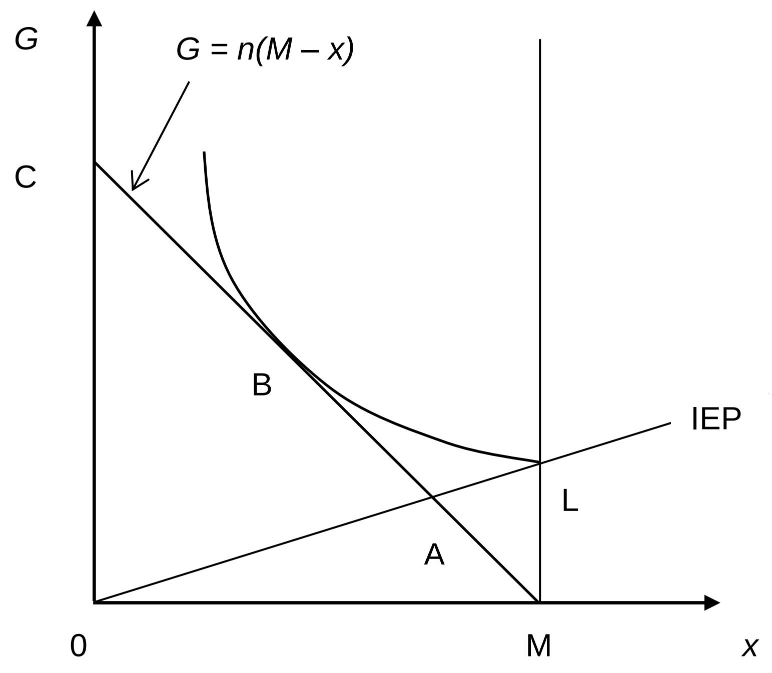
Figure 1

Determining the Minimum Size of Group



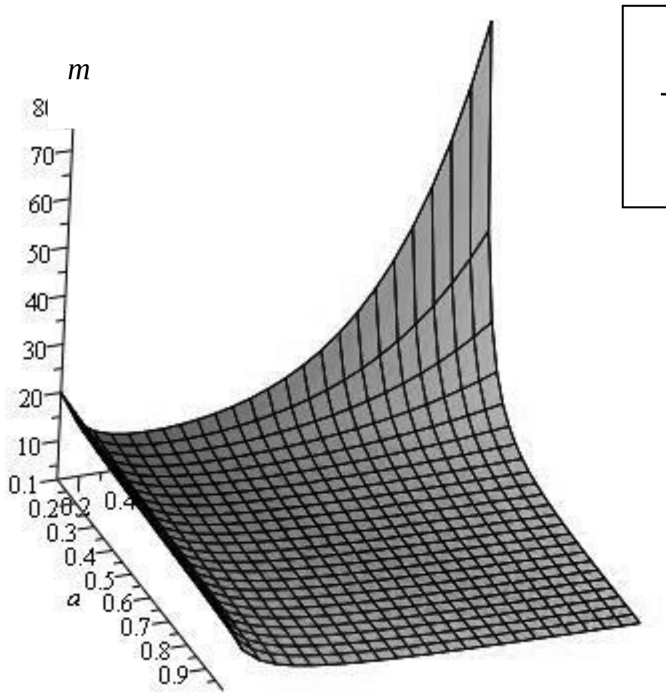


Figure 2

The shape of the *m* function

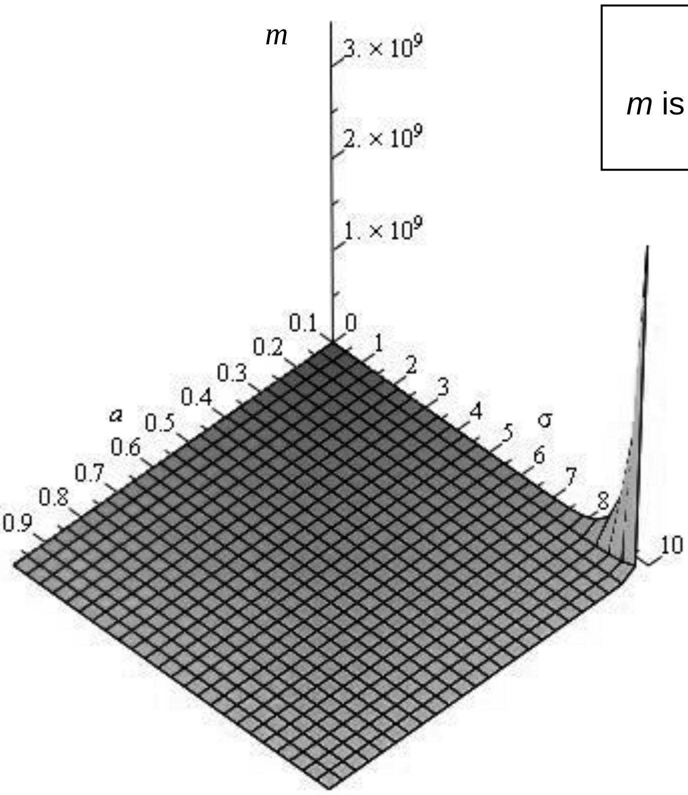


Figure 3

m is flat over most of its domain

Figure 4

Minimum Size of Trusting Group for Different Values of σ and *a*

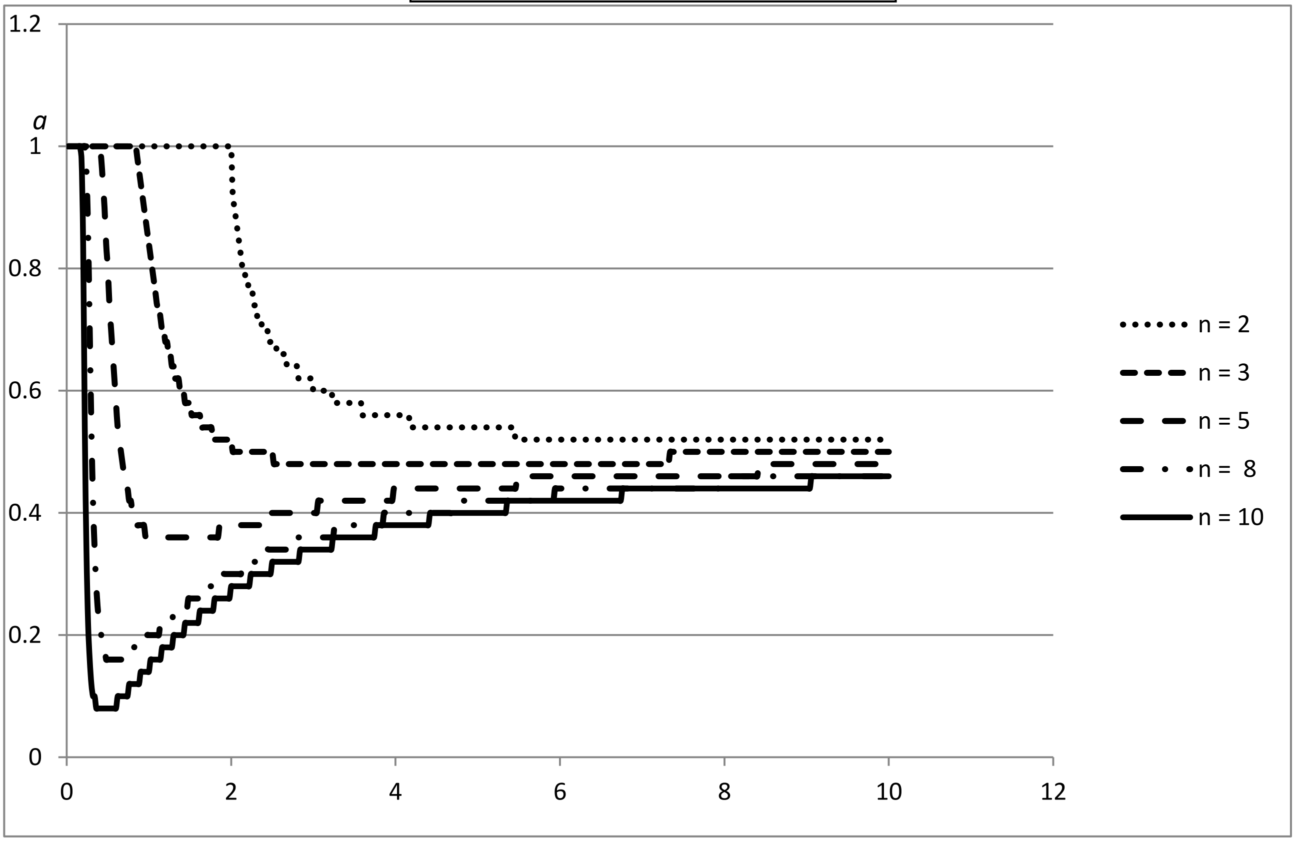


Figure 5

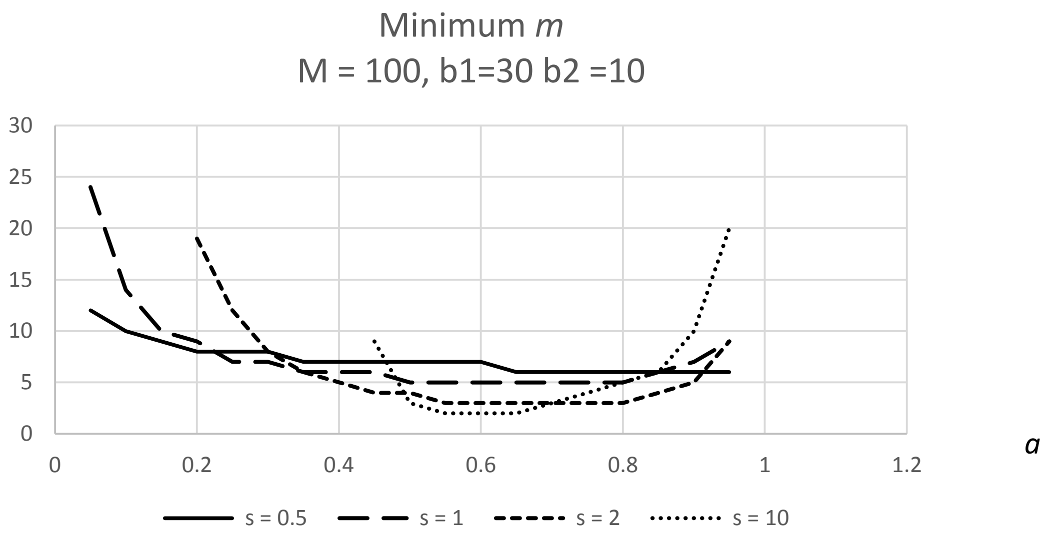


Figure 6

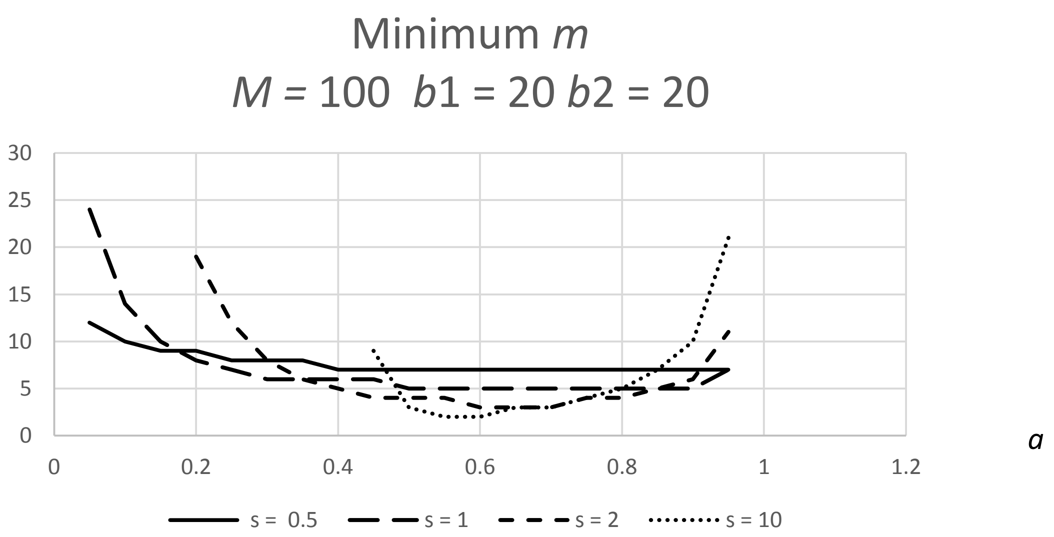


Figure 7

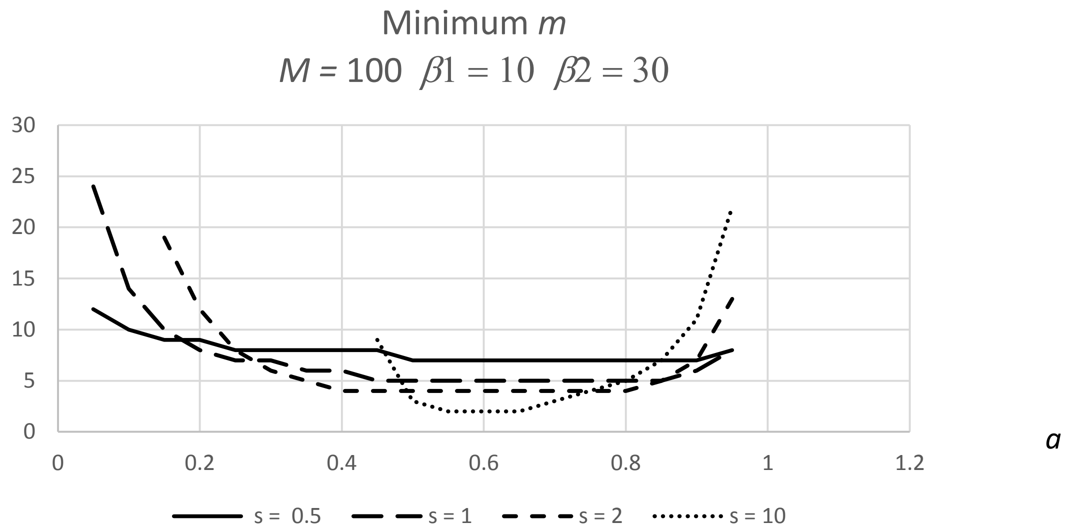


Figure 8

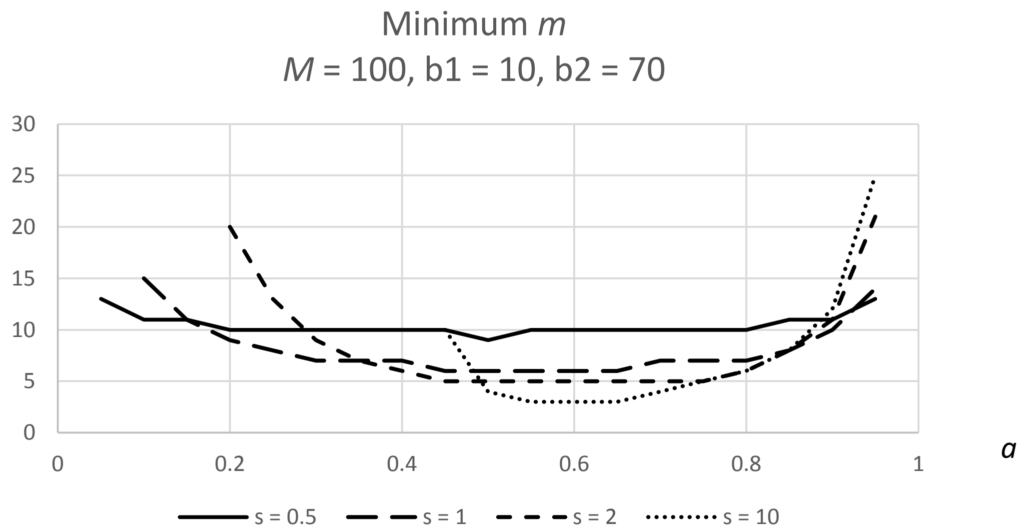


Figure 9