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Highlights

- A novel robust design of multiple-input multiple-output radar beamforming with full DoFs is proposed.
- A novel jointly estimating the desired signal steering vector and the covariance matrix is proposed.
- An iterative alternating method to solve two relaxed convex subproblems is derived, whose convergence is analytically proved.
- The covariance matrix estimation method is presented via the matrix rank-constrained minimization method.
- The proposed beamformer for MIMO radar can flexibly control the robust region and achieve high output performance.

Robust Adaptive Beamforming for Multiple-Input Multiple-Output Radar with Spatial Filtering Techniques

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Abstract

In this paper, we consider robust adaptive beamformer design for multiple-input multiple-output (MIMO) radar systems. The desired transmit-receive steering vector is estimated through maximizing the output power subject to constraints upon correlation coefficient and steering vector norm. The original nonconvex problem is reformulated as two reduced dimension semi-definite programming (SDP) problems. An iterative procedure is devised to tackle the two SDP problems, whose convergence is analytically proven. Based on the estimated desired signal, we are then able to obtain the interference covariance matrix via the matrix rank-constrained minimization method. Compared to other robust adaptive beamforming methods for MIMO radar, the proposed approach has the advantages of high efficiency and accuracy. Simulation results are presented to confirm the effectiveness and robustness of the proposed approach.

Keywords: `Robust beamforming, MIMO radar, iterative algorithm, convex quadratic program`

1. Introduction

Multiple-input multiple-output (MIMO) radar has received significant attention due to its various advantages over conventional radar systems, such as enhanced detection performance, improved parameter identifiability and angular resolution, providing more degrees of freedom (DOFs), and better spatial coverage [1]. MIMO radar is usually divided into statistical (or widely separated) MIMO radar [2] and colocated (or coherent) MIMO radar [3]. Statistical MIMO radar which is comprised of widely separated transmit and receive antennas, can achieve spatial diversity gain and enhance detection performance. On the other hand, colocated MIMO radar with waveform diversity can enhance parameter identifiability and increase the flexibility of transmit beampattern design, thereby improving spatial resolution via a great increase in DOFs of the system [4].

In practice, the adaptive beamforming algorithm is usually used to extract the desired signal and suppress simultaneously the interference as well as noise at the array output [5]. However, the conventional beamforming method often suffers severe performance degradation due to certain factors such as small number of training snapshots, corruption of training data by the desired signal in many practical applications, and the mismatch between the assumed and actual knowledge [6]. Thus, robust design techniques have been an active research topic. During the past decade, various robust adaptive beamformers have been proposed based on different principles to achieve high resolution in the framework of phased array receivers [7–9]. The diagonal loading technique is prevalent in enhancing the robustness of the beamformer. However, the limitation of the diagonal loading method is that the diagonal loading factor must be generally determined empirically [10]. The worst-case optimization-based technique developed in [11] delimits the uncertainty set by upper bounding the norm of the mismatch vector. In [12], a robust method is proposed by exploiting the specific structure of the matrix to enhance the robustness of adaptive arrays. In order to combat the effect of the desired signal in the sample covariance matrix, some robust methods based on covariance matrix estimation have been developed [13–17]. In the shrinkage method, an enhanced covariance matrix is obtained to improve the robustness against the signal mismatch problem. But the improvement in performance is limited [13, 14]. A spatial power spectrum sampling algorithm has been proposed to form an interference-plus-noise covariance matrix for further improvement in performance [15–17], while these techniques usually involve high computational complexity. The associated robust optimization techniques have also been considered for use in MIMO radar [18–20]. In [18], the worst-case optimization algorithm was used in MIMO radar. Moreover, in [19] a robust design has been proposed to mitigate signal mismatch with a certain selected probability distribution, which is a variant of the worst-case-based approach. An adaptive beamformer with magnitude response constraints is developed for MIMO radar [20], which employs the convex optimization method to obtain an exact robust solution.

In this paper, we consider a novel robust adaptive beamforming problem using full DOFs in the context of MIMO radar. The desired transmit-receive steering vector is estimated through maximizing the output power subject to spatial correlation coefficient and norm constraints. Then we devise an iteration procedure to tackle a relaxed version of the original nonconvex problem. Each iteration of the algorithm is handled via solving two low-dimensional convex optimisation problems [21, 22]. Then with the analyzed desired signal steering vector and the shrinkage estimator preprocessing, the interference covariance matrix can be estimated via the matrix rank-constrained minimization method. Compared with other high-performance algorithms for MIMO radar, the results indicate the effectiveness and robustness of the proposed algorithm.

The remainder of this paper is organized as follows. The MIMO signal model is described in Section 2. In Section 3, a novel steering vector method is proposed. Then, a new method to reconstruct the interference-plus-noise covariance matrix is introduced. In Section 4, we evaluate the performance via numerical simulations. Finally, conclusions are drawn in Section 5.

2. MIMO signal model

Consider a MIMO narrowband radar system composed of M_t transmit antennas and M_r receive antennas. We assume that all transmit and receive antennas are isotropic.

Each transmit element emits a different waveform and the baseband signal at the receiver can be written as

$$\mathbf{x}(t) = \alpha_0 \mathbf{a}_r(\phi_0)\, \mathbf{a}_t{}^T(\theta_0)\mathbf{s}(t) + \sum_{j=1}^{J} \alpha_j \mathbf{a}_r\left(\phi_j\right) \mathbf{a}_t{}^T\left(\theta_j\right)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where t is the time index, and $(\cdot)^T$ denotes the transpose operation. Parameters α_0 and α_j denote respectively the complex coefficient of the desired signal, and the complex coefficient of the jth interference. We assume that the desired signal, interference, and noise are statistically mutually independent, and the interference is neither close to nor in the mainlobe beam region of the array. The directions of departure (DODs) and directions of arrival (DOAs) of the desired signal and interferences with respect to the transmit and receive array normals are denoted respectively as $\left\{\theta_j, \varphi_j\right\}_{j=0}^{J}$. We also assume that the waveforms $\mathbf{s}(t) = \left[s_1(t), \ldots, s_{M_t}(t)\right]$ are mutually orthogonal with unit energy such that $\int_{T_N} \mathbf{s}(t)\,\mathbf{s}^H(t)dt = \boldsymbol{I}$, where $\boldsymbol{I}$ and T_N represent the identity matrix and the radar pulse width; $\mathbf{n}(t)$ is the additive white Gaussian noise vector; and $\mathbf{a}_t(\cdot)$ and $\mathbf{a}_r(\cdot)$ denote the corresponding $M_t \times 1$ and $M_r \times 1$ steering vectors, which have the following general forms

$$\begin{aligned} \mathbf{a}_t(\theta) &= \left[1\ e^{j2\pi d_t sin\theta/\lambda} \ldots e^{j2\pi(M_t-1)d_t sin\theta/\lambda}\right]^T, \\ \mathbf{a}_r(\phi) &= \left[1\ e^{j2\pi d_r sin\theta/\lambda} \ldots e^{j2\pi(M_r-1)d_r sin\theta/\lambda}\right]^T, \end{aligned} \tag{2}$$

where λ is the carrier wavelength. The interelement spacing in the transmit and receive arrays are denoted by d_t and d_r, respectively. By matched filtering the received data to the m_tth transmitted waveform at the receiver (i.e., $\mathbf{y}_{m_t}(t) = \int_{T_N} \mathbf{x}(t)\,\mathbf{s}_{m_t}{}^*(t)dt, m_t = 1, \ldots, M_t$, where $()^*$ denotes the conjugate operator), then the output of the matched filters of the MIMO radar can then be expressed as

$$\begin{aligned} \mathbf{y} &= \alpha_0 \mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0) + \sum_{j=1}^{J} \alpha_j \mathbf{a}_t\left(\theta_j\right) \otimes \mathbf{a}_r\left(\phi_j\right) + \mathbf{z} \\ &= \mathbf{y}_s + \mathbf{y}_j + \mathbf{z}, \end{aligned} \tag{3}$$

where $\mathbf{y}_s$, $\mathbf{y}_j$, $\mathbf{z}$ are the desired signal, interference, and noise vector components, respectively; and $\otimes$ denotes the Kronecker product; $\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)$ denotes the $M_t M_r \times 1$

transmit-receive steering vector. Under the assumption that both the signal steering vector and the data matrix are known precisely, the transmit-receive beamforming weight vector $\mathbf{w}$ can be obtained via maximizing the output signal-to-interference-plus-noise ratio (SINR)

$$\mathrm{SINR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{jn} \mathbf{w}} = \frac{\sigma_0^2 \left| \mathbf{w}^H \mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0) \right|^2}{\mathbf{w}^H \mathbf{R}_{jn} \mathbf{w}}, \tag{4}$$

where σ_0^2 denotes the desired signal power, and $|\cdot|$ is an absolute operator. $\mathbf{R}_s = \mathrm{E}\left[\mathbf{y}_s \mathbf{y}_s^H\right]$ and $\mathbf{R}_{jn} = \mathrm{E}\left[\left(\mathbf{y}_j + \mathbf{z}\right)\left(\mathbf{y}_j + \mathbf{z}\right)^H\right]$ represent the desired signal and the interference-plus-noise covariance matrices, respectively, where $\mathrm{E}\{\cdot\}$ is the statistical expectation operator. In practice, the interference-plus-noise covariance matrix is difficult to be obtained, thus, it is usually replaced by the sample covariance matrix $\hat{\mathbf{R}}$, which is calculated from the received signal vectors as

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{y}(l) \mathbf{y}^H(l), \tag{5}$$

where $\mathbf{y}(l)$ denotes the sample data at the lth snapshot and L denotes the number of snapshots. The transmit-receive beamforming weight vector is given by

$$\mathbf{w} = \frac{\hat{\mathbf{R}}^{-1}\left(\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)\right)}{\left(\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)\right)^H \hat{\mathbf{R}}^{-1}\left(\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)\right)}. \tag{6}$$

Note that $\hat{\mathbf{R}}$ contains the desired signal component. As stated in the last section, the calculated adaptive weight vector by using $\hat{\mathbf{R}}$ will obtain worse performance as compared with the one using the covariance matrix without any contribution from the desired signal. Based on the Capon spatial power spectrum estimator [23], the beamformer output power can be expressed as

$$P(\theta_0) = \frac{1}{\left(\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)\right)^H \hat{\mathbf{R}}^{-1}\left(\mathbf{a}_t(\theta_0) \otimes \mathbf{a}_r(\phi_0)\right)}. \tag{7}$$

3. Proposed method

As stated previously, adaptive beamformers are sensitive to steering vector mismatch, especially when the desired signal is present in the training data, which may cause the self-null phenomenon of the direction of the desired signal and result in dramatic performance degradation. In addition, the reconstructed interference-plus-noise covariance matrix may not be easily obtained. In this section, we propose a different approach to obtain the beamforming weight vector. The main idea is first to estimate the desired signal steering vector and then remove the actual desired signal information from the sample covariance matrix.

3.1. Steering Vector Estimation

According to the description in [13], shrinkage methods are suitable for high-dimensional covariance estimation with small number of samples. We use the shrinkage form as

$$\tilde{\mathbf{R}} = \hat{\alpha}\boldsymbol{I} + \hat{\beta}\hat{\mathbf{R}}, \tag{8}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are combination coefficients ($\hat{\beta} \in [0, 1]$, $\hat{\alpha} \geq 0$), which are solutions corresponding to the minimisation of the MSE function

$$\mathrm{MSE}\left(\tilde{\mathbf{R}}\right) = \mathrm{E}\left\{\left\|\tilde{\mathbf{R}} - \mathbf{R}\right\|_F^2\right\}, \tag{9}$$

where $\mathbf{R}$ denotes the theoretical covariance matrix of the array output vector; and $\|\cdot\|_F$ represents the Frobenius norm of a matrix. Note that as suggested in [13], the estimates of $\hat{\alpha}$ and $\hat{\beta}$ can be obtained as follows

$$\begin{aligned} \hat{\alpha} &= \min\left[\frac{\hat{\nu}\hat{\rho}}{\|\hat{\mathbf{R}}-\hat{\nu}\boldsymbol{I}\|_F^2}, \hat{\nu}\right], \\ \hat{\beta} &= 1 - \frac{\hat{\alpha}}{\hat{\nu}}, \end{aligned} \tag{10}$$

where $\hat{\rho} = \frac{1}{L^2}\sum_{l=1}^{L}\|\mathbf{y}(l)\|^4 - \frac{1}{L}\left\|\hat{\mathbf{R}}\right\|_F^2$, and $\hat{\nu} = \mathbf{tr}\left(\hat{\mathbf{R}}\right)\big/(M_t M_r)$. Substituting (10) into (8), we can obtain an estimate of the covariance matrix $\tilde{\mathbf{R}}$. The Capon spatial spectrum can be used for the direction of arrival estimation, and we aim to obtain a beamformer that maximizes the array output power, By considering that $\mathbf{a}_t \otimes \mathbf{a}_r$ represents the actual transmit-receive steering vector, our aim is to find

$$\min_{\mathbf{a}_t \otimes \mathbf{a}_r} (\mathbf{a}_t \otimes \mathbf{a}_r)^H \tilde{\mathbf{R}}^{-1} (\mathbf{a}_t \otimes \mathbf{a}_r). \tag{11}$$

Next, we assume that $\bar{\mathbf{a}}_t(\theta_0)$ and $\bar{\mathbf{a}}_r(\phi_0)$ denote the presumed transmitted steering vector and the presumed receive steering vector, respectively. Thus the presumed transmit-receive steering vector can be expressed as $\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0)$. In order to eliminate the ambiguity in the desired signal covariance term, let $\|\bar{\mathbf{a}}_t(\theta_0)\| = \sqrt{M_t}$, and $\|\bar{\mathbf{a}}_r(\phi_0)\| = \sqrt{M_r}$, where $\|\cdot\|$ denotes the Euclidean norm. We define the spatial correlation coefficient as the absolute value of the cosine of the angle between two vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, which can be expressed as

$$cor(\mathbf{a}_1, \mathbf{a}_2) = \frac{\left|\mathbf{a}_1^H \mathbf{a}_2\right|}{\|\mathbf{a}_1\| \, \|\mathbf{a}_2\|}, \tag{12}$$

where $0 \le cor(\mathbf{a}_1, \mathbf{a}_2) \le 1$. We also assume that the actual steering vectors satisfy the same norm constraint as the presumed steering vectors (i.e., $\|\mathbf{a}_t\| = \sqrt{M_t}$, $\|\mathbf{a}_r\| = \sqrt{M_r}$), which is reasonable for many scenarios including the cases of look direction error and phase perturbations. The norm constraint still holds approximately for the small gain perturbations [12]. Applying the spatial correlation methodology, we obtain

$$\mu \le \frac{\left|(\mathbf{a}_t \otimes \mathbf{a}_r)^H (\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0))\right|}{\|\mathbf{a}_t \otimes \mathbf{a}_r\| \, \|\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0)\|} \le 1, \tag{13}$$

where μ is an appropriate scalar factor. For a robust adaptive beamformer, we can use the constraint to distinguish the desired signal region from the interference region, and to guarantee that the desired signal can be covered by the region of interest [12], which offers efficient, and flexible choices for the robustness parameters. As a reference, μ can be defined as

$$\mu = \frac{\left|(\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0))^H \left(\bar{\mathbf{a}}_t(\theta_\mu) \otimes \bar{\mathbf{a}}_r(\phi_\mu)\right)\right|}{M_t M_r}, \tag{14}$$

where θ_μ can be set according to the robust region of interest, which means that the left side of the inequality constraint in (13) measures the relative correlation between the presumed transmit-receive steering vector and a reference transmit-receive steering vector. Then, we get

$$\nu \leq \left|(\mathbf{a}_t \otimes \mathbf{a}_r)^H \left(\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0)\right)\right| \leq M_t M_r, (\nu = \mu M_t M_r). \tag{15}$$

We next append the correlation coefficient and the norm constraint. Proceeding in this way, the transmit-receive steering vector of the MIMO radar can be calculated by solving the following optimization problem

$$\begin{aligned} &\min_{\mathbf{a}_t \otimes \mathbf{a}_r} (\mathbf{a}_t \otimes \mathbf{a}_r)^H \tilde{\mathbf{R}}^{-1} (\mathbf{a}_t \otimes \mathbf{a}_r), \\ &\text{s.t. } \nu \leq \left|(\mathbf{a}_t \otimes \mathbf{a}_r)^H \left(\bar{\mathbf{a}}_t(\theta_0) \otimes \bar{\mathbf{a}}_r(\phi_0)\right)\right| \leq M_t M_r, \\ &\quad \|\mathbf{a}_t \otimes \mathbf{a}_r\| = \sqrt{M_t M_r}. \end{aligned} \tag{16}$$

It is well-known that the transmit-receive steering vector of the MIMO radar is a special structured model of the virtual steering vector, i.e. the transmit-receive steering vector of the MIMO radar is the Kronecker product of the transmit and receive array steering vectors. In order to avoid solving the high-dimensional optimisation problem directly, the optimisation problem (16) can be reformulated as

$$\begin{aligned} &\min_{\mathbf{a}_r, \mathbf{a}_t} (\mathbf{a}_t \otimes \mathbf{a}_r)^H \tilde{\mathbf{R}}^{-1} (\mathbf{a}_t \otimes \mathbf{a}_r), \\ &\text{s.t. } \nu \leq \left|\mathbf{a}_t^H \bar{\mathbf{a}}_t(\theta_0)\, \mathbf{a}_r^H \bar{\mathbf{a}}_r(\phi_0)\right| \leq M_t M_r, \\ &\quad \|\mathbf{a}_r\| = \sqrt{M_r}, \\ &\quad \|\mathbf{a}_t\| = \sqrt{M_t}. \end{aligned} \tag{17}$$

The problem (17) is non-convex with respect to optimization variables $(\mathbf{a}_r, \mathbf{a}_t)$. It can be seen from (17) that the objective function can be transformed into a separable form with respect to one variable vector if either of the transmit and receive array steering vectors $\mathbf{a}_t$ and $\mathbf{a}_r$ is fixed. Thus, an iterative alternating method is presented to solve the problem, which is able to provide high quality solutions to the formulated non-convex problem [24, 25]. Assume the desired transmit steering vector $\mathbf{a}_t$ is known, the optimization problem (17) can be simplified as follows

$$\begin{aligned} &\min_{\mathbf{a}_r} \mathbf{a}_r^H \tilde{\mathbf{R}}_r^{-1} \mathbf{a}_r, \\ &\text{s.t.} \;\; \nu \leq \left| \mathbf{a}_t^H \bar{\mathbf{a}}_t(\theta_0) \mathbf{a}_r^H \bar{\mathbf{a}}_r(\phi_0) \right| \leq M_t M_r, \\ &\qquad \|\mathbf{a}_r\| = \sqrt{M_r}, \end{aligned} \tag{18}$$

where $\tilde{\mathbf{R}}_r^{-1} = [\mathbf{a}_t \otimes \boldsymbol{I}]^H \tilde{\mathbf{R}}^{-1} [\mathbf{a}_t \otimes \boldsymbol{I}] \in \mathbb{C}^{M_r \times M_r}$. Similarly, fix the receive steering vector $\mathbf{a}_r$ and (17) can be reduced to

$$\begin{aligned} &\min_{\mathbf{a}_t} \mathbf{a}_t^H \tilde{\mathbf{R}}_t^{-1} \mathbf{a}_t, \\ &\text{s.t.} \;\; \nu \leq \left| \mathbf{a}_t^H \bar{\mathbf{a}}_t(\theta_0) \mathbf{a}_r^H \bar{\mathbf{a}}_r(\phi_0) \right| \leq M_t M_r, \\ &\qquad \|\mathbf{a}_t\| = \sqrt{M_t}, \end{aligned} \tag{19}$$

where $\tilde{\mathbf{R}}_t^{-1} = [\boldsymbol{I} \otimes \mathbf{a}_r]^H \tilde{\mathbf{R}}^{-1} [\boldsymbol{I} \otimes \mathbf{a}_r] \in \mathbb{C}^{M_t \times M_t}$. The optimization problems (18) and (19) are nonconvex quadratic programs. The key difficulties with (18) and (19) are their nonconvexity due to the left side of the inequality constraint and the nonlinear equality constraints $\|\mathbf{a}_r\| = \sqrt{M_r}$ and $\|\mathbf{a}_t\| = \sqrt{M_t}$, which implies that the convex optimization technique cannot be applied directly with such constraints. Since $\left|\mathbf{a}_1^H \mathbf{a}_2\right|^2 = \left(\mathbf{a}_1^H \mathbf{a}_2\right)^* \left(\mathbf{a}_1^H \mathbf{a}_2\right)$, then we have $\left|\mathbf{a}_1^H \mathbf{a}_2\right|^2 = \mathbf{tr}\left\{\mathbf{a}_1 \mathbf{a}_1^H \mathbf{a}_2 \mathbf{a}_2^H\right\} = \mathbf{tr}\left\{\mathbf{R}_{\mathbf{a}_1} \mathbf{R}_{\mathbf{a}_2}\right\}$, where $\mathbf{tr}\{\cdot\}$ represents the trace of the matrix within the braces. For the problem (18), the transmit steering vector $\mathbf{a}_t$ and $\bar{\mathbf{a}}_t(\theta_0)$ are known, which means that $\gamma = \mathbf{a}_t^H \bar{\mathbf{a}}_t(\theta_0)$ is constant. We exploit the equivalent matrix formulation

$$\begin{aligned} &\min_{\mathbf{A}_r} \mathbf{tr}\left(\tilde{\mathbf{R}}_r^{-1} \mathbf{A}_r\right), \\ &\text{s.t.} \;\; \nu^2 \big/ |\gamma|^2 \leq \mathbf{tr}\left(\mathbf{R}_{\bar{\mathbf{a}}_r} \mathbf{A}_r\right) \leq (M_t M_r)^2 \big/ |\gamma|^2, \\ &\qquad \mathbf{tr}\left(\mathbf{A}_r\right) = M_r, \\ &\qquad \mathbf{A}_r = \mathbf{a}_r \mathbf{a}_r^H, \end{aligned} \tag{20}$$

where $\mathbf{R}_{\bar{\mathbf{a}}_r} = \bar{\mathbf{a}}_r(\phi_0) \bar{\mathbf{a}}_r^H(\phi_0)$. All the nonconvexity of problem (18) is now confined in the rank-one constraint $\mathbf{A}_r = \mathbf{a}_r \mathbf{a}_r^H$. The problem (20) can be relaxed into a convex SDP optimization problem, i.e., neglecting the rank-one constraint [26]. By doing so, we obtain an enlarged relaxed quadratic problem

$$\begin{aligned}
&\min_{\mathbf{A}_r} \mathbf{tr}\left(\tilde{\mathbf{R}}_r^{-1}\mathbf{A}_r\right),\\
&\text{s.t.}\ \ \nu^2\Big/|\gamma|^2 \leq \mathbf{tr}\left(\mathbf{R}_{\bar{\mathbf{a}}_r}\mathbf{A}_r\right) \leq (M_t M_r)^2\Big/|\gamma|^2,\\
&\qquad \mathbf{tr}\left(\mathbf{A}_r\right) = M_r,\\
&\qquad \mathbf{A}_r \succeq \mathbf{0},
\end{aligned} \tag{21}$$

where $\mathbf{A} \succeq \mathbf{0}$ denotes $\mathbf{A}$ is a positive semidefinite (PSD) matrix. Analogously, if the receive steering vector $\mathbf{a}_r$ and $\bar{\mathbf{a}}_r(\phi_0)$ are known, $\rho = \mathbf{a}_r^H \bar{\mathbf{a}}_r(\phi_0)$ is constant. The optimization problem (19) can be recast equivalently as

$$\begin{aligned}
&\min_{\mathbf{A}_t} \mathbf{tr}\left(\tilde{\mathbf{R}}_t^{-1}\mathbf{A}_t\right),\\
&\text{s.t.}\ \ \nu^2\Big/|\rho|^2 \leq \mathbf{tr}\left(\mathbf{R}_{\bar{\mathbf{a}}_t}\mathbf{A}_t\right) \leq (M_t M_r)^2\Big/|\rho|^2,\\
&\qquad \mathbf{tr}\left(\mathbf{A}_t\right) = M_t,\\
&\qquad \mathbf{A}_t = \mathbf{a}_t\mathbf{a}_t^H,
\end{aligned} \tag{22}$$

where $\mathbf{R}_{\bar{\mathbf{a}}_t} = \bar{\mathbf{a}}_t(\theta_0)\,\bar{\mathbf{a}}_t^H(\theta_0)$. Neglecting the rank-one constraint, the SDP relaxation of the problem (22) is given by

$$\begin{aligned}
&\min_{\mathbf{A}_t} \mathbf{tr}\left(\tilde{\mathbf{R}}_t^{-1}\mathbf{A}_t\right),\\
&\text{s.t.}\ \ \nu^2\Big/|\rho|^2 \leq \mathbf{tr}\left(\mathbf{R}_{\bar{\mathbf{a}}_t}\mathbf{A}_t\right) \leq (M_t M_r)^2\Big/|\rho|^2,\\
&\qquad \mathbf{tr}\left(\mathbf{A}_t\right) = M_t,\\
&\qquad \mathbf{A}_t \succeq \mathbf{0}.
\end{aligned} \tag{23}$$

These semidefinite programming problems (21) and (23) can be solved efficiently by the interior point method [27]. Once one pair of optimal solutions $\{\mathbf{A}_r, \mathbf{A}_t\}$ is obtained, we can check the rank of $\mathbf{A}_r$ and $\mathbf{A}_t$. If the rank of $\mathbf{A}_r^\star = \mathbf{a}_r^\star(\mathbf{a}_r^\star)^H$ and $\mathbf{A}_t^\star = \mathbf{a}_t^\star(\mathbf{a}_t^\star)^H$ equal to one, then $\mathbf{a}_r^\star$ and $\mathbf{a}_t^\star$ can be obtained exactly [26, 28]. Equations (21) and (23) are a relaxation of (18) and (19) obtained through dropping the rank-one constraint, respectively. If $\mathbf{A}_r$ or $\mathbf{A}_t$ has rank higher than one, we can construct a rank-one optimal solution via the matrix decomposition theorem [29], and thus the solutions $\mathbf{a}_r^\star$ and $\mathbf{a}_t^\star$ are optimal for (18) and (19). Specifically, in order to construct the rank-one optimal solution, we exploit a specific rank-one matrix decomposition theorem [29, Theorem 2.2], which is cited as the following lemma.

Lemma 1: Let $\mathbf{X}$ be a non-zero $M \times M$ ($M \geq 3$) complex Hermitian positive semidefinite matrix of rank r and $\mathbf{A}_i$ be Hermitian matrices, $i = 1, 2, 3$. Then,

• if $r \geq 3$, one can find, in polynomial time, a rank-one matrix $\mathbf{x}\mathbf{x}^H$ such that $\mathbf{x}$ (synthetically denoted as $\mathbf{x} = \mathcal{D}_1(\mathbf{X}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3)$) is in range $(\mathbf{X})$, and

$$\mathbf{x}^H \mathbf{A}_i \mathbf{x} = \mathbf{tr}(\mathbf{X}\mathbf{A}_i), i = 1, 2, 3 \tag{24}$$

with $\mathbf{X} - \frac{1}{r}\mathbf{x}\mathbf{x}^H \geq \mathbf{0}$ and $\operatorname{rank}\left(\mathbf{X} - \frac{1}{r}\mathbf{x}\mathbf{x}^H\right) \leq r - 1$.

• if $r = 2$, for any $\mathbf{z}$ not in the range space of $\mathbf{X}$, one can find a rank-one matrix $\mathbf{x}\mathbf{x}^H$ such that $\mathbf{x}$ (synthetically denoted as $\mathbf{x} = \mathcal{D}_2(\mathbf{X}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3)$) is in the linear subspace spanned by range $\{\mathbf{z}\} \cup$ range $(\mathbf{X})$, and

$$\mathbf{x}^H \mathbf{A}_i \mathbf{x} = \mathbf{tr}(\mathbf{X}\mathbf{A}_i), i = 1, 2, 3 \tag{25}$$

with $\mathbf{X} + \mathbf{z}\mathbf{z}^H - \frac{1}{r}\mathbf{x}\mathbf{x}^H \geq \mathbf{0}$ and $\operatorname{rank}\left(\mathbf{X} + \mathbf{z}\mathbf{z}^H - \frac{1}{r}\mathbf{x}\mathbf{x}^H\right) \leq 2$.

We next consider the optimization of receive steering vector $\mathbf{a}_r^{\#}$ with fixed transmit steering vector. The solution of the relaxed SDP problem (21) can be obtained firstly. Then, resorting to Lemma 1, we can find an optimal solution $\mathbf{a}_\mathbf{r}^{\#}$ of problem (18). We then use the same approach for the transmit steering vector. With fixed receive steering vector, we can solve $\mathbf{A}_t^{\#}$ via the relaxed SDP problem (23). Then an optimal solution $\mathbf{a}_t^{\#}$ of (19) can be obtained based on Lemma 1. **Algorithm 1** and **Algorithm 2** summarize respectively the procedures to determine the optimal solution $\mathbf{a}_r$ of (18) and $\mathbf{a}_t$ of (19).

Algorithm 1 Algorithm for Receive Steering Vector Design

Require: ν, $\tilde{\mathbf{R}}_r^{-1}$, $\mathbf{R}_{\bar{\mathbf{a}}_r}$.

Ensure: An optimal solution $\mathbf{a}_r^\#$.

Solve the problem (21), obtain $\mathbf{A}_r^\#$;
if rank $\left(\mathbf{A}_r^\#\right) = 1$ **then**
    Perform an eigen-decomposition $\mathbf{A}_r^\# = \tilde{\mathbf{a}}_r^\# \left(\tilde{\mathbf{a}}_r^\#\right)^H$;
else if rank $\left(\mathbf{A}_r^\#\right) = 2$ **then**
    Find $\tilde{\mathbf{a}}_r^\# = \mathcal{D}_2\left(\mathbf{A}_r^\#, \tilde{\mathbf{R}}_r^{-1}, \boldsymbol{I}_{M_r}, \mathbf{R}_{\bar{\mathbf{a}}_r}\right)$;
else
    Find $\tilde{\mathbf{a}}_r^\# = \mathcal{D}_1\left(\mathbf{A}_r^\#, \tilde{\mathbf{R}}_r^{-1}, \boldsymbol{I}_{M_r}, \mathbf{R}_{\bar{\mathbf{a}}_r}\right)$;
end if
Output $\mathbf{a}_r^\# = \frac{\sqrt{M_r}}{\|\tilde{\mathbf{a}}_r^\#\|}\tilde{\mathbf{a}}_r^\#$.

Algorithm 2 Algorithm for Transmit Steering Vector Design

Require: ν, $\tilde{\mathbf{R}}_t^{-1}$, $\mathbf{R}_{\bar{\mathbf{a}}_t}$.

Ensure: An optimal solution $\mathbf{a}_t^\#$.

Solve the problem (23), obtain $\mathbf{A}_t^\#$;
if rank $\left(\mathbf{A}_t^\#\right) = 1$ **then**
    Perform an eigen-decomposition $\mathbf{A}_t^\# = \tilde{\mathbf{a}}_t^\# \left(\tilde{\mathbf{a}}_t^\#\right)^H$;
else if rank $\left(\mathbf{A}_t^\#\right) = 2$ **then**
    Find $\tilde{\mathbf{a}}_t^\# = \mathcal{D}_2\left(\mathbf{A}_t^\#, \tilde{\mathbf{R}}_t^{-1}, \boldsymbol{I}_{M_t}, \mathbf{R}_{\bar{\mathbf{a}}_t}\right)$;
else
    Find $\tilde{\mathbf{a}}_t^\# = \mathcal{D}_1\left(\mathbf{A}_t^\#, \tilde{\mathbf{R}}_t^{-1}, \boldsymbol{I}_{M_t}, \mathbf{R}_{\bar{\mathbf{a}}_t}\right)$;
end if
Output $\mathbf{a}_t^\# = \frac{\sqrt{M_t}}{\|\tilde{\mathbf{a}}_t^\#\|}\tilde{\mathbf{a}}_t^\#$.

Here we use the discrete-time index k to represent the iteration number in the optimization problem, we get the beamformer output power $P^{(k)}$ at the kth iteration

$$P^{(k)} = \frac{1}{\left(\mathbf{a}_r^{(k)} \otimes \mathbf{a}_t^{(k)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_r^{(k)} \otimes \mathbf{a}_t^{(k)}\right)}. \tag{26}$$

Thus, the objective function of (17) is the reciprocal of the array output power, i.e., $\frac{1}{P^{(k)}}$. We perform the optimization alternately by using **Algorithm 1** and **Algorithm 2** until the estimates satisfy the convergence condition which is expressed as

$$\left|\frac{1}{P^{(k)}} - \frac{1}{P^{(k+1)}}\right| \leq \delta. \tag{27}$$

where δ is a threshold to determine that the difference between two consecutively obtained P values is small enough. Based on the above analysis, the transmit and receive array steering vectors $\mathbf{a}_t$ and $\mathbf{a}_r$ can be obtained according to **Algorithm 3**.

Algorithm 3 Iterative Procedure for Transmit-Receive Design

Require: ν, $\tilde{\mathbf{R}}_r^{-1}$, $\mathbf{R}_{\bar{\mathbf{a}}_r}$, $\tilde{\mathbf{R}}_t^{-1}$, $\mathbf{R}_{\bar{\mathbf{a}}_t}$, δ.

Ensure: An optimal solution $\left(\mathbf{a}_t^{\#}, \mathbf{a}_r^{\#}\right)$ to problem (17).

Set $k = 0$, $\mathbf{a}_t^{(k)} = \bar{\mathbf{a}}_t(\theta_0)$.

repeat

$k := k + 1$;

Solve the problem (18) to find the $\mathbf{a}_r^{\#}$ through **Algorithm 1**;

Set $\mathbf{a}_r^{(k)} = \mathbf{a}_r^{\#}$;

Use $\mathbf{a}_r^{(k)}$ to update $\mathbf{a}_r$ in (19);

Solve the problem (19) to find the $\mathbf{a}_t^{\#}$ through **Algorithm 2**;

Set $\mathbf{a}_t^{(k)} = \mathbf{a}_t^{\#}$;

Replace $\mathbf{a}_t$ in (18) with $\mathbf{a}_t^{(k)}$;

Until $\left|\frac{1}{P^{(k)}} - \frac{1}{P^{(k+1)}}\right| \leq \delta$;

Report $\mathbf{a}_t^{\#} = \mathbf{a}_t^{(k+1)}$, $\mathbf{a}_r^{\#} = \mathbf{a}_r^{(k+1)}$.

Proposition 1: The proposed iterative procedure for solving the problem (17) is convergent.

Proof: From **Algorithm 3**, we consider the iterative alternating routine

$$\mathbf{a}_t^{(0)} \to \mathbf{a}_r^{(1)} \to \ldots \mathbf{a}_t^{(k)} \to \mathbf{a}_r^{(k+1)} \to \mathbf{a}_t^{(k+1)} \to . \tag{28}$$

We first prove that the objective function, i.e., $\frac{1}{P^{(k)}}$ is a monotone non-increasing

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sequence. For a complex-valued problem, the relaxed SDP problem is equivalent to the original quadratically constrained quadratic program (QCQP) problem when the number of constraints is no more than 3, i.e., SDR is not just a relaxation. It is tight [21]. Thus, the two problems (18) and (19) are equivalent to two convex SDP optimization problems. For the optimization of $\mathbf{a}_r^{(k+1)}$ with fixed $\mathbf{a}_t^{(k)}$, i.e., $\mathbf{a}_t^{(k)} \rightarrow \mathbf{a}_r^{(k+1)}$. Since the problem (18) shares hidden convexity properties. It implies that the $\mathbf{a}_r^{(k+1)}$ is an optimal solution to (18) with fixed $\mathbf{a}_t^{(k)}$, then

$$\left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k+1)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k+1)}\right) \leq \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k)}\right) = \frac{1}{P^{(k)}}. \tag{29}$$

Similarly, for the optimization of $\mathbf{a}_t^{(k+1)}$ with fixed $\mathbf{a}_r^{(k+1)}$, i.e., $\mathbf{a}_r^{(k+1)} \rightarrow \mathbf{a}_t^{(k+1)}$. The problem (19) is hidden convex. As a result, in the $(k+1)$th iteration, the $\mathbf{a}_t^{(k+1)}$ is an optimal solution to (19) in the feasible set, and we get

$$\frac{1}{P^{(k+1)}} = \left(\mathbf{a}_t^{(k+1)} \otimes \mathbf{a}_r^{(k+1)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_t^{(k+1)} \otimes \mathbf{a}_r^{(k+1)}\right) \leq \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k+1)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k+1)}\right). \tag{30}$$

According to (29) and (30), we obtain $\frac{1}{P^{(k+1)}} \leq \frac{1}{P^{(k)}}$, i.e., the objective function is
95 non-increasing during the alternating iterations. Moreover, the objective function is positive, i.e., $\frac{1}{P^{(k)}} = \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k)}\right)^H \tilde{\mathbf{R}}^{-1} \left(\mathbf{a}_t^{(k)} \otimes \mathbf{a}_r^{(k)}\right) > 0$, then it is bounded below. Hence, according to the monotone convergence theorem[30], the alternating optimization is guaranteed to converge.

3.2. *Covariance Matrix Estimation*

In order to remove the information related to the desired signal in the sample covariance matrix, a rank-constrained problem arises. If the number of interference signals is known to be less than or equal to J, the interference covariance matrix $\tilde{\mathbf{R}}_i$ can be determined by solving the optimization problem

$$\begin{aligned} &\min_{\tilde{\mathbf{R}}_i} \left\|\tilde{\mathbf{R}} - \mathbf{R}_s - \tilde{\mathbf{R}}_i\right\|_F^2, \\ &\text{s.t. } \operatorname{rank}(\tilde{\mathbf{R}}_i) \leq J, \\ &\qquad \tilde{\mathbf{R}}_i \succeq \mathbf{0}. \end{aligned} \tag{31}$$

Then, with the analyzed transmit and receive array steering vectors $\mathbf{a}_t$ and $\mathbf{a}_r$, the optimization in (31) can be recast as

$$\begin{aligned} &\min_{\tilde{\mathbf{R}}_i} \left\|\tilde{\mathbf{R}} - \tilde{\sigma}_0^2(\mathbf{a}_t \otimes \mathbf{a}_r)(\mathbf{a}_t \otimes \mathbf{a}_r)^H - \tilde{\mathbf{R}}_i\right\|_F^2, \\ &\text{s.t. } \operatorname{rank}(\tilde{\mathbf{R}}_i) \leq J, \\ &\qquad \tilde{\mathbf{R}}_i \succeq \mathbf{0}, \end{aligned} \tag{32}$$

where $\tilde{\sigma}_0^2$ denotes the estimated desired signal power. According to the updated $\mathbf{a}_r$ and $\mathbf{a}_t$, the desired signal power can be estimated as

$$\tilde{\sigma}_0^2 = \frac{1}{(\mathbf{a}_r \otimes \mathbf{a}_t)^H \tilde{\mathbf{R}}^{-1} (\mathbf{a}_r \otimes \mathbf{a}_t)}. \tag{33}$$

With matrix variable $\tilde{\mathbf{R}}_i$, the solution for the rank-constrained problem (32) can be obtained as follows [31]

$$\begin{aligned} \tilde{\mathbf{R}}_i &= \sum_{i=1}^{J} \lambda_i \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i^H \left(\tilde{\mathbf{R}}_a\right), \\ \tilde{\mathbf{R}}_a &\triangleq \tilde{\mathbf{R}} - \tilde{\sigma}_0^2 (\mathbf{a}_t \otimes \mathbf{a}_r)(\mathbf{a}_t \otimes \mathbf{a}_r)^H, \end{aligned} \tag{34}$$

where λ_i and $\mathbf{e}_i$ denote the ith largest eigenvalue and the corresponding normalized eigenvector of the matrix within the braces. Furthermore, only the noise covariance matrix term σ_n^2 remains unknown; in order to eliminate the effect of noise perturbation, the eigenvalues of noise should be equal. Thus σ_n^2 can be replaced by the smallest eigenvalues of sample covariance matrix (i.e., $\tilde{\sigma}_n^2 = 1/((M_t M_r - J - 1) \sum_{i=J+1}^{(M_t M_r} \lambda_i \left(\tilde{\mathbf{R}}\right))$. Thus, the noise covariance matrix can be modified as $\tilde{\sigma}_n^2 \boldsymbol{I}$. Finally, the reconstructed interference-plus-noise covariance matrix can be expressed as

$$\begin{aligned} \tilde{\mathbf{R}}_{\text{in}} &= \sum_{i=1}^{J} \lambda_i \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i^H \left(\tilde{\mathbf{R}}_a\right) + \sum_{i=J+1}^{M_t M_r} \tilde{\sigma}_n^2 \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i \left(\tilde{\mathbf{R}}_a\right) \mathbf{e}_i^H \left(\tilde{\mathbf{R}}_a\right), \\ &= \mathbf{A} \mathbf{\Lambda} \mathbf{A}^H, \end{aligned} \tag{35}$$

where $\mathbf{A} = \left[\mathbf{e}_1 \left(\tilde{\mathbf{R}}_a\right), \ldots, \mathbf{e}_{M_t M_r} \left(\tilde{\mathbf{R}}_a\right)\right]$, and $\mathbf{\Lambda} = \mathbf{Diag}\left[\lambda_1, \ldots, \lambda_J, \tilde{\sigma}_n^2, \ldots, \tilde{\sigma}_n^2\right]$, where $\mathbf{Diag}(\cdot)$ denotes the diagonal matrix formed by the entries of the vector argument. Once we get the transmit-receive steering vector and the interference-plus-noise matrix, the transmit-receive beamforming weight vector for the MIMO radar can be expressed as

$$\mathbf{w} = \frac{\tilde{\mathbf{R}}_{\text{in}}^{-1}(\mathbf{a}_t \otimes \mathbf{a}_r)}{(\mathbf{a}_t \otimes \mathbf{a}_r)^H \tilde{\mathbf{R}}_{\text{in}}^{-1}(\mathbf{a}_t \otimes \mathbf{a}_r)} = \frac{\mathbf{A}\mathbf{\Lambda}^{-1}\mathbf{A}^H(\mathbf{a}_t \otimes \mathbf{a}_r)}{(\mathbf{a}_t \otimes \mathbf{a}_r)^H \mathbf{A}\mathbf{\Lambda}^{-1}\mathbf{A}^H(\mathbf{a}_t \otimes \mathbf{a}_r)}. \tag{36}$$

3.3. Discussion

In our work, we assume that the desired signal region and interference region can be distinguished via the correlation coefficient and norm constraints, which can be also exploited to restrict the desired signal to the region of interest. Thus, we can obtain the corrected transmit-receive steering vector based on the Capon spatial spectrum estimator, which offers efficiency because the extended Capon beamformer can determine accurately the power of the desired signal. Then, based on the estimated information about the desired signal, we reconstruct the interference-plus-noise covariance matrix via the matrix rank-constrained minimization method to remove the information corresponding to the desired signal from the sample covariance matrix. Thus, with the reconstructed interference-plus-noise matrix and the corrected steering vector, the proposed algorithm is robust to the steering vector mismatch.

In our approach, the main computational complexity lies in the estimation of the interference-plus-noise covariance matrix and the processing of **Algorithm 3**. Since the proposed beamformer employs the full DOFs of the MIMO radar (i.e., the adaptive dimension of the beamformer is $(M_t M_r)$), the computational complexity of the covariance matrix estimation and eigenvalue decomposition is $O\left((M_t M_r)^3\right)$. The computational complexity of the **Algorithm 3** is linear with respect to the number of iterations k, and the computational burden associated with each iteration. Finally, each iteration comprises the solution of two low-dimensional SDP problems (21) and (23). The former corresponds to complexity of the order of $O\left(M_r^{4.5}\log(1/\eta)\right)$ (where $\eta > 0$ is a prescribed accuracy, e.g., see [32, p. 251]), and the complexity of the rank-one decomposition requires $O\left(M_r^3\right)$ operations. Similarly, the complexity of solving the problem (23) is given by $O\left(M_t^{4.5}\log(1/\eta)\right)$ and the complexity of the rank-one decomposition procedure is $O\left(M_t^3\right)$. Compared to other full DOFs algorithms, the computational complexity of the sample matrix inverse (SMI) beamformer and the diagonally loaded SMI (LSMI) is $O\left((M_t M_r)^3\right)$ [10]. The computational complexity of the full DOFs Second-order cone programming (F-SOCP) algorithm using convex optimisation is $O\left((M_t M_r)^{3.5}\right)$ in [18]. The proposed beamforming algorithm is therefore comparable to other full DOFs robust beamforming algorithms. For the Kronecker beamformer in [20], the computational complexity can be reduced to the order of $O\left((M_t M_r)^2\right)$. The two low-dimensional sub-problems in each iteration have complexity $O\left(M_t^{3.5} + 2PM_t^{2.5}\right)$ and $O\left(M_r^{3.5} + 2PM_r^{2.5}\right)$, where P denotes the number of sampled points in the region of interest. Nevertheless, the Kronecker beamformer is a restricted beamformer whose adaptive dimension is $(M_t + M_r)$, which reduces its DOFs used for interference suppression. In summary, the computational complexity of the aforementioned beamformers is listed in Table 1.

Table 1: Comparisons of Computational complexities

Beamformer	Proposed	MRC [20] (each iteration)	F-SOCP [18]	SMI/LSMI
Computational complexity	$O\left(k\left(M_t^{4.5}\log(1/\eta)+M_t^3\right)\right)$ $+O\left(k\left(M_r^{4.5}\log(1/\eta)+M_r^3\right)\right)+O((M_t M_r)^3)$	$O(M_t^{3.5}+2PM_t^{2.5})+O(M_r^{3.5}+2PM_r^{2.5})$ $+O((M_t M_r)^2)$	$O((M_t M_r)^{3.5})$	$O((M_t M_r)^3)$

4. Simulations

In this section, several numerical examples are exhibited to investigate the above deterministic analysis. Consider a colocated MIMO radar system shares a uniform linear array (ULA) with M_r=M_t=7 elements spaced half a wavelength apart. Assume two interference directions are set to be −35° and 30°, respectively, and the interference-to-noise-ratio (INR) equals to 35dB for both interferences. The presumed desired signal direction is set to be 0°, and the signal-to-noise-ratio (SNR) is fixed at 5dB (except the scenarios where SNR varies). All the simulations are carried out on a PC with Intel Core i3 CPU and 4 GB memory. The number of snapshots is set to be $N = 200$ (except the scenarios where the snapshot number varies). 100 Monte Carlo trials are performed in each configuration. Finally, we consider the exit conditions $\delta = 10^{-3}$ for Algorithm 3.

The proposed method is compared with the SMI method, the LSMI method [10], the F-SOCP optimization beamformer in [18], and the MRC Kronecker beamformer in [20]. The optimal output SINR is provided for comparison. The DL factor is assumed as 10 times the noise power. The value $\varepsilon = 5$ is used for the robust beamformer in [18]. In [20], the ripple of the magnitude response constraints approach is set as 3dB and a fixed beamwidth of 6° is employed, and the relative regularization factor $\gamma = 6$. For simplicity and without loss of generality, we assume that $\mu = 0.8$ in the proposed method.

Fig. 1 shows the output SINR of the beamformers versus input SNR when the actual direction of the desired signal is fixed at 3°. It can be found that the proposed method performs best among the tested beamformers. As expected, the proposed beamformer can obtain the best output SINR in a large range of SNR, and this improvement is especially remarkable at high SNRs. The main reason is that the estimation of the interference-plus-noise covariance matrix via matrix rank-constrained optimization is effective in this situation. It implies that the proposed method is not sensitive to the power of the desired signal.

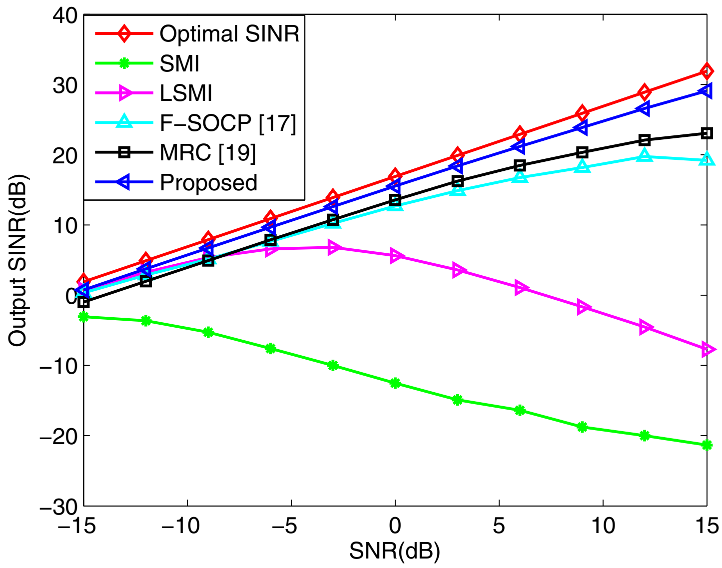

Figure 1: Output SINR versus input SNR in the case of signal direction error.

In the second example, the SINR performance of these methods is shown with respect to the number of training snapshots. The other parameters remain the same as for Fig.1 except the number of snapshots. The result is shown in Fig.2. Compared with these existing algorithms, the proposed method provides a faster convergence rate and higher output performance than the others. This performance behavior implies that the proposed covariance estimator is well conditioned under small sample sizes. The MRC method also has excellent sample convergence rate, which is because the Kronecker beamformer can reduce the number of training samples required.

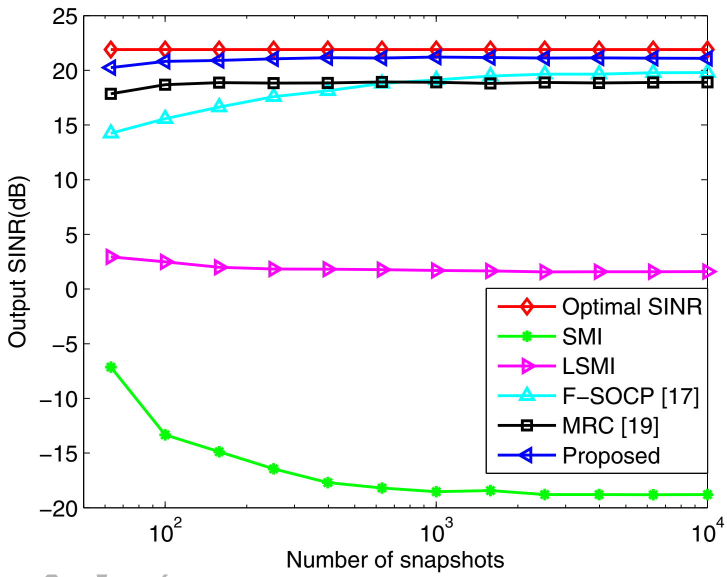

Figure 2: Output SINR versus number of snapshots in the case of signal direction error.

In Fig.3, we analyze the effect of the iteration number on the system performance. It can be observed that the proposed algorithm has a fast convergence rate and less than ten cycles are needed to achieve a satisfactory SINR in the scenario where only signal direction error exists, which is very useful for fast computation in practical applications. Moreover, the CPU run times are usually used as a performance metric. We present the run time of three typical robust algorithms, and the parameters are the same as those for Fig.1. Run times averaged over 100 trials are shown in Table 2.

180 As expected, the MRC method is by far the most efficient. This is mainly because it is a Kronecker beamformer, which reduces its DOFs in order to decrease the computational complexity. For the full DOFs robust beamforming methods, the proposed algorithm has a run time that is slightly more than the MRC method in this scenario. It is also observed that the F-SOCP design is generally slowest as a result of its highest
185 computational complexity.

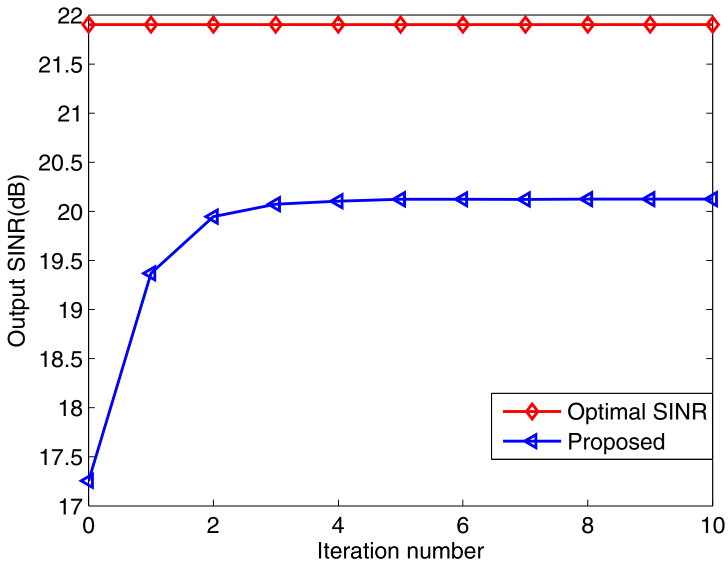

Figure 3: Output SINR versus iteration number.

Table 2: Comparison of Average Run Time (in seconds)

Beamformer	Proposed	MRC [20]	F-SOCP [18]
Computation time (Sec)	1.72	1.26	2.37

In this example, a more practical scenario with direction mismatch is investigated. The pointing error changes from $-5°$ to $5°$. We assume that $\mu = 0.7$ in the proposed method, and the beamwidth of [20] is set to be $10°$. Other parameters remain the same

as those used for Fig.1. The results are presented in Fig.4. As expected, a wider range
190 of mismatch angle leads to a worse SINR. It is clear that even small pointing error can lead to severe performance degradation for the SMI beamformer. Meanwhile, other approaches are more robust against the pointing error. The results indicate that the proposed and the MRC methods can remain robust over a large pointing error range since these methods can flexibly control the robust region.

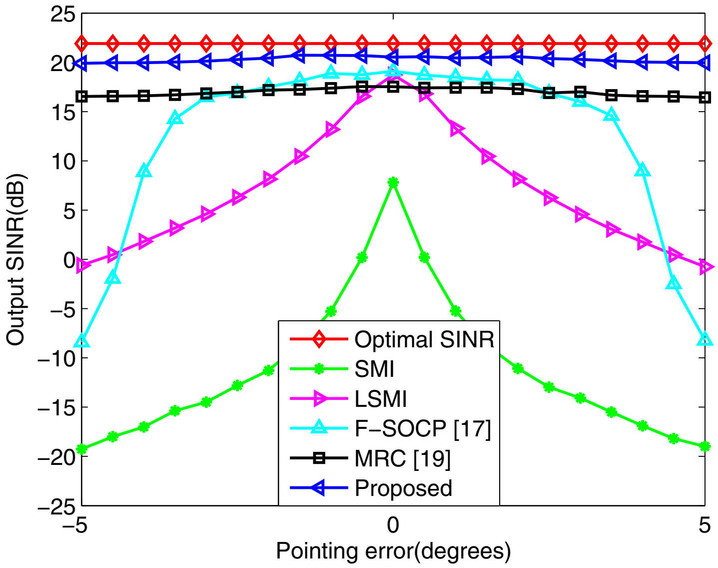

Figure 4: Output SINR versus pointing error.

195 In Fig.5, we analyze the impact of μ on the performance of the proposed algorithm. The direction mismatch is assumed to be random and uniformly distributed over $[-5°, 5°]$, other parameters are the same as for generating Fig.3. In this scenario, it has been shown that the proposed method suffers from performance degradation in output SINR when μ is large. It is because if μ is too large, the constraint region may not
200 cover all the uncertainly regions, and thus, the output performance will degrade. It should be noted that for smaller μ, the larger the constraint region becomes. If the region of interest includes the interference signal, it may result in the interference being regarded as the desired signal, and the beamformer may attempt to suppress the desired

signal as if it was interference. In other words, due to the large constraint set, the calculated steering vector may converge to an interference steering vector or corresponding linear combination. Thus, the proposed method may also suffer severe performance degradations. Therefore, if possible, μ should not be chosen too small.

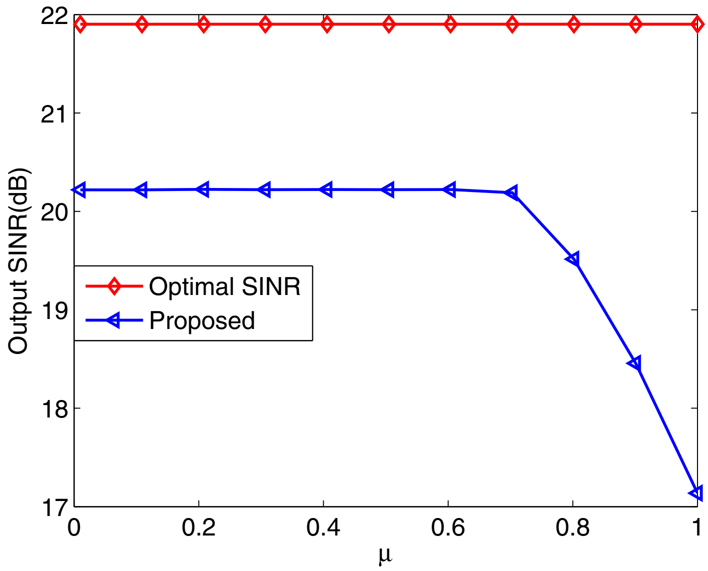

Figure 5: Output SINR versus μ.

From the above arguments, we only consider the direction mismatch of the desired signal. For further insight, we consider a more general type of mismatch, and the mismatched transmit and receive steering vectors are modeled as $\mathbf{a}_t = \bar{\mathbf{a}}_t(\theta_0) + \mathbf{e}$ and $\mathbf{a}_r = \bar{\mathbf{a}}_r(\phi_0) + \mathbf{e}$, respectively, where $\mathbf{e}$ is a random vector with i.i.d. zero-mean complex Gaussian random variables for both transmit and receive signals, components $e_i \sim CN\left(0, \sigma_e^2\right)$ for all i. In this example, σ_e^2 is chosen to be 0.1. Other parameters are the same as those used in Fig.1 and Fig.2 for Fig.6 and Fig.7, respectively. The output SINR performance of these algorithms versus the SNR and the number of snapshots are shown in Fig.6 and Fig.7, respectively. As shown, the proposed algorithm still has satisfactory performance when a general type of steering vector mismatch occurs, which implies that the proposed method is a good candidate for real applications.

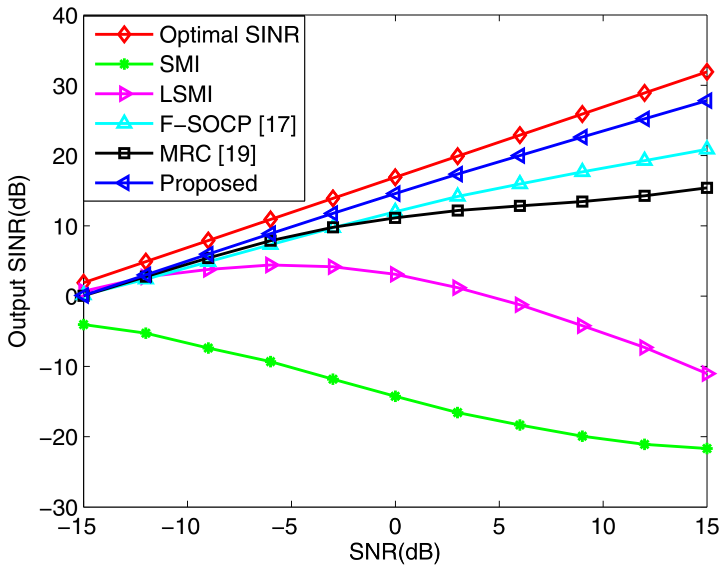

Figure 6: Output SINR versus SNR for general type mismatch.

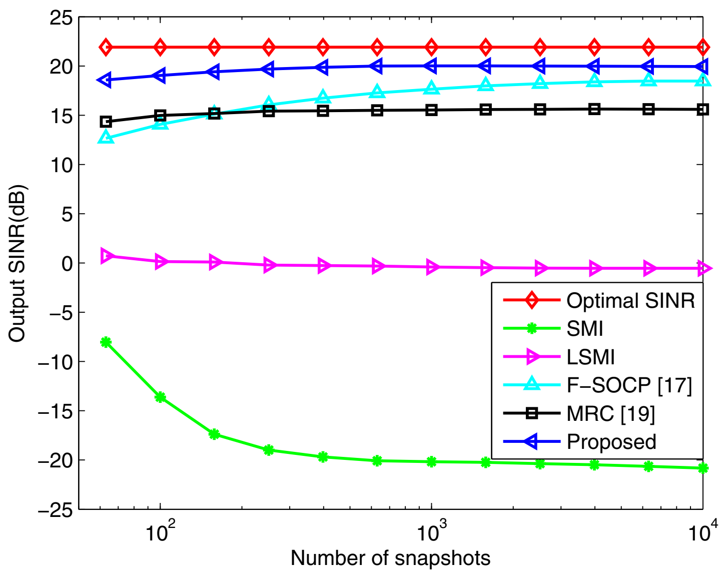

Figure 7: Output SINR versus number of snapshots for general type mismatch.

In this experiment, we study the sensitivity of the proposed algorithm to the steering vector random error. The performance of the proposed algorithm is displayed versus σ_e^2 in Fig.8. We can see that the proposed algorithm is sensitive to the steering vector error. The main reason is that the proposed algorithm imposes the norm constraints to the steering vectors, which may lead to an inaccurate approximation and the performance degradation of the beamformer. In addition, the proposed algorithm is based on the norm constraints, so the correlation coefficient constraint can be used to distinguish the desired signal region and interference region effectively. If the steering vector norms are violated severely, the correlation coefficient constraint may also be invalid. Therefore, the proposed algorithm is more suitable for the applications where steering vectors satisfy or approximate the norm constraint scenarios.

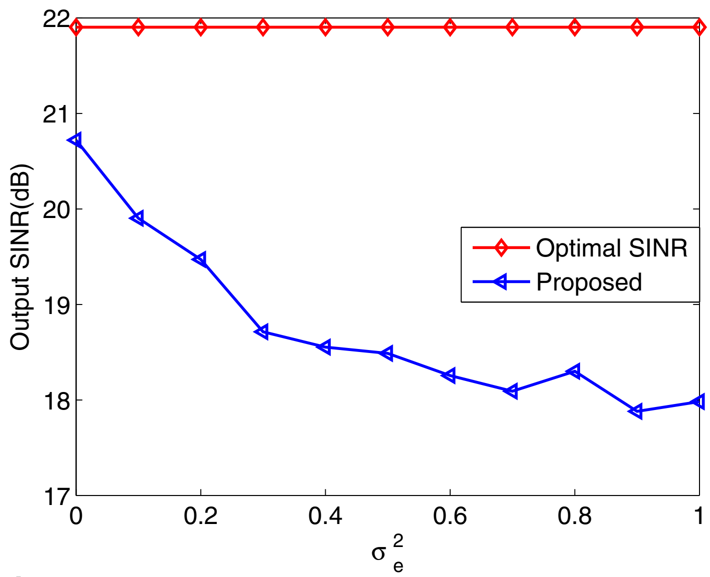

Figure 8: Output SINR versus σ_e^2 in the case of steering vector error.

5. Conclusion

In this paper, a novel robust adaptive beamforming technique with full DOFs for MIMO radar has been developed. We exploited an iterative algorithm to tackle the transmit/receive steering vector design problem. The cyclic optimization-based method involves the solutions of two relaxed problems. Based on the analyzed desired signal steering vectors and the shrinkage estimator processing, the interference-plus-noise covariance matrix can be effectively estimated via the matrix rank-constrained minimization method. Simulation results demonstrate that the proposed technique offers a better performance than several state-of-the-art algorithms.

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