Rapid Communications

Violations of a Leggett-Garg inequality without signaling for a photonic qutrit probed with ambiguous measurements

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We realize a quantum three-level system with photons distributed among three different spatial and polarization modes. Ambiguous measurements of the state of the qutrit are realized by blocking one out for the three modes at any one time. Using these measurements we construct a test of a Leggett-Garg inequality as well as tests of no-signaling-in-time for the measurements. We observe violations of the Leggett-Garg inequality that cannot be accounted for in terms of signaling. Moreover, we tailor the qutrit dynamics such that both ambiguous and unambiguous measurements are simultaneously nonsignaling, which is an essential step for the justification of the use of ambiguous measurements in Leggett-Garg tests.

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I. INTRODUCTION

Macrorealism, as defined by Leggett and Garg [1], posits that a macroscopic system will exist in a well-defined state at all times, and that this state can be measured without disturbing it (the assumption of noninvasive measurability). From these assumptions follow the Leggett-Garg inequalities (LGIs) [1–3], which hold under macrorealism but can be violated by quantum mechanics [4–11]. The same assumptions also imply the *no-signaling-in-time* (NSIT) equalities, which demonstrate the absence, on the statistical level, of signaling between measurements [12–15]. Having NSIT hold completes the formal similarity between the temporal LGI and spatial Bell tests [16]. Violations of a LGI without NSIT, however, provides a convenient loophole for a macrorealist to explain the experiment in terms of the signaling of invasive measurements.

It has been shown theoretically that when unambiguous, projective measurements are used, violations of LGIs are always accompanied by violations of NSIT [17,18], and thus the use of projective measurements is generally problematic in this context. In Ref. [17], however, George *et al.* realized LGI violations without signaling through use of measurements that were *ambiguous*, i.e., measurements where the individual results do not completely reveal the state of the system [19]. Quantum mechanically, such measurements are sometimes described as "semiweak." LGI violations with ambiguous measurements were also discussed in Refs. [5,20,21]. In Ref. [18], a general framework for LGI tests with ambiguous measurements was discussed. There it was shown that the derivation of LGIs that use data from ambiguous measurements rely on an assumption that equates the invasive influence of the ambiguous measuring device to that of an unambiguous one acting on the same system. While it is perhaps hard to see how this assumption might hold in general, it has the clear implication that an LGI test in which ambiguous measurements are observed to be nonsignaling is only consistent with its own assumptions if the corresponding set of unambiguous measurements on the same system is also observed to be nonsignaling.

In this Rapid Communication, we report on LG experiments with single photons that implement a three-level quantum system measured with both ambiguous and unambiguous measurements. We test LGIs and NSIT equalities with both sets of measurements. In the case of unambiguous measurements, we confirm that all observed LGI violations can explained in terms of signaling. In the ambiguous case, however, we show that it is possible arrange the time evolution of our three-level system such that the ambiguously measured LGI is violated while at the same time NSIT is satisfied for both measurement types. In this case, we obtain an LGI violation that is consistent both with assumption of noninvasive measurability as well as the assumptions implicit in the usage of ambiguous measurements in this type of test.

This Rapid Communication proceeds as follows. In Sec. II we describe what is meant here by ambiguous measurements and in Sec. III we describe their experimental realization for our photonic qutrit. In Sec. IV we consider the nonviolations of the LGI with unambiguous measurements when signaling is taken into account. Section V contains our main results where we employ ambiguous measurements to violate a LGI while all no-signaling constraints are fulfilled. We conclude with discussions in Sec. VI.

II. AMBIGUOUS MEASUREMENTS

We begin by discussing the meaning of unambiguous and ambiguous measurements following Ref. [19], which

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establishes these concepts identically in both quantum and classical contexts.

In our three-level system, unambiguous measurements reveal one of three distinct results $n \in \{A,B,C\}$, and since these results are repeatable, we associate n with the "realistic" system state. Let us denote the probability that we measure result n as $P(n)$.

On the other hand, ambiguous measurements do not reveal complete information about the state of the system and are nonrepeatable. The particular scheme we will consider here is a set of three individual measurements, each of which serves to exclude one of the three system states. Thus, the measurement outcomes are $\alpha \in \{B \cup C, A \cup C, A \cup B\}$ and our experiments record probabilities such as $P(B \cup C)$, etc.

The probabilities obtained with the two different measurement setups are clearly related. Elementary probability theory gives $P(B \cup C) = P(B) + P(C)$, for example, which could easily be verified experimentally. Given the complete set of three ambiguous probabilities, a macrorealist would have no qualms inferring the probabilities that the system "really was in" such as A, by calculating

$$P'(A) = \tfrac{1}{2}P(A \cup B) + \tfrac{1}{2}P(A \cup C) - \tfrac{1}{2}P(B \cup C), \quad (1)$$

and so on [22]. Here we maintain the notation P' for a probability inferred from ambiguous measurements, as opposed to one that is measured directly.

Quantum mechanically, unambiguous and ambiguous measurements are realized respectively as a complete set of projection operators and a more general positive operator-valued measure (POVM) that implements a semiweak measurement [23]. In the case of a measurement of the systems state at a single time, calculating either with quantum mechanics or classically, $P'(A)$ and $P(A)$ will clearly agree. However, when sequential measurements are made on the same system, the difference in the quantum case between directly measured probabilities (P) and those that are inferred (P') becomes critical.

III. EXPERIMENTAL SETUP

Our experiment realizes a quantum three-level system, or qutrit, with single photons traveling through the apparatus depicted in Fig. 1. As the general setup is similar to previous work [10], we refer the reader to these for more details on the implementation of the various components discussed below.

The basis states of the qutrit, $|A\rangle = (1,0,0)^{\mathrm{T}}$, $|B\rangle = (0,1,0)^{\mathrm{T}}$, and $|C\rangle = (0,0,1)^{\mathrm{T}}$, are respectively encoded by the horizontal polarization of the heralded single photons in the upper mode $|HU\rangle$, the vertical polarization of the photons in the upper mode $|VU\rangle$, and the horizontal polarization of the photons in the lower mode $|HD\rangle$. For this experiment, the photons are prepared in the initial state $|C\rangle$. The unitary evolution of the qutrit state is realized by a sequence of half-wave plates (HWPs) and subsequent birefringent calcite beam displacers (BDs) that realize two unitary operators $U_{21}(\theta_1,\chi_1,\phi_1)$ and $U_{32}(\theta_2,\chi_2,\phi_2)$ that are nominally identical and can be decomposed as [24,25]

$$U(\theta,\chi,\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\chi & 0 & \sin\chi \\ 0 & 1 & 0 \\ -\sin\chi & 0 & \cos\chi \end{pmatrix} \times \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

Throughout the experiment, measurement of the photon state at t_3 is always performed projectively. This is accomplished by BD_9 that maps the basis states of qutrit into three spatial modes followed by single-photon avalanche photodiodes (APDs), in coincidence with the trigger photons. The probability of the photons being measured in $|A\rangle$, $|B\rangle$, or $|C\rangle$ is obtained by normalizing photon counts in the certain spatial mode to total photon counts. The count rates are corrected for differences in detector efficiencies and losses before the detectors. We assume that the lost photons would have behaved the same as the registered ones (fair sampling) [26]. Experimentally this trigger-signal photon pair is registered by a coincidence count at APD with a 3 ns time window. Total coincidence counts are about 14 000 over a collection time of 7 s.

In the forms we consider them here, the Leggett-Garg and NSIT tests require two different types of measurement of time t_2, i.e., between the two unitary evolution operations. The unambiguous measurement is realized by placing blocking elements into the optical paths [10,27]. With, for example, the channels B and C blocked, the joint probability $P(n_3,n_2 = A)$ is obtained without the measurement apparatus having interacton with the photon. In our experiment, this blocking is realized by a polarizing beam splitter (PBS) followed by beam stoppers. The PBS is used to map the basis states of qutrit to three spatial modes and the beam stoppers are used to block photons in two of the three spatial modes and let the photons in the remaining one pass through. By inserting the HWPs before and after the PBS, we can block any two of the channels and let the photons in the remaining one pass through for the next evolution.

The ambiguous measurement is realized in a similar fashion but this time we block just one mode and let photons propagate forward from the remaining two. With channel C blocked, for example, and with projective measurements at t_3, we obtain the joint probability $P(n_3,n_2 = A \cup B)$, where the inference that the photon must have occupied either state A or B at time t_2 is the essential ambiguity in this scheme.

IV. LGI WITH UNAMBIGUOUS MEASUREMENTS

We first consider an LGI test with unambiguous measurements. In the case where the state preparation is elected to coincide with the first measurement [8,10,10,28,29], the LGI correlator reads

$$K = \langle Q_2\rangle + \langle Q_3 Q_2\rangle - \langle Q_3\rangle. \quad (3)$$

The expectation value $\langle Q_3\rangle$ is obtained using time-evolution operators U_{21} and U_{32} applied sequentially, followed by a projective measurement. This yields the probabilities $P(n_3)$ and $\langle Q_i\rangle = \sum_{n_i} q(n_i)P(n_i)$. Here the quantities $q(A) =$

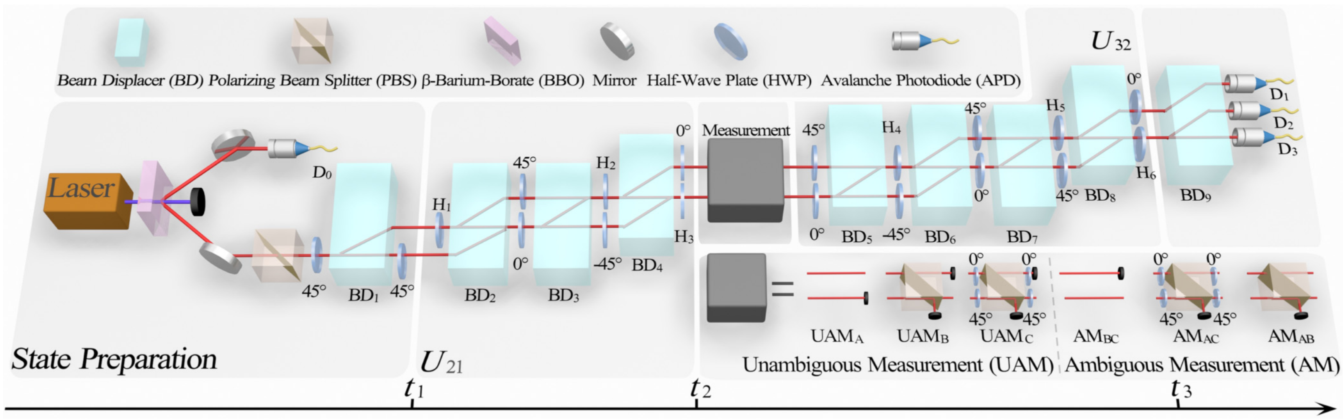

FIG. 1. Experimental setup for the test of LGI. The heralded single photons are created via type-I spontaneous parametric down-conversion in a β-barium-borate (BBO) nonlinear crystal and are injected into the optical network (see figure for acronyms). The first polarizing beam splitter (PBS), half-wave plates (HWPs) at 45° and BD_1 are used to generate the initial qutrit state. The evolution operations U_{21} and U_{32} are realized by HWPs and beam displacers (BDs). The projective measurement at time t_3 is realized via the last BD which maps the basis states of the qutrit into three spatial modes. Detecting heralded single photons means in practice registering coincidences between the trigger detector D_0 and each of the detectors for measurement D_1, D_2, and D_3. The unambiguous and ambiguous measurements at time t_2 are realized by blocking two channels or one channel at a time.

$-q(B) = q(C) = 1$ define a mapping from observed state n to dichotomic variable Q [30]. The remaining correlation functions are obtained as $\langle Q_3 Q_2\rangle = \sum_{n_3,n_2} q(n_3)q(n_2)P(n_3,n_2)$ and $\langle Q_2\rangle = \sum_{n_3,n_2} q(n_2)P(n_3,n_2)$ with the joint probabilities $P(n_3,n_2)$ being obtained from experimental runs in which evolution operators U_{21} and U_{32} have projective measurements situated both between and after them.

Under the standard LG assumptions, this correlator obeys $K \leqslant 1$. However, we will consider the form of the LGI given in Ref. [18], which avoids the noninvasive-measurability assumption. In this case, we obtain the "modified LGI," which reads

$$K \leqslant 1 + \Delta, \quad \Delta \equiv \sum_{n_3} |\delta(n_3)|. \tag{4}$$

Here

$$\delta(n_3) = P(n_3) - \sum_{n_2} P(n_3,n_2) \tag{5}$$

describes the amount of signaling from time t_2 to t_3. Under assumption of noninvasive measurability, this would be zero such that we have

$$\delta(n_3) = 0; \quad \forall n_3. \tag{6}$$

These are the NSIT equalities [12] and if they are satisfied, the modified LGI reduces to its original form $K \leqslant 1$. In the approach we pursue here, however, we take the quantities $\delta(n_3)$ to be obtained from experiment, and consider the modified LGI, Eq. (4), in this light.

Figure 2 shows a comparison of the measured values of K and $1 + \Delta$ from our experiment with unambiguous measurements. We have selected a particular choice of evolution operators such that, for these parameters, we find analytically that $K = (3 - \cos 2\theta_2)/2$, with θ_2 an adjustable evolution parameter. As Fig. 2 shows, this result is very closely matched by experiment. Error bars in this figure include both the statistical uncertainty and the error due to the inaccuracy of the wave plates [31]. The statistical errors based on the assumption of Poissonian statistics are relatively small. However, about 20 wave plates are used and each of them has an angle error of approximately 0.1°. These errors accumulate in a cascaded setup and we have simulated numerically their total effect with

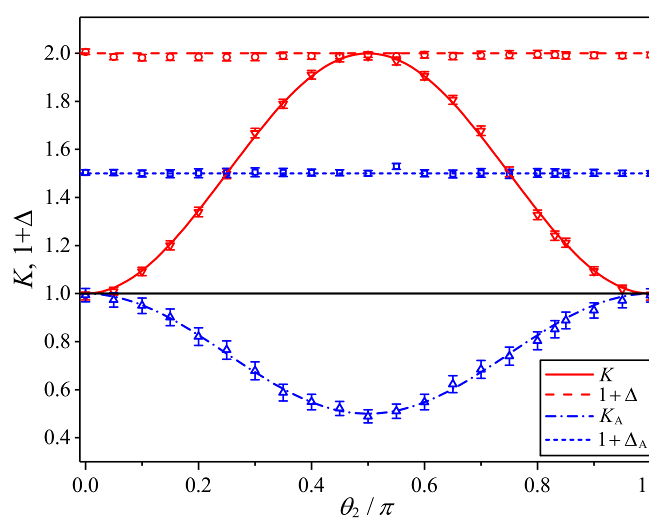

FIG. 2. Experimentally determined values of the LG correlator K and the corresponding right-hand side $1 + \Delta$ of the modified LGI, Eq. (4), with evolution parameters $\theta_1 = 0$, $\chi_1 = \chi_2 - \pi/2 = \pi/4$, $\phi_1 = \phi_2 = 0$ and a range of θ_2 values. These parameters are chosen to maxmize the value of the unambiguously measured K. However, although we have $K \geqslant 1$ for all values of θ_2, we have $K \leqslant 1 + \Delta$ throughout the tested range. Thus when the observed signaling is taken into account, the modified LGI, Eq. (4), is never violated. For completeness, we also plot the ambiguously measured K_A and $1 + \Delta_A$ for the same parameters. Here, too, we observe $K_A \leqslant 1 \leqslant 1 + \Delta_A$ and no violations are recorded. Theoretical predictions are represented by curves and lines, and the experimental results by symbols. Error bars include both the statistical uncertainty and the error due to the inaccuracy of the wave plate alignment.

a Monte Carlo method. These inaccuracies are sufficient to explain deviations from theoretical predictions.

The maximum value of K that we observe is 1.988 ± 0.016 at $\theta_2 = \pi/2$, which agrees well with the theoretical prediction of 2. This value represents an enhanced violation of the LGI, above the bound set for genuinely dichotomic measurements, as described in [30] and observed experimentally in [10]. It is clearly far in excess of the usual LGI macrorealistic bound of $K \leqslant 1$. For these parameters, we obtain the right-hand side of the LGI as $1 + \Delta = 2$ analytically, which is constant as a function of θ_2. This behavior is recovered by experiment and for $\theta_2 = \pi/2$ we obtain $1 + \Delta = 1.995 \pm 0.011$. Thus, while the observed value of K is clearly in excess of the standard bound, when the observed degree of signaling is taken into account, we find that the modified LGI, Eq. (4), still holds. This is in line with the theoretical results of Refs. [17,18] which forbid violations of Eq. (4) with projective measurements.

V. LGI WITH AMBIGUOUS MEASUREMENTS

Following Ref. [18], an LGI constructed with the ambiguous measurements has exactly the same form as before:

$$K_A \leqslant 1 + \Delta_A, \quad \Delta_A = \sum_{n_3} |\delta_A(n_3)|, \tag{7}$$

where the subscript A denotes quantities obtained from ambiguous measurements. These quantities have forms identical to those considered previously but with probabilities $P(n_3,n_2)$ replaced with those inferred from the ambiguous measurements. In particular, we obtain the joint probabilities $P'(n_3,A)$ in the same way as Eq. (1) and write

$$P'(n_3,A) = \tfrac{1}{2}P(n_3,A\cup B) + \tfrac{1}{2}P(n_3,A\cup C) - \tfrac{1}{2}P(n_3,B\cup C), \tag{8}$$

and similarly for the other two probabilities. The ambiguously measured no-signaling quantities are then $\delta_A(n_3) \equiv P(n_3) - \sum_{n_2} P'(n_3,n_2)$, and the correlation functions in K_A are the same as before with the replacement $P \rightarrow P'$. The ambiguously measured probabilities $P(n_3,\alpha)$ are obtained experimentally in exactly the same way as before, but with ambiguous measurements replacing the unambiguous one at t_2. Theoretically, they are obtained with a POVM as outlined in Ref. [18]. Note that the algebraic bound of $K = 3$ is never violated, irrespective of measurement type [5,30].

Results for K_A and $1 + \Delta_A$ for the parameter set in Sec. IV are shown in Fig. 2. In this case $K_A \leqslant 1 \leqslant 1 + \Delta_A$ and no violations of the ambiguously measured LGI are observed.

Figure 3, however, shows these quantities for a different set of evolution parameters, namely, $\theta_1 = 0.831\pi$, $\chi_1 = \chi_2 = 0.688\pi$, $\phi_1 = \phi_2 = 0.423\pi$, and $0 \leqslant \theta_2 \leqslant \pi$. For a significant range of θ_2 values, we obtain $K_A > 1$. Moreover, for $0.677\pi \leqslant \theta_2 \leqslant 0.983\pi$, we find that $K_A \geqslant 1 + \Delta_A$, and thus we find violations of the modified ambiguously measured LGI. The maximum violation is found at $\theta_2 = 0.831\pi$ with values $K_A = 1.483 \pm 0.031$, in close agreement with the theoretical prediction 1.464. Most importantly, at this value of θ_2 the signaling quantities are $\Delta = 0.019 \pm 0.020$ and $\Delta_A = 0.013 \pm 0.018$, both of which are, to within experimental uncertainty, essentially zero in accordance with theory which gives $\Delta = \Delta_A = 0$ exactly at this point.

While our experiment therefore satisfies no signaling for both sets of measurements, and therefore also shows equality of signaling between them, we can understand the origins of the LGI violations in our scheme by looking at the individual inferred probabilities $P'(n_3,n_2)$. The complete set of these is plotted in Fig. 4 for the parameters of Fig. 3. Crucially, for all values of θ_2 at least three of the inferred probabilities are *negative*. For example, $P'(A,A) = -0.109$ for all θ_2.

These results make it clear that, in quantum-mechanical terms, these inferred probabilities are quasiprobabilities. The role of negative quasiprobabilities in violations of LGIs has been discussed a number of times [2,16,32–34]. In addition, anomalous weak values have been directly connected to LGI violations [5,35], and such values imply the existence of negative quasiprobabilities [36]. The link between negative quasiprobabilities and contextuality reported in Refs. [37,38] also implies a connection between the LGI violations and contextuality.

VI. DISCUSSION

We have described here the experimental violation of the LGI using a realization of a three-level system with single photons. We have shown that it is possible to obtain violations of the modified LGI, Eq. (7), that takes into account the observed degree of signaling. Violations of this inequality were observed for a range of our evolution parameter θ_2. Moreover, at the particular point $\theta_2 = 0.831\pi$, both signaling quantities Δ and Δ_A were found to be zero. At this point, then, NSIT is obeyed

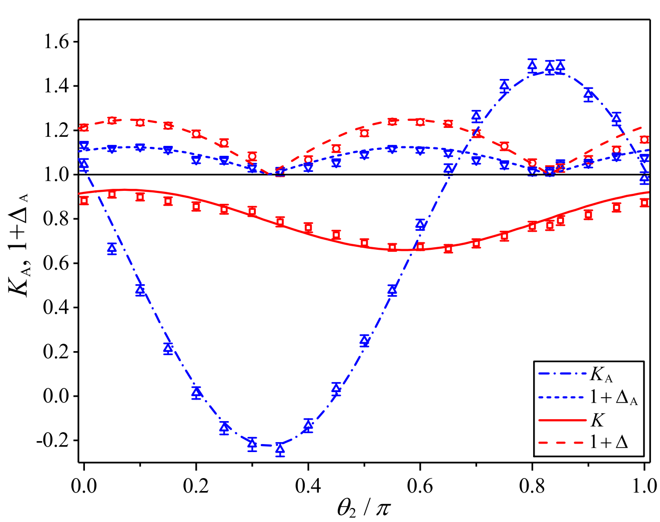

FIG. 3. Experimentally determined values of the LG correlator and upper bound for a second set of parameters: $\theta_1 = 0.831\pi$, $\chi_1 = \chi_2 = 0.688\pi$, $\phi_1 = \phi_2 = 0.423\pi$ and a range of values of θ_2. In this case, the focus is on the ambiguously measured correlator K_A and its bound $1 + \Delta_A$. Results are also shown for the unambiguously measured K and $1 + \Delta$. For $\theta_2 = 0.831\pi$, we observe a value of $K_A = 1.483 \pm 0.031$ while both $1 + \Delta_A$ and $1 + \Delta$ are close to 1, within experimental certainty. At this point, then, we observe LGI violations in the absence of signaling for both measurement types. Other details as in Fig. 2.

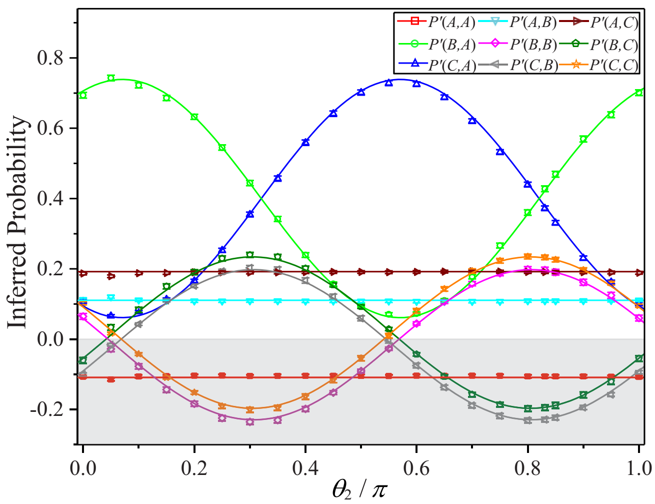

FIG. 4. Experimentally determined values of the inferred joint probabilities $P'(n_3,n_2)$ (for $n_2,n_3 = A,B,C$) as a function of the parameter θ_2 (the other parameters fixed as in Fig. 3). Theoretical predictions are represented by curves and lines, and the experimental results by symbols. That P' takes on negative values is indicative of the quantum-mechanical quasiprobabilistic nature of these quantities.

by both the ambiguous and unambiguous measurements. This is particularly important because, according to Ref. [18], the derivation of Eq. (8) and hence Eq. (7) relies on the assumption that both unambiguous and ambiguous measurements are "equally invasive" and therefore must exhibit the same degree of signaling, i.e., $\Delta = \Delta_A$ [39]. Only at the point $\theta_2 = 0.831\pi$, are the dynamics of our three-level system such that we have $\Delta = \Delta_A$. At this point then the use of the ambiguous measurements to construct the LGI for "macrorealistic" state n_i is justified.

Due to its use of photons, this is a proof-of-principle experiment and cannot be viewed as a test of *macroscopic* realism, as originally envisaged by Leggett and Garg but rather of *microscopic* realism [40,41] as has been famously tested in Bell-type experiments [42]. Nevertheless, the general principle used for constructing ambiguous LGI tests without signaling could potentially be scaled up to larger, massive objects, perhaps most directly in molecular interference experiments [43].

Despite the enhanced no-signaling features of our experiment, and in common with all known Leggett-Garg-type tests, possible loopholes exist for a macrorealist determined to hold their position. The finding that some of the inferred probabilities, $P'(n_3,n_2)$, are negative would presumably lead the macrorealist to reject the possibility that it is possible to learn anything about the unambiguous state of the system from the ambiguous setup. This position, however, would require a significant degree of contrivance given that both measurements are known to be individually nonsignaling.

ACKNOWLEDGMENTS

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