A study of the rupture behavior of a ship side plate laterally punched by a full-shape bulbous bow indenter

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Abstract

The rupture behaviors of a ship side plate, including fracture initiation and crack propagation, subjected to lateral impact loading by a full-shape bulbous bow indenter are studied in this paper through analytical methods and verified by experimental methods. Analytical expressions are built to assess the whole collapse resistance, especially the initial fracture prediction and crack propagation resistance of the side plate. Experiments are conducted to validate the proposed analytical method. The specimens are quasi-statically punched at the mid-span by a combine-shaped bulbous indenter, which is specifically designed to distinguish the fracture initiation and crack propagation behaviors. The results from these two methods match well in terms of the resistance-penetration responses, thus proving the rationality of the proposed analytical method. In addition, the experimental observations demonstrate that the crack propagation resistance is not related to the crack number generated in the plate because the resistance arises from bending and tension effects, which are relevant to the circumferential parameters, not the crack number. Moreover, in the crack propagation process, material failure of the plate is due to the tension effect at the crack tips with high stress concentration, and the plate can still absorb energy significantly after rupture.

Keywords: Collision; Ship side plate; Rupture behavior; Analytical method; Experiment; Crack propagation.
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>half-length of the rectangular plate</td>
</tr>
<tr>
<td>$b_0$</td>
<td>half-width of the rectangular plate</td>
</tr>
<tr>
<td>$F_c$</td>
<td>vertical crack propagation resistance of the plate</td>
</tr>
<tr>
<td>$F(\phi_c)$</td>
<td>punch force of the plate before fracture</td>
</tr>
<tr>
<td>$k$</td>
<td>strength coefficient of the material</td>
</tr>
<tr>
<td>$n$</td>
<td>work hardening exponent of the material</td>
</tr>
<tr>
<td>$r_0$</td>
<td>rolling radius of the plate</td>
</tr>
<tr>
<td>$R_b$</td>
<td>radius of the spherical punch</td>
</tr>
<tr>
<td>$R_{bp}$</td>
<td>radius of the circumferential bending hinge line</td>
</tr>
<tr>
<td>$R_{tp}$</td>
<td>radius of the circumferential tension plate</td>
</tr>
<tr>
<td>$t_p$</td>
<td>initial plate thickness</td>
</tr>
<tr>
<td>$u$</td>
<td>in-plane displacement of the plate</td>
</tr>
<tr>
<td>$w_c$</td>
<td>vertical displacement of the outmost contact point</td>
</tr>
<tr>
<td>$w_i$</td>
<td>indentation depth of the indenter</td>
</tr>
<tr>
<td>$w_{ic}$</td>
<td>indentation depth of the indenter in the crack propagation process</td>
</tr>
<tr>
<td>$w_p$</td>
<td>penetration depth of the plate</td>
</tr>
<tr>
<td>$w_{p,f}$</td>
<td>critical penetration depth of the plate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>half spreading angle of the cone</td>
</tr>
<tr>
<td>$\theta$</td>
<td>rotation angle of the ruptured plate</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>rotation angle of the plate from radial bending to circumferential bending</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>indenter wrapping angle at the outmost contact point</td>
</tr>
<tr>
<td>$\phi_{cf}$</td>
<td>critical wrapping angle to fracture, corresponding to $w_{p,f}$</td>
</tr>
<tr>
<td>$\phi_{c,f}$</td>
<td>wrapping angle when $\partial F(\phi_c)/\partial \phi_c=0$</td>
</tr>
<tr>
<td>$\varepsilon_\theta$</td>
<td>circumferential strain</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>radial tension strain</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>horizontal distance from the outmost contact point to plate edge</td>
</tr>
<tr>
<td>$\sigma_{0p}$</td>
<td>flow stress of the plate</td>
</tr>
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</table>

### 1. Introduction

During a head-on collision with a ship side structure, the ship side shell will very likely endure rupturing behavior. In some serious situations, as shown in Fig. 1, the striking bulbous bow will induce dramatic rupturing of the side shell. When the striking energy is high, the rupture behavior can be divided into two processes: fracture initiation and crack propagation. This accidental scenario will cause serious plating rupture and lead to the opening of a large hole that is consistent not only with the shape of the bulbous bow but also with the bow structure behind the bulbous nose. In most studies of plating rupture due to bulbous bow striking, the rupturing behavior was investigated by primarily focusing on the fracture initiation, which conformed to
the shape of a bulbous nose; however, the subsequent crack propagation behaviors related to the bow structure behind the bulbous nose were often neglected (Calle and Alves, 2015; Marinatos and Samuelides, 2015; Calle et al., 2017; Gruben et al. 2017). Head-on collision accidents have clearly demonstrated that the ultimate rupturing shapes of the side shell are normally determined by the combination of the bulbous nose and the bow structure behind the bulbous nose, see in Fig. 1. In addition, the rupture behavior of the ship side plate is more complicated when the two processes are taken into consideration. To further clarify the rupture behavior, this paper addresses a study of shell plating rupture behavior during the whole deformation process and refers to three key points that are often considered during crashworthiness structural design.

(1) How can the crack initiation and crack propagation of shell plating be accurately described by analytical calculations?

(2) Does the uncertainty of crack occurrence play an important role in affecting the energy dissipation capability of the side structure?

(3) What is failure mechanism and energy dissipation ability of the ship side plate when suffering a head-on collision with the structure behind the bulbous nose?

To answer these three questions, in this paper, the rupturing behavior of the side shell under the scenario of a high-energy head-on collision by a bulbous bow is analyzed, and analytical methods for estimating the crack initiation and crack propagation are proposed. The structural behavior of the side shell will then be summarized based on the results of the investigation.

The commonly used approaches in ship collision investigations are classified as experiment, numerical simulation and simplified analytical method (Hong, 2009). Compared with the first two methods, simplified analytical method is the preferred tool in the pre-design stage because these methods can most rapidly assess the crashworthiness of ship structures (Amdahl, 1983; Zhang, 1999; Gao et al., 2014; Sun et al., 2015; Yu et al., 2015). Extensive analytical studies have been conducted to forecast the lateral resistance and fracture initiation of a single plate suffering lateral indentation by a sphere indenter (Simonsen and Lauridsen, 2000; Lee et al., 2004; Gong et al. 2015). Nevertheless, there are no globally acknowledged methods for plating rupture initiation in ship collision and grounding analysis. Moreover, analytical solutions for crack propagation resistance after plate fracture have seldom
been directly proposed. One method proposed by Wang et al. (2000) is based on the plate tearing mechanisms, where the force needed to drive the cone into the perforated plate is related with the crack number and the instantaneous crack length. Therefore, the main purposes of the analytical method in present study is to build analytical equations for fracture prediction of the ship side plate and crack propagation resistance after plate fracture, and to evaluate how much the crack number influences the resistance in crack propagation process.

When it comes to the crack propagation behavior of a ship side plate laterally punched by a bulbous bow, previous studies have mostly produced qualitative conclusions. For instance, based on stiffened plate punching experiments, Alsos and Amdahl (2009) stated that a ship side plate can exhibit significant post-fracture resistance after the onset of fracture. Alves et al. (2014) reported that subtle differences in the crack shape have a strong influence on the energy absorption capability of the ruptured structure after crack initiation. Recently, Morin et al. (2017) clarified that crack propagation in the plate is dominated by slant (shear) failure, which differs from plate initial fracture caused by ductile failure. The phenomenon of crack propagation in the side plate can be clearly observed in double hull penetration experiments in which the outer hull was torn into several petals due to excessive indentation. Nevertheless, in simulating the failure mechanisms of fracture initiation and crack propagation, the corresponding failure criteria were generally not distinguished (Paik et al. 1999; Karlsson et al., 2009; Zhang et al. 2018). All efforts have indicated that the crack propagation behavior of the plate warrants attention. However, this behavior has not been adequately resolved because the crack propagation behavior occurs immediately after plate fracture and the failure mode of crack propagation varies greatly compared to that of crack initiation.

For simplicity, the energy dissipation due to crack propagation and the failure mechanism are studied in a relatively simpler manner, e.g., in-plane tension. Large-scale ductile crack propagation experiments were conducted by Simonsen and Törnqvist (2004) with the purpose of developing and validating macroscopic crack propagation criteria in plate structures. Good agreement between the experiments and numerical simulations proved that a constant average critical fracture strain modeled with shell elements can adequately predict the crack propagation resistance of plates. Moreover, the experimental data were further used to validate novel numerical
simulation techniques by other researchers (Kõrgesaar and Romanoff, 2013; Woelke et al., 2015). Current investigations of crack propagation in the plate mostly refer to the in-plane loads, whereas such idealistic conditions rarely occur in real situations due to various load conditions and inconsistencies in structural arrangements, such as the form of stiffeners, welds, etc. Moreover, the crack propagation behavior of ship side plate due to lateral punch in terms of the three key points previously mentioned still requires further in-depth investigation. Therefore, experiments are performed in this study to specifically investigate the rupture behavior of a ship side plate punched by the bow structure behind the bulbous nose and to validate the accuracy of the proposed analytical method, as experiments can provide reliable data in terms of resistance response, deformation mode and fracture patterns (Kiliclar et al. 2016; Gruben et al., 2017; Morin et al., 2017; Al-Tamimi et al. 2018; Cheng and Lee 2018).

The present work is undertaken to investigate the whole rupture behavior of a scaled ship side shell laterally punched by a full-shape bulbous bow indenter by using a simplified analytical method and experimental method. The rest of this paper is structured as follows. Section 2 presents analytical predictions for the resistance of the plate for the whole collapse process. In section 3, experimental details of the punch indentation are described. Section 4 describes the verification of analytical predictions by experiments. In section 5, several aspects related to the questions proposed in the Instruction are discussed. Conclusions are drawn in section 6.

2. Analytical predictions

This chapter presents analytical predictions for the whole collapse process of a ship side plate suffering bulbous bow impact, including the processes of large deformation, fracture, and out-of-plane crack propagation.

In a severe ship bulbous bow-side collision scenario, as shown in Fig. 2, ship side plate will be punched and penetrated by the bulbous bow. In developing the analytical solutions, several assumptions are made:

1) The bulbous bow is assumed to vertically impact on ship side plate at the mid-span between the web girders;

2) The web girders are assumed to be stiff enough to constrain the boundary of the outer side plate;

3) The bulbous bow is assumed to be rigid and the shape of the bulbous bow is simplified as a conical shape.
Based on the assumptions, Theoretical deformation modes and the derived formulae for the plate due to large deformation and crack propagation are described below.

2.1. Large stretching deformation resistance

The deformation resistance of the plate is described in this section. In the theoretical analysis, a rectangular plate with dimensions of $2a_0 \times 2b_0$ ($a_0$ and $b_0$ are the half-length and half-width of the rectangular plate, respectively) constrained by the web girders is extracted as the struck object, and the top of the bulbous bow is treated as a sphere. Moreover, the plate-sphere indentation system can be simplified as an axis-symmetric problem (Simonsen and Lauridsen, 2000; Lee et al. 2004; Gong et al. 2015). Thus, the deformation mode of a clamped thin circular plate punched by a spherical indenter is shown in Fig. 3. The radius of the plate is the half-width of the rectangular plate $b_0$, and the radius of the sphere is denoted as $R_b$. The total penetration depth of the plate is denoted as $w_p$. Point C is the outermost contact point between the plate and the indenter, and $\phi_c$ is the angle from the center of the indenter to point C.

The plate is assumed to suffer mostly membrane stretching, and the bending effect is neglected. In addition, the material element of the plate displaces only vertically in the large deformation process. Thus, the circumferential strain can be neglected ($\varepsilon_\theta = 0$) (Simonsen and Lauridsen, 2000; Lee et al. 2004; Gong et al. 2015). And in the present paper, the radial tension strain of the plate $\varepsilon_r$ is expressed as

$$\varepsilon_r = 1 / \cos \phi - 1$$

where $\phi$ is the angle between the horizontal line and the tangent line of a point on the plate.

Considering the power-law relationship of the plate material, the tension force $F_{ct}$ supplied by the plate at point C can be expressed as

$$F_{ct} = 2\pi R_b \sin \phi_c t_p \cos \phi_c k(\varepsilon_r^n)$$

where $k$ and $n$ are the strength coefficient and work hardening exponent of the material, respectively, and $t_p$ is the initial plate thickness.

Thus, substituting Eq. (1) into Eq. (2), the indentation force supplied by the plate in the vertical direction $F(\phi_c)$ can be obtained as

$$F(\phi_c) = F_{ct} \sin \phi_c = 2\pi k t_p R_b (1 / \cos \phi_c - 1)^n \cos \phi_c \sin^2 \phi_c$$

Moreover, the deformed plate is assumed to be expressed by a parabola in the $\rho$-$w$ plane.
coordinate system in current study, where \( \rho \) and \( w \) are the horizontal and vertical distances from any point on the plate to the plate boundary. Moreover, the symmetrical axis of the parabola is assumed to be horizontal to the plate tip. Thus, the deformation shape can be expressed as

\[
\rho(w) = aw^2 - 2aww
\]  \( (4) \)

where \( a \) is the coefficient determined in Eq. (8).

The coordinate values at point C can be obtained as

\[
w_\rho = w_\rho - R_e + R_e \cos \varphi_c
\]  \( (5) \)

\[
\rho_c = h_0 - R_e \sin \varphi_c
\]  \( (6) \)

In addition, point C is the tangent point of the plate and the indenter. Thus, Eq. (4) satisfies the following relation:

\[
\rho(w_c) = \frac{1}{\tan \varphi_c}
\]  \( (7) \)

\( a \) can be obtained by substituting Eq. (5) and Eq. (7) into Eq. (4):

\[
a = \frac{1}{2R_e(\cos \varphi_c - 1)\tan \varphi_c}
\]  \( (8) \)

Point C is also the intersection point of the plate and the indenter, and thus Eq. (4) also satisfies the relation

\[
\rho(w_c) = \rho_c
\]  \( (9) \)

Substituting Eqs. (5), (6), (8) and (9) into Eq. (4), the total penetration depth of the plate can be obtained as

\[
w_\rho(\varphi_c) = \sqrt{2(h_0 - R_e \sin \varphi_c)R_e(1 - \cos \varphi_c)\tan \varphi_c + R_e^2(1 - \cos \varphi_c)^2}
\]  \( (10) \)

By using Eq. (3) and Eq. (10), the penetration depth and corresponding values of the vertical resistance can be determined with a wrapping angle \( \varphi_c \).

2.2. Fracture prediction

Analytical fracture prediction of the ship side plate is crucial for estimating energy dissipation and structural resistance. This section proposes an analytical method that can be used to predict the fracture initiation of a side shell under a bulbous bow striking scenario.

There are two main methods for forecasting the failure of the plate. One is to find the maximum material strain that can give the penetration to fracture. However, this method requires extra calibrations by numerical simulations (Lee et al. 2004) or experiments (Gong et al. 2015), which is not convenient. The other method is to consider failure from a stability point of view (Yu, 1996); in this method, the point of plate failure is assumed to be relevant to the peak of the resistance-penetration curves,
The latter method is adopted in this study to derive the critical penetration depth. The accuracy of this method is strongly related to the function applied to express the tension strain. And the difference of the current method is the solution of the tension strain, which can lead to different expressions of the critical penetration depth. Therefore, a correction coefficient is added in the current method to ensure the accuracy of the prediction result.

Note that Eq. (3) is given as a function of the wrapping angle \( \varphi_c \). Thus, the critical penetration depth \( w_{p,f} \) is determined by the critical value of \( \varphi_{c,f} \), which satisfies \( \frac{\partial F}{\partial \varphi_c} = 0 \). The equation can be obtained as

\[
3\cos \varphi_{c,f}^2 + n \cos \varphi_{c,f} + n - 1 = 0 \tag{11}
\]

The solution to Eq. (11) can be approximated by the expression

\[
\cos \varphi_{c,f} = \frac{1 - n}{\sqrt{3}} \tag{12}
\]

Substituting Eq. (12) into Eq. (10) and removing the relatively small quantities, \( w_{p,f} \) can be approximated and expressed as

\[
w_{p,f} = c_1 \sqrt{(7n + 0.76)h_n R_p} \tag{13}
\]

where \( c_1 \) is the correction coefficient.

\( c_1 \) has been calibrated by model tests in recent years with a wide range of plate size and indenter scantling values. Table 1 summarizes the critical penetration depth from the experiments and analytical predictions. When \( c_1 \) is equal to 0.5, the deviation of the critical penetration depth between the experiment and analytical prediction is less than 8%. Importantly, the experiments conducted by Liu et al. (2009) were low-velocity impact tests, and the experimental data for an aluminum plate were used for comparison due to its strain-rate insensitivity. Moreover, in the experiments conducted by Kõrgesaar et al. (2018) and Gruben et al. (2017), the tested specimens were stiffened plates. The influence of the attached stiffeners on the critical penetration depth for the plate was considered to be small. In general, the selected experiments are reasonable for calibrating the analytical method. The acceptable match between the experimental and analytical results illustrates that the calibrated value of \( c_1 \) is appropriate.

2.3. Crack propagation resistance

After the plate is fractured due to indentation by the bulbous bow nose, further
indentation by the structure behind the bulbous nose can prompt crack propagation and the opening of a large hole in the ship side plate. The shape of the bow structure after the bulbous nose is simplified as a cone. Thus, the energy dissipation mechanism and lateral resistance force for the ruptured plate punched by a cone are examined in this section.

Similar to the large stretching deformation mode of the plate shown in Fig. 3, the crack propagation mode for the side plating is also treated as axis-symmetric. Fig. 4 depicts the deformation mode of the ruptured plate. The two deformation modes are bridged by two values as shown in Fig. 4. One is the radius of the necking circle when the plate is fractured, denoted as \( R_i \), which can be expressed as \( R_i = b_0 - \rho_c \), where \( \rho_c \) is obtained by Eq. (6). The other is the rotation angle of the plate when the plate is initially fractured, denoted as \( \theta \), which is assumed to maintain constant in the perforating process. According to the geometric relation, \( \theta \) can be expressed as

\[
\tan \theta = \frac{w_{p_f} - R_b(1 - \cos \varphi_{cf})}{b_0 - R_b \sin \varphi_{cf}},
\]

(14)

where \( \varphi_{cf} \) is the corresponding wrapping angle of \( w_{p_f} \).

Moreover, Fig. 4 illustrates the deformation and crack propagation process of the ruptured plate laterally punched by a cone. Initially, radial bending of the plate is generated at point \( K_1 \), and the subsequent bending of the plate can lead to in-plane displacement \( u \), which will result in a circumferential tension. In addition, point \( K_2 \) is assumed to be horizontal to the crack tips, which means that the tension effect is limited in the arc of \( K_1K_2 \). Beyond point \( K_2 \), the ruptured plate is separated into several petals by the cracks. When the petals contact the cone (point \( K_3 \)), each petal of the plate will suffer circumferential bending because the ruptured plate will finally deform according to the shape of the indenter based on observations of actual collision scenarios (Fig. 1) and model tests (Wang et al. 2000; Zhang et al. 2018). For reference, a sketch for the top view of the ruptured plate is plotted in Fig. 5. The solution process for the lateral resistance is described below.

The rate of membrane energy can be expressed as

\[
\dot{E}_{\text{m,p}} = \int \int \sigma_{op} \dot{e}_\theta dS_{mp},
\]

(15)

where \( S_{mp} \) is the area of the plate that suffers tension, \( \dot{e}_\theta \) is the circumferential tension strain rate, \( \sigma_{op} \) is the flow stress of the plate and can be calculated by \( \sigma_{op} = (\sigma_y + \sigma_u)/2 \) (Hong, 2009; Jones, 2014). \( \sigma_y \) and \( \sigma_u \) are initial yield stress and ultimate tensile stress.
of the material, respectively. Note that the rigid-plastic material instead of the strain-hardening material is used to derive the resistance in the crack propagation process for calculation convenience.

The circumferential tension strain can be expressed as

$$\varepsilon_{\theta} = \frac{u}{R_p}$$  \hspace{1cm} (16)

where $R_p$ is the radius of the plate that experiences circumferential stretching and is appropriated as a constant distance from point $T'$ to the $w$-axis. $u$ and $R_p$ can be obtained through geometric relations and they are expressed as

$$u = r_0 \left(1 - \cos \beta\right) \sin \alpha \cos \zeta,$$

$$R_p = R + \frac{w_i}{\tan \alpha - \tan \theta}$$  \hspace{1cm} (17)

where $r_0$ is the radius of the curved surface, $\alpha$ is the half spreading angle of the cone, $\beta$ is the angle between lines $OK_1$ and $OT$, $w_{ic}$ is the indentation depth at the crack propagation process, $\zeta$ is the rotation angle of the plate from radial bending to circumferential bending, and $\zeta = \alpha - \theta$.

Integrating Eq. (16) and Eq. (17), the strain rate $\dot{\varepsilon}_{\theta}$ can be approximated as

$$\dot{\varepsilon}_{\theta} = \frac{\sin \beta}{R_p} \sin 2\zeta \cdot \dot{w}_w$$  \hspace{1cm} (18)

Thus, the membrane energy rate can be obtained by substituting Eq. (18) into Eq. (15):

$$\dot{E}_{m,p} = \frac{2\pi \sigma_0 t_p}{\sin 2\zeta} \sin 2\alpha \left(1 - \cos \beta_{\max}\right) r_0 \cdot \dot{w}_w$$  \hspace{1cm} (19)

where $\beta_{\max}$ is the angle between lines $OK_1$ and $OK_2$. According to the geometric relation, $1 - \cos \beta_{\max}$ can be approximated as

$$1 - \cos \beta_{\max} = \frac{1.1 \sin \theta (1 - \cos \zeta)}{\sin \alpha \cos \zeta}$$  \hspace{1cm} (20)

For the radial bending effect, the bending energy rate can be expressed as follows:

$$\dot{E}_{b_{1,p}} = M_{bp} \dot{\beta} l_{b_{1,p}}$$  \hspace{1cm} (21)

where $M_{bp}$ is the bending moment per unit length of the plate and can be obtained as $M_{bp} = \sigma_0 t_p^2 / 4$ and $l_{b_{1,p}}$ is the length of the bending hinge lines and can be expressed as

$$l_{b_{1,p}} = 2\pi R_{bp}$$  \hspace{1cm} (22)

where $R_{bp}$ is radius of the circular hinge line and can be expressed as

$$R_{bp} = R_p + r_0 \tan \frac{\zeta}{2} \cos \theta$$  \hspace{1cm} (23)

In addition, the instantaneous rotation angle rate $\dot{\beta}$ can be expressed as
\[ \dot{\beta} = \frac{\cos \alpha}{r_0 \sin \zeta} \dot{w}_w \]  

(24)

Thus, the radial bending energy rate can be obtained by substituting Eqs. (22)-(24) into Eq. (21):

\[ \dot{E}_{b1} = \frac{\pi \sigma \rho t^2 p R_{bp} \cos \alpha}{2 \sin \zeta} \frac{\dot{w}_w}{r_0} \]  

(25)

In addition, the circumferential bending hinge line rate can be expressed as

\[ \dot{i}_{\alpha p} = \dot{\beta} r_0 = \frac{\cos \alpha}{\sin \zeta} \dot{w}_w \]  

(26)

The circumferential bending energy rate can be expressed as

\[ \dot{E}_{b2} = M_{bp} 2 \pi i_{\alpha p} = \frac{\pi \sigma \rho t^2 p \cos \alpha}{2 \sin \zeta} \dot{w}_w \]  

(27)

The equilibrium of the ruptured plate-cone system can be expressed via the principle of virtual work

\[ F_p \dot{w}_w = \dot{E}_{sw} + \dot{E}_{s1} + \dot{E}_{s2} \]  

(28)

where \( F_p \) is the plastic resistance force. Substituting Eqs. (19), (25) and (27) into Eq. (28), \( F_p \) is obtained as

\[ F_p = \frac{\pi \sigma \rho t^2 p \cos \alpha}{2 \sin \zeta} \left( \frac{R_{bp}}{r_0} + 1 \right) \frac{2 \pi \sigma \rho t^2 p \cos \alpha \sin \theta (1 - \cos \zeta)}{\sin \zeta \cos \zeta} \frac{1}{r_0} \]  

(29)

where \( R_{bp} \) and \( \theta \) can be obtained from Eq. (23) and Eq. (14), respectively.

Minimizing the plastic force with respect to the rolling radius \( r_0 \) yields the following:

\[ r_0 = \frac{\cos \zeta}{2} \sqrt{\frac{t_p R_{bp}}{\sin \theta (1 - \cos \zeta)}} \]  

(30)

The contribution of friction is also considered in the perforating process, and it is related to the friction coefficient \( \mu \) (estimated as 0.35 by Zhang (2002)) and the half spreading angle of the cone \( \alpha \). According to Ohtsubo and Wang (1995) and Zhang (2002), the total resistance force \( F_c \) can be expressed as

\[ F_c = F_p (1 + \mu \tan \alpha) \]  

(31)

### 3. Penetration test design

One of the purposes for this model tests is to validate the derived analytical expressions. Another purpose is to investigate the collapse characteristics of a ship side plate subjected to a head-on collision by a full-shape bulbous bow, especially the crack propagation behavior after plate fracture.

The quasi-static indentation experiments are performed at Huazhong University of Science and Technology. The setup used in the experiments is presented in Fig. 6. The
specimens are clamped between a bottom flange and an upper flange, which are made of Q345 steel with a thickness of 25 mm. They are fixed together by M20 bolts. Since the boundary conditions in experimental settings can affect strongly the tested results (Villavicencio and Guedes Soares, 2011; Liu et al., 2018), current fixtures have been proven to provide clamped boundary constraints through validation by numerical simulations with solid elements considering all fixtures.

The deformation of the specimens is enforced at a rate of ~10 mm/min on the middle span by two hydraulic cylinders, which are connected in series to obtain sufficient loading distance. A 100-ton load cell fixed between the hydraulic cylinder and the indenter and two displacement sensors jointed on the indenter are utilized to obtain the force-time and displacement-time curves, respectively. To visualize the deformations, 50×50 mm grids are drawn on the front and rear sides of the specimens. Moreover, two cameras are placed under the bottom flange to capture the deformation process of the specimens.

In general, in model tests, the bulbous bow is treated as rigid and simplified as a conical indenter with the top radius (Wang et al. 2000; Alsos and Amdahl, 2009). Nevertheless, in this study, in order to investigate the fracture initiation and crack propagation behaviors separately, the indenter is designed as a combined shape, as depicted in Fig. 7. The hemispherical end cap of the indenter is used to punch the specimen to fracture. The cylinder part provides an interval from crack initiation to propagation. The cone part enables the cracks on the plate to spread to a large area under lateral indentation. The geometry of the indenter is also shown in Fig. 7.

The dimensions of the specimen is illustrated in Fig. 8, where the central 600×600 mm square is the exposed area of the panels and the surrounding areas with a width of 155 mm are applied to clamp the specimens. Moreover, replicate tests for the specimen are performed to ensure the reliability of the experimental results because the crack number and crack propagation paths produced in the plate may be uncontrolled.

Standard tensile test is performed to determine the elastic-plastic behavior of the specimens. The material used for the plates is grade B normal structural steel qualified by CCS (China Classification Society). These steel plates are from the same batch supplied by the WISCO company (Wuhan Iron and steel (Group) Company) and are 3.15 mm in thickness. To obtain the mechanical properties of the steel, quasi-static
tensile tests are conducted using standard tensile specimens and procedures. The dimensions of the machined tension test pieces are shown in Fig. 9. Based on the displacement-prescribed tensile tests performed on the universal testing machine, the engineering stress-strain behavior of the material can be obtained. Moreover, the equivalent stress-strain relationship is represented by a power-law relation:

\[
\sigma_{eq} = \begin{cases} 
\sigma_y & \text{if } \varepsilon_{eq} \leq \varepsilon_{plat} \\
ke_{eq} & \text{otherwise}
\end{cases}
\]  

(32)

where \(\sigma_{eq}\) and \(\varepsilon_{eq}\) are the equivalent stress and the equivalent strain, respectively. \(\varepsilon_{plat}\) denotes the equivalent plastic strain at the plateau exit. The mechanical properties of the plate material are summarized in Table 2, and the tensile stress-strain curves are presented in Fig. 9.

4. Verification and application of the analytical method

In this section, the proposed analytical method is verified with respect to the critical penetration depth, resistance after plate rupture, and crack propagation resistance by comparing the results from the analytical method and experiment. The resistance-penetration responses are shown in Fig. 11. The whole collapse processes of the specimen can be categorized into two rupture processes: the fracture initiation process and crack propagation process. To further clarify the accuracy of the analytical method, five specific moments of the resistance-penetration curve are analyzed. Moreover, the corresponding deformation shapes at these moments for the specimen are depicted in Fig. 12.

At moment a, necking appears on the plate (Fig. 12(a)). The plate is subsequently fractured, accompanied by a rapid load decrease. The compared curves shown in Fig. 11 demonstrate that the analytical method can adequately predict the resistance due to the large deformation and fracture initiation of the plate.

At moment b, disparities can be observed when the ruptured specimen is punched by the spherical top of the indenter. The experimental response curve decreases gradually when the resistance force drops to some extent. In this process, several cracks will be generated on the plate around the necking circle (Fig. 12(b)), which illustrates that a portion of the energy contributes to the initiation of the cracks. The plate will subsequently experience crack propagation. But, this plastic behavior is neglected in the analytical method, which may lead to a lower amount of energy dissipation prediction.
After the top hat of the specimen is pushed aside by the indenter, the cylinder part of the indenter will penetrate the specimen to form a large hole. In this process, some lateral resistance remains (Fig. 11) that is originated from the sliding effect between the top hat of the ruptured plate and the indenter (Fig. 12(c)). In the analytical method, the solution of the resistance in this process is not considered. However, the sliding distance $w_{slide}$ can be obtained according to the deformation shape of the plate expressed by Eq. (4) and the geometry relation shown in Fig. 13. The sliding distance calculated for the specimen is 161.03 mm.

From moment c to moment d, the cone part of the indenter will press the ruptured plate, and the corresponding resistance will increase remarkably. In the analytical solution, it is assumed that the resistance increases linearly. The ruptured plate will be gradually transformed into the deformation mode depicted in Fig. 4, where the penetration depth needed in the transformation process is the vertical distance between point $K_1$ and point $K_3$ ($w_{K}$). The distance is obtained as 2.76 mm according to the geometric relation. At moment d, the crack propagation mode of the plate is generated, and the subsequent process is the crack propagation process (moment e). According to Eq. (31), the resistance-penetration response can be obtained. The detailed calculation process is shown in Fig. 14. Note that the length of the circumferential bending hinge line in the curved plate will gradually increase with the punch of the cone, which can lead to an increase in the resistance. The compared curves in Fig. 11 demonstrate that the analytical method predicts the crack propagation resistance of the plate well.

In general, the proposed analytical method can adequately predict most of the plastic resistance in the current experiment, except the resistance needed to start the cracks (moment b). The current experiments are designed to separate the crack initiation and crack propagation processes for a ship side plate impacted by a full-shape bulbous bow to investigate the energy dissipation mechanisms more clearly and assess the amounts of energy dissipated in the crack propagation process and that in the crack initiation process individually. However, in an actual ship bow-side collision scenario, the process of the load decreasing nearly to zero and subsequent load increasing, i.e., from moment b to moment d in Fig. 11, will not appear because crack propagation follows plate fracture, see the experiments performed by Wang et al. (2000), Alsos and Amdahl (2009) and Zhang et al. (2018). Therefore, the results
demonstrate that the proposed analytical method can well forecast the overall collapse behavior of a ship side plate impacted by a full-shape bulbous bow.

Finally, the applicability of the proposed analytical method is presented. Current equations can be applied to assess the resistance of ship side plate suffering head-on collision by a striking ship equipped with a bulbous bow. In this case, ship side plate will experience large stretching deformation, initial fracture and subsequent crack propagation. Nevertheless, the assumed case is quite limited compared with actual collision scenarios as stated in section 2, where the boundary of the collided plate is fixed, the impact position on the plate is at the mid-span, the impact angle is vertically, the bulbous bow is undeformable and the shape of the bulbous bow is conical. The application of proposed method to these cases should be further researched.

5. Experimental observations and discussion

In addition to verifying the proposed analytical method, the experiments can clarify the other two key points mentioned in the Introduction.

5.1. Influence of crack number

Replicate tests are performed for the specimen, and the corresponding crack numbers generated are found to be different. The influence of the crack number on the resistance response is studied in this section.

The resistance-penetration responses are shown in Fig. 15, which indicates excellent test repeatability as the curves practically overlap. However, the numbers of cracks observed in each damaged specimen are not the same, as shown by the permanent deformation shapes in Fig. 16, with 4 to 6 cracks produced in the specimens.

The energy dissipation mechanisms for out-of-plane crack propagation (section 2.3) can explain why the crack propagation resistances are nearly identical but the numbers of cracks are different. In the crack propagation process, the energy dissipated by the specimens involves radial and circumferential bending in each petal and stretching before the plate experiences fracture. These deformation behaviors are related to the circumferential parameters instead of the crack number. These results illustrate that the number of cracks has little influence on the total out-of-plane crack propagation resistance.

Moreover, in order to clarify the reason why the number of cracks generated on the
plate is different, the damage forms of the plates at moments before and after cracks generation are researched, as shown in Fig. 17. The initial fracture of the plate around the necking circle can lead to not only a top hat connected with adjacent plate, but also an arc of free edge (marked with blue lines in Fig. 17). With the punch of the indenter, cracks will initiate around the free edges. As the length of the arc lines can be judged by the central angle, it can be found that larger central angle corresponds to more crack number (Fig. 17). Before cracks are generated, the free edges of the plates suffer circumferential tension effect due to the punch of the indenter. When exceeding the maximum bearing limit, cracks initiate around the free edges and longer free edge will be separated by more cracks. Thus, it demonstrates that the number of crack produced on the plate is related with the length of the free edge produced when the plate is initially fractured.

5.2. Behavior of a ship side plate in crack propagation process

The behaviors of a ship side plate subjected to a head-on collision by a bulbous nose, including large deformation and ductile tension failure, have been extensively investigated in previous studies. By contrast, the behavior of the ship side plate driven by the structure behind the bulbous nose is seldom referred to.

In the current experiments, the ruptured specimens are separated into several petals due to the punching of the cone part of the indenter. The failure mode of the plate is shown in Fig. 18, where model “I” cracks are produced in the plate. The material fails due to in-plane tension effect at the crack tips. In the crack initiation process, plate fracture is also caused by the tension effect. However, the stress concentration at the crack tips in the crack propagation process can lead to a lower breaking strength, which indicates that the force required to advance the cracks is lower than the plate without cracks. In addition, the tension effect concentrates in the circumferential area before the cracks arrive rather than in the whole plate. Thus, the absorbed energy contributed by the tension effect in the crack propagation process is much smaller than that in the crack initiation process.

However, by adding the bending effects in the plate, a significant portion of the energy can be dissipated in the crack propagation process compared with that in the crack initiation process, as shown in Fig. 15. Current analytical method demonstrates that the energy dissipated due to tension effect is about 0.44 to that dissipated due to bending effects. It illustrates that both bending and tension effects play an important
role in crack propagation process. Moreover, the deformation energy can be obtained by integrating the area beneath the curve of resistance vs. penetration. Treating moment b as the division of the two processes, the experimental values of the energy dissipated (\(E_1\) and \(E_2\)) by each specimen in the two rupture processes are summarized in Table 3, including the energy dissipated by the experiment and replicate tests denoted as US, US_r1 and US_r2. In the specimens, the ratios of energy absorbed in the crack propagation process to that in the crack initiation process reach ~1. These results illustrate that a large portion of the impact energy can be absorbed by the ship side plate with the punch of the bow structure behind the bulbous nose in an accident of a ship suffering serious collisions.

6. Conclusions

This paper investigates the whole collapse behavior of a ship side plate impacted by a full-shape bulbous bow, including large deformation to fracture and crack propagation. Analytical expressions are developed to assess the impact resistance, especially fracture initiation and crack propagation. The proposed analytical method is proved to be reliable by validation in quasi-static tests with unstiffened plates laterally punched by a combine-shaped indenter. Moreover, several conclusions can be drawn based on the experimental observations. They are listed as follows:

1) In the crack propagation process, the numbers of cracks generated on the specimens are random between 4 and 6. The crack propagation resistance is attributed to radial and circumferential bending in each petal of the specimen and circumferential stretching in the area before the cracks approach. When these effects are summed, the total crack propagation resistances of the specimens are nearly identical even though the number of cracks differs.

2) The failure of the perforated plate laterally punched by a cone is due to the tension effect at the crack tips with high stress concentration (Model “I” crack).

3) The punch of the structure behind the bulbous bow nose can lead to energy dissipation of the ruptured plate due to bending and tension effects. Current experiments show that the amount of energy dissipated by the punching of the structure behind the bulbous nose is nearly equal to that dissipated by the punch of the bulbous bow nose. Neglecting this portion of energy produced by the structure behind the bulbous nose will no doubt result in an underestimate of the energy dissipation ability of a ship side plate in a head-on collision.
Acknowledgments

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<th>References</th>
<th>$b_0$ (mm)</th>
<th>$R_0$ (mm)</th>
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<th>Experiment (mm)</th>
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Table 2

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Table 3

<table>
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<td>US_r2</td>
<td>14.76</td>
<td>15.49</td>
<td>1.049</td>
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- G-Gauge length: 50 mm
- W-Width: 12.5 mm
- T-Thickness: 3.15 mm
- R-Radius of fillet: 12.5 mm
- L-Overall length: 200 mm
- A-Length of reduced section: 85 mm
- B-Length of grip section: ~50 mm
- C-Width of grip section: 20 mm
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