

# Convex Programming Energy Management and Components Sizing of A Plug-in Fuel Cell Urban Logistics Vehicle

Xiaohua Wu<sup>a</sup>, Xiaosong Hu<sup>b,\*\*</sup>, Xiaofeng Yin<sup>a</sup>, Lei Li<sup>a</sup>, Zhaowei Zeng<sup>a</sup>, Volker Pickert<sup>1</sup>

<sup>a</sup>*School of Automobile and Transportation, Xihua University, Chengdu, 610039, China;*

<sup>b</sup>*State Key Laboratory of Mechanical Transmissions, Department of Automotive Engineering, Chongqing University, Chongqing, 400044, China*

<sup>c</sup>*School of Electrical and Electronic Engineering, Newcastle University, UK*

---

## Abstract

This paper devises an optimization framework for efficient energy management and components sizing of a plug-in fuel cell urban logistics vehicle (PFCULV). Based on the structure and system models of the PFCULV, a convex programming (CP) problem is formulated to simultaneously optimize both the control decision and parameters of power sources, including a fuel cell pack and a battery pack. This paper seeks to minimize a summation of energy cost and power sources cost, while satisfying vehicle power demand and battery health requirements. Considering different drive cycles, the optimal parameters and energy costs are systematically investigated. As a result, the optimal battery rated power and energy capacity are not affected by the different drive cycles, given an electric-only range between 40 km and 60 km. Finally, based on the developed CP control law and optimal parameters, we examine the power distribution of the PFCULV in different drive cycles.

*Keywords:* Plug-in Fuel Cell Vehicle, Urban Logistics Vehicle, Energy Management, Parameters Matching, Convex Programming, Optimization

---

\*This work was supported in part by Science and Technology Department of Sichuan Province (Grant Nos. 2017GZ0104, 2016HH0010), Education Department of Sichuan Province (Grant No. 18ZA0450), Xihua University Fund (Grant No. szjj2018-133).

\*\*Corresponding author (X. Wu and X. Hu equally contributed to this research work.)

*Email address:* [xiaosonghu@ieee.org](mailto:xiaosonghu@ieee.org) (Xiaosong Hu)

---

## 1. Introduction

### 1.1. Motivation

With the acceleration of economic development and urbanization, China has put forward higher requirements for energy-saving and emission-reduction performance of urban logistics vehicles (ULVs), an important type of commercial vehicles. Researchers and automobile industries have to develop low-fuel consumption and high-efficiency ULVs. Due to a limited driving range and a long charging time of a battery electric vehicle (EV), growing attention has been paid to fuel cell electric vehicles (FCEVs) [1, 2]. This paper devises an optimization framework for efficient energy management and components sizing of a plug-in fuel cell urban logistics vehicle (PFCULV).

### 1.2. Literature review

There is a rich literature on FCEVs energy management approaches, which can be generally categorized into sequential quadratic programming (SQP), adaptive control approach, logic threshold control strategy, fuzzy logic control strategy, dynamic programming, predictive control, tracking control, and convex programming (CP).

An SQP based equivalent consumption minimum strategy for a FCEV powered by a fuel cell (FC) system, a battery pack, and a supercapacitor pack is presented in [3], which minimizes its hydrogen consumption and prolongs the FC lifetime. In terms of fuel economy and drivability, the energy management strategy (EMS) of a FC/supercapacitor hybrid electric bus is considered in [4]. Based on the ensemble of well-established methods, high-fidelity vehicle dynamic simulations, and real-world vehicle test data, energy use and air emissions of hydrogen FC electric trucks are provided in [5]. Due to the passive and active EMS, the output characteristics of each power source of a solar cell/FC/battery hybrid unmanned aerial vehicle are investigated in [6]. The EMS of a vehicle powered by an FC system and a Li-ion battery is proposed in [7], based on an

29 adaptive control approach with fuzzy logic parameter tuning. The impact of FC  
30 performance and control strategies on the benefits of hybridization are studied  
31 in [8]. The offline strategy (dynamic programming), and online strategy (opti-  
32 mized fuzzy logic controller) for a real FCEV are described in [9]. To coordinate  
33 energy sources and power components of a FCEV, an optimal EMS based on  
34 specific fuel consumption due to load shifting is defined in [10]. Based on a novel  
35 fractional-order extremum seeking method, an online EMS is presented in [11],  
36 which can improve both the FC efficiency and durability. An overview of power  
37 conditioning system architectures for an FCEV propulsion system, as well as the  
38 control strategy to reduce the power losses, is presented in [12]. To minimize hy-  
39 drogen consumption and simultaneously protect FC health, a two-stage energy  
40 management controller (including predictive controller, and tracking controller)  
41 is formulated and investigated for a series plug-in FC/Li-ion battery hybrid  
42 midsize sedan in [13].

43 Considering different constraints of maximum velocity and driving range,  
44 components sizes (including the electric machine, power electronics, batteries,  
45 etc.,) of an FCEV are optimized in [14], where the impact of the FC size on  
46 the vehicle's performance is particularly examined. A logic threshold control  
47 strategy and a fuzzy logic control strategy are provided to control a hybrid power  
48 supply of an aluminum air FC stack and a supercapacitor pack for a FCEV in  
49 [15], where component parameters are analyzed and calculated. Based on a  
50 statistical description of driving cycles, energy sources of a FC based collection  
51 truck are sized in [16], which shows that a 20 kW FC stack is sufficient for a 13  
52 000 kg collection truck. The impact of different levels of modeling details on the  
53 total cost of ownership of a FCEV is investigated through convex optimization in  
54 [17], where sizes of its FC system, battery pack, and electric motor are optimized  
55 simultaneously. In [18], CP is extended to rapidly and efficiently optimize both  
56 the EMS and components (the battery pack and the FC system) sizes for an  
57 FC electric bus.

58 The structure and key parameters for power sources of FCEVs in the avail-  
59 able literature are listed in Table 1. The main power source structure is fuel

60 cell/battery (FC+B), and a few of setups use fuel cell/supercapacitor (FC+S)  
61 or fuel cell/battery/supercapacitor (FC+B+S). Most of the existing literature  
62 focuses on energy management problems of cars or buses, whereas few are about  
63 trucks. Almost all of plug-in (P) batteries are used for cars. However, hydrogen  
64 stations are limited, and hydrogen price is expensive at present. The prices of  
65 battery and electricity are cheaper than those of FC and hydrogen. Therefore,  
66 plug-in fuel cell electric powertrains with relatively large battery sizes could be a  
67 more affordable and sustainable solution, particularly for economy-critical truck  
68 applications. However, few studies consider optimal component sizes and control  
69 strategy (see Table 1, both means sizing and EMS) simultaneously for a plug-  
70 in FC/battery truck. The additional plug-in functionality could significantly  
71 change parametric optimization and EMS design decisions, versus non-plug-in  
72 powertrains. Hence, this paper devises an optimization framework for efficient  
73 energy management and components sizing for a PFCULV.

### 74 *1.3. Contributions*

75 To overcome the downsides of the previous studies, this paper delivers two  
76 key contributions to the relevant literature. First, CP is leveraged to optimize  
77 both the control decision and parameters of a PFCULV at the same time. To the  
78 best knowledge of the authors, this is the first study on the CP-driven joint op-  
79 timization of control strategy and component sizes for a PFCULV. Considering  
80 the PFCULV's electric-only driving distance constraint and the battery lifetime  
81 requirement, we optimize the battery energy capacity for the PFCULV. Second,  
82 based on different drive cycles and hydrogen prices, we perform a sensitivity  
83 study for the power-source parameters and energy cost.

### 84 *1.4. Outline of paper*

85 The remainder of the paper proceeds as follows. Section 2 details the system  
86 structure and models of the PFCULV. The CP problem is formalized in Section  
87 3. The optimization results are discussed in Section 4, followed by conclusions  
88 summarized in Section 5.

Table 1: FCEVs literature analysis.

References	Mass kg	Power source strcuture	FC kW	Battery kWh	Other
Ahmadi[19]	1500	FC+B+S	50	7.5	car EMS
Sarioglu[10]	1800	FC+B	70	1.2	car EMS
Xu[20]	2000	FC+B	70	3	car EMS
Martel[21]	1349	FC+B (P)	-	-	car EMS
Geng[13]	1500	FC+B (P)	30	-	car EMS
Song[22]	1600	FC+B (P)	6.5	5.12	car EMS
Fernandez[23]	2290	FC+B (P)	-	16	car EMS
Jensen[14]	1508	FC+B	62.0	17.4	car sizing
Pourabdollah[17]	1600	FC+B (P)	-	-	car sizing
Geng[24]	8000	FC+B (P)	30	-	truck EMS
Ravey[16]	19000	FC+B	20	25	truck sizing
Carignano[4]	13650	FC+S	48	-	bus EMS
Carignano[25]	14100	FC+S	48	-	bus EMS
Hu[18]	134800	FC+B	58	5.6	bus both
Hu[26]	14500	FC+B+S	100	-	bus both

## 89 2. Structure and models

### 90 2.1. Structure and parameters

91 We consider the propulsion system of a PFCULV shown in Fig. 1. The  
92 ULV is impelled by a traction motor. A DC-DC converter is applied to regulate  
93 the FC system current flow into the DC bus, to which a Li-ion battery pack  
94 is connected in parallel. The difference between the current demand from the  
95 motor inverter and the current flow from the FC system DC-DC converter is  
96 offset by the Li-ion battery pack, which can be charged through plugging-in to  
97 the grid.

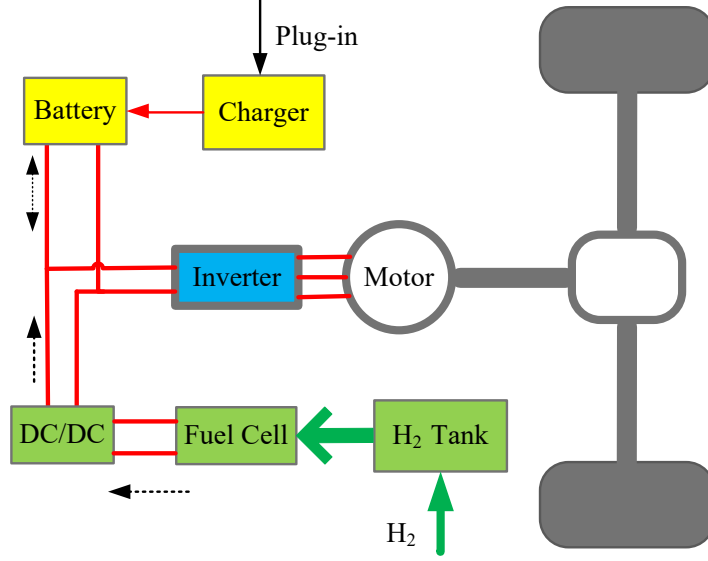


Figure 1: Architecture of the PFCULV propulsion system.

98 Performance specifications for the PFCULV are listed in Table 2. The key  
 99 parameters of the PFCULV are listed in Table 3. The parameters of power  
 100 sources, including the rated power of the FC system, the rated power and energy  
 101 capacity of the battery pack, need to be matched and optimized.

## 102 2.2. FC system modeling

The FC system is the main power source of the PFCULV. The FC system rated power is higher than average driving resistance power ( $u_a=50$  km/h), but lower than the required power when the vehicle is driving at the maximum speed ( $u_{\max}=80$  km/h), to avoid a overly large and costly FC system. So, the rated power of the FC system  $P_{FC}$  should be satisfied as follows

$$(m_t g f + \frac{C_D A}{21.15} u_a^2) \frac{u_a}{3600 \eta_T} \leq P_{FC} \leq (m_t g f + \frac{C_D A}{21.15} u_{\max}^2) \frac{u_{\max}}{3600 \eta_T}, \quad (1)$$

where  $m_t$  is the total mass of the vehicle chassis, the battery, and the FC system. In this paper, the FC system with a power rating of  $P_{FCb}=50$  kW in ADVISOR

Table 2: Performance specifications for the PFCULV.

Specification description	Symbol	Value	Unit
Maximum velocity	$u_{\max}$	$\geq 80$	km/h
Average velocity	$u_a$	50	km/h
0~50 km/h acceleration time	$t_m$	$\leq 25$	s
Maximum grade	$\alpha_{\max}$	$\geq 20$	%
Driving distance of using only hydrogen fuel	$d_h$	$\geq 200$	km
Driving distance of using only the battery with average velocity	$d_b$	$40 \leq d_b \leq 60$	km
Service period	$y_t$	5	years

(FC-ANL50h2) is considered as the baseline FC system. Its hydrogen consumption is a function of the FC system net power, which is approximated by a quadratic function below [18]:

$$P_h = a_0 P_{fc}^2 + a_1 P_{fc} + a_2, \quad (2)$$

103 where  $a_0$ ,  $a_1$ , and  $a_2$  are nonnegative power-dependent coefficients.  $P_h$  and  
 104  $P_{fc}$  are the hydrogen power and the net power of the baseline FC system,  
 105 respectively. The original and approximate hydrogen consumption curves are  
 106 indicated in Fig. 2.

We assume that the rated power of the FC system targeted in the PFCULV is

$$P_{FC} = P_{FCb} x_{fc}, \quad (3)$$

$$x_{fc,\min} \leq x_{fc} \leq x_{fc,\max}, \quad (4)$$

where  $x_{fc}$  is the optimal design parameter of the FC system.  $x_{fc,\min}$  and  $x_{fc,\max}$  are the minimal and maximal values of  $x_{fc}$ , respectively. The hydrogen con-

Table 3: Key parameters of the PFCULV.

Parameter description	Symbol	Value	Unit
Frontal area	$A$	4.4	m <sup>2</sup>
Drag coefficient	$C_D$	0.75	-
Rolling resistance coefficient	$f$	0.015	-
Rotation mass conversion factor	$\delta$	1	-
Gross vehicle mass	$m_v$	6400	kg
Ratio of final drive	$i_0$	12.9	-
Wheel radius	$r$	0.367	m
Motor peak power	$P_{m \max}$	90	kW
Motor rated power	$P_{me}$	45	kW
Motor rated speed	$n_{me}$	1900	rpm
Motor maximum speed	$n_{m \max}$	9000	rpm
Efficiency of motor and final drive	$\eta_T$	0.9	-
Power of air conditioning	$P_{air}$	1	kW

sumption of the PFCULV is thus

$$P_{h,k} = a_0 P_{fc,k}^2 + a_1 P_{fc,k} + a_2, \quad k = 0, \dots, N - 1, \quad (5)$$

where the subscript  $k$  is the time index, and  $N$  is the final time step of drive cycle. Then,  $P_{h,k}$  and  $P_{fc,k}$  are the hydrogen power and the net power of the PFCULV FC system, respectively. And  $P_{fc,k}$  should respect the following constraint

$$0 \leq P_{fc,k} \leq P_{FC}. \quad (6)$$

### 107 2.3. Battery modeling

The battery pack is another power source for the PFCULV. Main principles for battery parameters matching include: (i) it can recover most of the braking energy; (ii) it can work with the FC system to meet the power requirement for



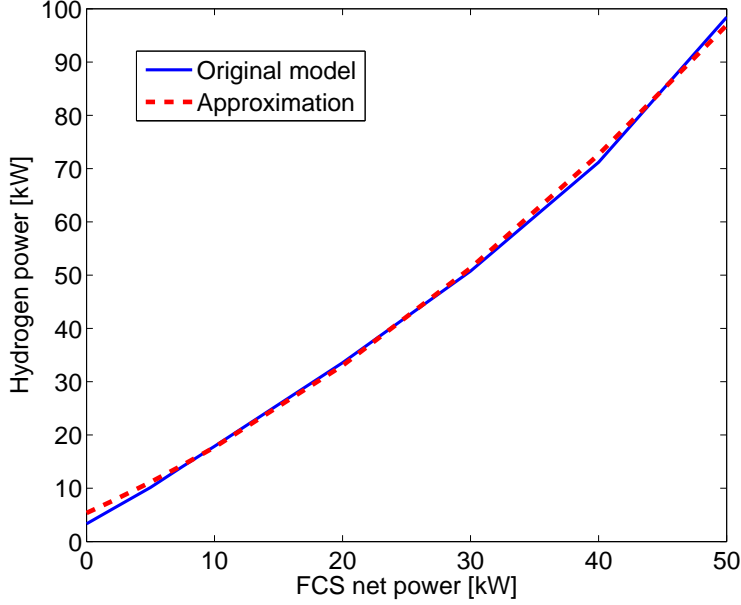


Figure 2: FCS modeling.

vehicle propelling and auxiliary electrical systems in a hybrid mode; (iii) it can meet the requirement for the electric-only driving distance. So, the rated power of the battery pack  $P_B$  should be constrained as follows:

$$\frac{P_{me}}{\eta_T} + P_{aux} - P_{FC} \leq P_B \eta_b \leq \frac{P_{me}}{\eta_T}, \quad (7)$$

108 where  $P_{me}$  is the rated power of the motor, and  $P_{aux}$  is the power demand of  
 109 vehicular auxiliary electrical systems. Usually,  $P_{aux} = \frac{P_{air}}{0.65}$ , with  $P_{air}$  being  
 110 the power for air conditioning [27]. Moreover,  $\eta_b$  is the average battery system  
 111 efficiency.

To meet the requirement for the electric-only driving distance  $d_b$  (40 km  $\leq d_b \leq 60$  km), the battery energy should be constrained by

$$40 \left( \frac{F_t}{3600\eta_T} + \frac{P_{aux}}{u_a} \right) \leq E_b D_{oD} \eta_b \leq 60 \left( \frac{F_t}{3600\eta_T} + \frac{P_{aux}}{u_a} \right), \quad (8)$$

where  $E_b$  is the battery energy capacity (kWh),  $D_{oD}$  is the depth of discharge (%), and  $F_t$  is the traction force (N) with an average velocity ( $u_a=50$  km/h),

as calculated by

$$F_t = m_t g f + \frac{C_D A}{21.15} u_a^2. \quad (9)$$

We assume that the battery energy capacity  $E_b$  is

$$E_b = \frac{n_0 x_b V_{nom} Q}{3600000}, \quad (10)$$

$$x_{b,\min} \leq x_b \leq x_{b,\max}, \quad (11)$$

112 where  $n_0$  is the base number of battery cells,  $x_b$  is the optimal design parameter  
 113 of the battery pack,  $V_{nom}$  is the nominal voltage of the battery cell, and  $Q$  is  
 114 the battery cell's capacity. Moreover,  $x_{b,\min}$  and  $x_{b,\max}$  are the minimal and  
 115 maximal values of  $x_b$ , respectively.

The controller also must maintain the battery energy and power within allowable bounds [28, 29],

$$SOC_{\min} E_b \leq E_k \leq SOC_{\max} E_b, \quad k = 0, \dots, N, \quad (12)$$

$$-P_B \leq P_{b,k} \leq P_B, \quad k = 0, \dots, N - 1, \quad (13)$$

where  $E_k$  and  $P_{b,k}$  are the battery energy and internal power at time  $k$ , respectively. The discharge power is assumed to be positive, by convention.  $SOC_{\min}$  and  $SOC_{\max}$  are the battery's minimal SOC (state of charge) and maximal SOC, respectively. The dynamics of the battery are governed by the following equations:

$$E_{k+1} = E_k + \Delta t P_{b,k}, \quad k = 0, \dots, N - 1, \quad (14)$$

$$E_0 = E_{init}, \quad (15)$$

$$0 \leq E_0 - E_{end} \leq E_b D_{oD}, \quad (16)$$

$$P_{loss,k} = (1 - \eta_b) |P_{b,k}|, \quad k = 0, \dots, N - 1, \quad (17)$$

116 where  $\Delta t$  is the time interval, and  $E_{init}$  and  $E_{end}$  are the initial and final battery  
 117 energy, respectively. Further,  $P_{loss,k}$  is the power loss of the battery system.

Battery degradation always occurs during realistic vehicle operations, and its fading rate depends on a multitude of factors. The energy throughput-based

battery cell state-of-health (SOH) model from [30] is used in this paper, and the derivative of SOH is approximated by a function of the battery cell power  $P_{bc,k}$  as follows [31],

$$SOH_{c,k+1} = SOH_{c,k} + \Delta SOH_{c,k}, \quad k = 0, \dots, N - 1, \quad (18)$$

$$\Delta SOH_{c,k} = \min(b_{j,1} + b_{j,2}|P_{bc,k}|), \quad j = 1, 2, 3, 4, \quad (19)$$

where  $b_{j,1}, b_{j,2}$  with  $j = 1, 2, 3, 4$ , are coefficients approximating the SOH derivative, and  $SOH_{c,k}$  is the battery cell SOH at time  $k$ . The original and approximate battery cell wear models are indicated in Fig. 3. According to the number of battery replacements  $n_r$ , the battery cell SOH decrease over a drive cycle is constrained by

$$1 - \Delta SOH_{c,\max} \leq SOH_{c,k} \leq 1, \quad (20)$$

$$SOH_{c,1} - SOH_{c,\text{end}} \leq \Delta SOH_{c,\max}, \quad (21)$$

$$\Delta SOH_{c,\max} = \frac{L_d(n_r + 1)}{L_a y_t}. \quad (22)$$

118 where  $\Delta SOH_{c,\max}$  is the maximum allowed change in SOH over a drive cycle,  
 119  $y_t$  is the vehicle service period,  $L_a$  is the average distance traveled annually,  
 120 and  $L_d$  is the length of drive cycle. Furthermore,  $SOH_{c,1}$  and  $SOH_{c,\text{end}}$  are the  
 121 initial and final battery cell SOH values over a drive cycle, respectively.

Since the battery cell has a very flat OCV curve in the allowed SOC window [26],  $P_{b,k} \approx \frac{n_0 x_b P_{bc,k}}{1000}$ . Using a variable change,  $\overline{SOH}_k = n_0 x_b SOH_{c,k}$ . Eq. (17) can be rewritten as

$$\Delta \overline{SOH}_k = \min(b_{j,1} n_0 x_b + b_{j,2} |1000 P_{b,k}|), \quad j = 1, 2, 3, 4. \quad (23)$$

Since the maximum allowed change in SOH over a drive cycle  $\Delta SOH_{c,\max}$ , is very small, an additional variable change is required to scale the SOH within a certain range [0, 1]. Let  $SOH_k$  be a new variable [32], such that

$$\overline{SOH}_k = SOH_k n_0 x_{b,\max} \Delta SOH_{c,\max} + n_0 x_b (1 - \Delta SOH_{c,\max}). \quad (24)$$

Differentiate Eq. (24) to get

$$\Delta SOH_k = \frac{\Delta \overline{SOH}_k}{n_0 x_{b,\max} \Delta SOH_{c,\max}}. \quad (25)$$

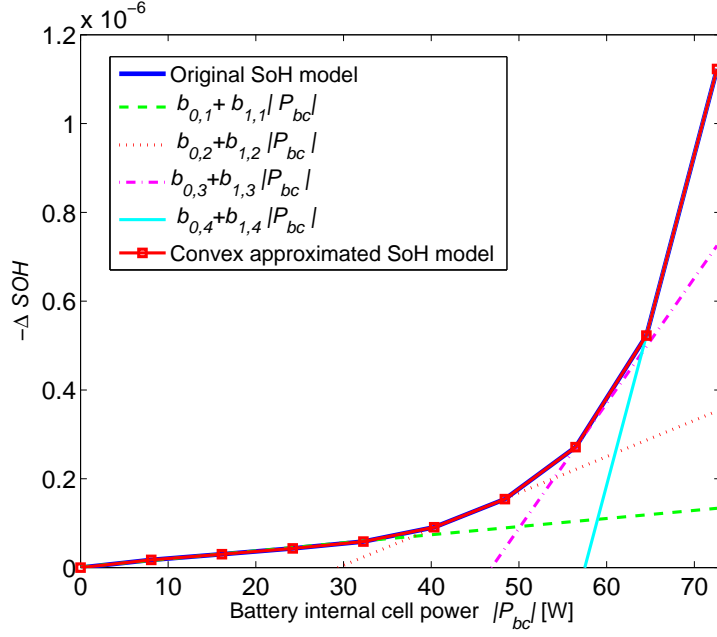


Figure 3: Battery cell wear model[31].

And  $SOH_k$  is constrained by

$$\Delta SOH_k = SOH_{k+1} - SOH_k, \quad (26)$$

$$0 \leq SOH_k \leq \frac{x_b}{x_{b,max}}. \quad (27)$$

#### 122 2.4. System model

The PFCULV satisfies the dynamic equation of the vehicle during driving, and fully supplies the required energy from the battery and FC systems. The power balance is depicted by

$$P_{b,k} + P_{fc,k} = P_{u,k} + P_{aux} + P_{bloss,k}, \quad (28)$$

where the PFCULV running power demand is expressed as

$$P_{u_k} = (m_t g f + \frac{C_D A}{21.15} u_k^2 + \delta m_t \frac{\Delta u_k}{3.6 \Delta t}) \frac{u_k}{3600 \eta_T}, \quad (29)$$

where  $u_k$  is the velocity of the PFCULV at time  $k$ . The total mass of the chassis, battery, and FC systems can be expressed as

$$m_t = m_v + m_b n_0 x_b + m_{fc} x_{fc}, \quad (30)$$

123 where  $m_v$ ,  $m_b$ , and  $m_{fc}$  are the gross vehicle mass, the battery cell mass, and  
124 the baseline FC system mass, respectively.

### 125 3. Optimization problem formulation

The convex objective function  $F(x)$ , which is of great interest to the PFCULV owner, is formulated to minimize a summation of the total energy cost, and the power source cost, for which we mainly consider the costs of the FC system and the battery pack:

$$F = C_{bf} + B_c + F_{cc}, \quad (31)$$

126 where  $C_{bf}$  is the total energy cost over a drive cycle, including both hydrogen  
127 and electricity costs,  $B_c$  is the battery cost, and  $F_{cc}$  is the FC system cost. Costs  
128 of the FC system and the battery are first normalized per kilometer (km), and  
129 then multiplied with the length of drive cycle.

It is assumed that the payment for the FC system is divided in equal amounts over a period of the PFCULV service period,  $y_t = 5$  years, with a  $r_y = 5\%$  yearly interest rate. The equivalent FC system cost related to a drive cycle is obtained by multiplying the length  $L_d$  with the FC price per km, given the average annual mileage  $L_a$ :

$$F_{cc} = c_f P_{FC} \left(1 + r_y \frac{y_t + 1}{2}\right) \frac{L_d}{L_a y_t}, \quad (32)$$

130 where  $c_f$  is the FC price per kW.

The battery equivalent cost,  $B_c$ , is calculated similarly:

$$B_c = c_b E_b \left(1 + r_y \frac{1 + \frac{y_t}{n_r + 1}}{2}\right) \frac{L_d (n_r + 1)}{L_a y_t}, \quad (33)$$

131 where  $c_b$  is the battery price per kWh.

The energy cost  $C_{bf}$  includes costs of hydrogen and electricity:

$$C_{bf} = \frac{c_h}{C_h} \sum_{k=0}^{N-1} P_{h,k} + c_e(E_{init} - E_{end}), \quad (34)$$

132 where  $c_e$  is the electricity price,  $c_h$  is the hydrogen cost per kg in (yuan), and  
 133  $C_h$  is the lower heating value of hydrogen in (J/g).

134 The optimization variables include the state variables  $E_k$ ,  $SOH_k$ , the control  
 135 variables  $P_{fc,k}$ ,  $P_{b,k}$ , and the optimal design parameters  $P_B$ ,  $x_b$ , and  $x_{fc}$ . It is  
 136 easy to see that the objective function  $F$  is convex with respect to  $P_{fc,k}$  and  
 137  $E_k$ .

138 According to the FC system model, the FC system constraints are Eqs. (1),  
 139 (4), and (6), with  $m_t = m_v + m_b n_0 x_b + m_{fc} x_{fc}$  and  $P_{FC} = P_{FCb} x_{fc}$ . It is easy  
 140 to see that the FC system constraints are linear inequality functions, and thus  
 141 convex.

According to the battery model, the battery constraints are Eqs. (7), (8),  
 (11)-(16), (25), and (27), with  $F_t = m_t g f + \frac{C_{DA}}{21.15} u_a^2$ ,  $E_b = \frac{n_0 x_b V_{nom} Q}{3600000}$ ,  $\Delta SOH_{c,max} =$   
 $\frac{L_d(n_r+1)}{L_a y_t}$ ,  $\Delta \overline{SOH}_k = \min(b_{j,1} n_0 x_b + b_{j,2} |1000 P_{b,k}|)$ , and  $\Delta SOH_k = SOH_{k+1} -$   
 $SOH_k$ . It is easy to see that Eqs. (7), (8), (11)-(13), (16), and (27) are linear  
 inequality functions, and thus convex. Eqs. (14) and (15) are linear equality  
 functions, and thus ensure convexity. Eq. (25) is an absolute equality function,  
 which are not affine. In a standard convex optimization problem, only affine  
 equality constraints are tolerated. However, relaxing Eq. (25) to an inequality  
 gives a convex problem without qualitatively altering the original problem as  
 follows [33, 34],

$$\Delta SOH_k \leq \min(b_{j,1} n_0 x_b + b_{j,2} |1000 P_{b,k}|) / n_0 / x_{b,max} / \Delta SOH_{c,max}. \quad (35)$$

According to Eqs. (28)-(30), the PFCULV's power balance constraints can  
 be expressed as

$$P_{b,k} + P_{fc,k} = (m_t g f + \frac{C_{DA}}{21.15} u_k^2 + \delta m_t \frac{\Delta u_k}{3.6 \Delta t}) \frac{u_k}{3600 \eta_T} + P_{aux} + (1 - \eta_b) |P_{b,k}|. \quad (36)$$

Eq. (36) is an absolute equality function, which is not affine. However, relaxing Eq. (36) to an inequality gives a convex problem without qualitatively altering the original problem as follows [33, 34],

$$P_{b,k} + P_{fc,k} \geq (m_t g f + \frac{C_{DA}}{21.15} u_k^2 + \delta m_t \frac{\Delta u_k}{3.6 \Delta t}) \frac{u_k}{3600 \eta_T} + P_{aux} + (1 - \eta_b) |P_{b,k}|. \quad (37)$$

142 The overall convex optimization framework is summarized in Table 4. Thanks  
 143 to the convexity, a globally optimal solution with arbitrary initialization can be  
 144 readily accomplished.

Table 4: Convex optimization framework for combined the PFCULV dimensioning and control.

---

(1)	The driving cycle, vehicle parameters, and key parameters of power sources are firstly specified.
(2)	For $k = 1, \dots, N$ , do the optimization below Optimization variables: $E_k, SOH_k, P_{fc,k}, P_{b,k}, P_B, x_b, x_{fc}$ . Expressions: $m_t, P_{FC}, P_{h,k}, F_t, E_b, \Delta SOH_k, C_{bf}, B_c, F_{cc}$ . Objective function: Eq. (31). Inequality constraints: Eqs. (1), (4), (6)-(8), (11)-(13), (16), (27), (35), (37). Equality constraints: Eqs. (14), (15).

---

#### 145 4. Results & discussion

146 This section analyses the properties of the proposed CP approach. The  
 147 CVX-tool is employed to parse the optimization problem. All the simulations  
 148 are run on a PC with a 2.50 GHz Intel Core i5-6300u CPU and 8 GB of internal  
 149 memory. Thanks to the mentioned advantages of the proposed method, the  
 150 optimal components size and control strategy are simultaneously obtained in  
 151 the Matlab environment. The basic parameters of the FC system, and the  
 152 battery utilized in the optimization for the PFCULV are listed in Table 5.

Table 5: Key parameters of power sources for the PFCULV.

Specification description	Symbol	Value	Unit
Hydrogen lower heating value	$C_h$	120000	J/g
Baseline FC system rated power	$P_{FCb}$	50	kW
Baseline FC system mass	$m_{fc}$	223.38	kg
Nominal capacity	$Q$	8280	As
Nominal voltage	$V_{nom}$	3.3	V
Battery cell mass	$m_c$	0.07	kg
Depth of discharge	$D_{oD}$	0.7	-
Average battery efficiency	$\eta_b$	0.92	-
Maximum SOC	$SOC_{\max}$	0.9	-
Minimum SOC	$SOC_{\min}$	0.2	-
Initial SOC	$SOC_{init}$	0.9	-
Base number of battery cells	$n_0$	3000	-
Minimum $x_b$	$x_{b,\min}$	0.01	-
Maximum $x_b$	$x_{b,\max}$	2	-
Minimum $x_{fc}$	$x_{fc,\min}$	0.1	-
Maximum $x_{fc}$	$x_{fc,\max}$	2	-
Number of battery replacements	$n_r$	1	-

153 *4.1. Parameters optimization of power sources*

154 Referring to the Bloomberg New Energy Finance (BNEF) survey, the average  
155 price of a Li-ion battery pack might vary from 680 yuan/kWh(100 \$/kWh)  
156 to 1421 yuan/kWh (209 \$/kWh) in 2025 [35, 36]. In light of the report of  
157 US Department of Energy, hydrogen fuel cell per kW varies from 204 yuan  
158 (30 \$) to 6800 yuan (1000 \$) [37]. In China, the electricity price for EVs  
159 is about 1 yuan/kWh, and the hydrogen price is 40 yuan/kg at present. In  
160 Chengdu, if hydrogen is produced by distributed energy, such as hydropower,  
161 its price will be about 15 yuan/kg. If the battery pack price is 1000 yuan/kWh,



162 the FC price is 10000 yuan/kW, the electricity price is 1 yuan/kWh, and the  
 163 hydrogen price is 40 yuan/kg, the optimal values of the battery and the FC  
 164 systems for four drive cycles are shown in Table 6. The four drive cycles include,  
 165 (i) the WVUSUB drive cycle represents typical suburban driving for a heavy  
 166 vehicle; (ii) the CSHVR drive cycle represents typical city-suburban driving for  
 167 a heavy vehicle; (iii) the WVUCITY drive cycle represents typical city driving  
 168 for a heavy vehicle; (iv) the CBDTRUCK drive cycle represents central business  
 169 district test cycle for trucks. For the four drive cycles, the optimal battery rated  
 170 power maintains constant, equaling to 54.35 kW. The reason for this result may  
 171 be a cheap price of the battery and the constraint  $P_B \eta_b \leq \frac{P_{me}}{\eta_T}$ . And the optimal  
 172 battery energy capacity is approximately 29 kWh. This is due to the PFCULV's  
 173 constraint that the electric-only driving distance with an average velocity (50  
 174 km/h) is more than 40 km, but less than 60 km. It is easily observed that  
 175 varying drive cycles only affect the size of the FC system.

Table 6: Optimal results of the FC system rated power, battery rated power, and energy capacity for the PFCULV in different drive cycles.

Drive cycles	$P_{FC}$ kW	$P_B$ kW	$E_b$ kWh	$V_{\max}$ km/h	$L_a$ km	$acc_{\max}$ m/s <sup>2</sup>
WVUSUB	44.81	54.35	29.06	72.10	11.97	1.29
CSHVR	36.59	54.35	28.96	70.49	10.81	1.16
WVUCITY	30.07	54.35	28.87	57.65	5.32	1.14
CBDTRUCK	21.62	54.35	28.76	32.19	3.51	0.36

#### 176 4.2. Sensitivity to parameters of FC system

177 In this subsection, we account for a simple but nontrivially useful sensitivity  
 178 analysis to parameters of the FC system for the PFCULV, where sub-optimal  
 179 parameters of the FC system are contrasted to the optimal parameters of CP.  
 180 Given sub-optimal parameters of the FC system meeting necessary design re-  
 181 quirements (Eqs. (1), (4), and (6)), and  $E_b=29$  kWh,  $P_B = 54.35$  kW,  $c_e=1$

182 yuan/kWh, and  $c_h=40$  yuan/kg, the control results of CP for different drive  
 183 cycles are shown in Table 7. In the table,  $C_T$ ,  $C_e$ , and  $C_h$  are the total energy  
 184 cost, electricity cost, and hydrogen cost, respectively. Since the hydrogen price  
 185 is more expensive than the electricity price, and the length of drive cycle is less  
 186 than 40 km, electricity is the main energy for the vehicle propulsion. Increasing  
 187 the FC rated power leads to increased hydrogen consumption. For example, in  
 188 the WVUCITY cycle, when the FC rated power is 50 kW, the total cost increase  
 189 is up to 61.08%.

#### 190 4.3. Energy cost based on different hydrogen prices and drive cycle lengths

191 This subsection presents the PFCULV energy cost in concatenated CSHVR  
 192 drive cycles with different prices of hydrogen. The single CSHVR drive cycle  
 193 and vehicle power demand are described in Fig. 4, which represents typical  
 194 city-suburban driving for a heavy vehicle, and is suitable for ULV. Based on  
 195 optimal results of the battery energy capacity  $E_b=28.96$  kWh, the rated power  
 196  $P_B = 54.35$  kW, and the FC system rated power  $P_{FC}=36.59$  kW, the total  
 197 energy cost, electric cost, and hydrogen cost for different numbers of CSHVR  
 198 drive cycles are shown in Table 8. Two different hydrogen prices, including 40  
 199 yuan/kg and 15 yuan/kg are considered. When hydrogen is the main energy of  
 200 the PFCULV in a long trip, the price change of hydrogen will significantly affect  
 201 the fuel economy of the PFCULV. If the hydrogen price is 40 yuan/kg, when  
 202 the number of CSHVR drive cycle is not more than 5, electricity is the main  
 203 energy, and the total energy cost per km is about 0.73 yuan. When the number  
 204 of CSHVR drive cycle is more than 5, the electricity cost is constant, equal to  
 205 20.18 yuan. However, if the hydrogen price is 15 yuan/kg, hydrogen is the main  
 206 energy, and the total energy cost per km is about 0.35 yuan, and the electricity  
 207 cost is always zero. The energy cost decrease is approximately 52.05% less for  
 208 the  $c_h=15$  yuan/kWh case, relative to the  $c_h=40$  yuan/kWh case.

#### 209 4.4. Example of energy management strategy

210 Based on the optimal results of the battery energy capacity  $E_b=28.96$  kWh,  
 211 the rated power  $P_B = 54.35$  kW, the FC system rated power  $P_{FC}=36.59$  kW, the

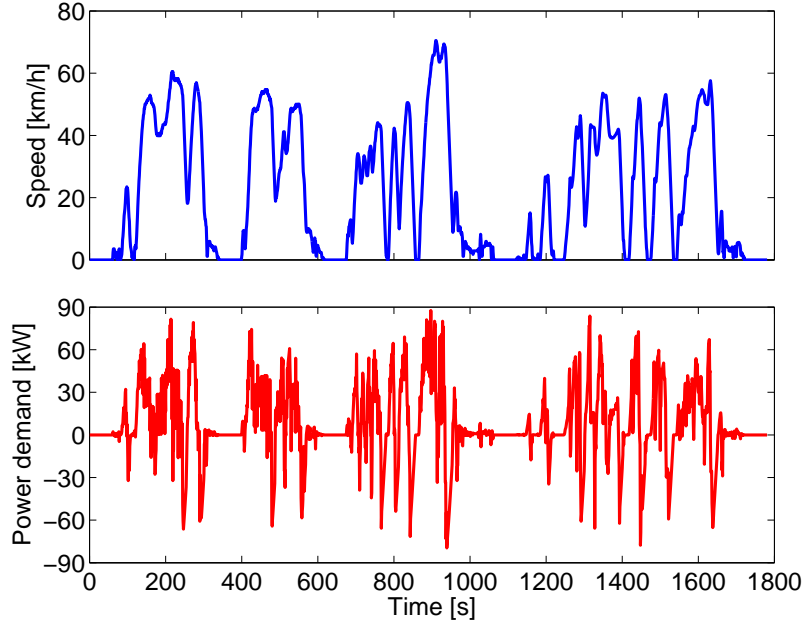


Figure 4: Speed and power demand for a single CSHVR drive cycle.

212 electricity price  $c_e=1$  yuan/kWh, and two different hydrogen prices, including  
 213 40 yuan/kg and 15 yuan/kg, this subsection presents the CP control law for the  
 214 PFCULV with different lengths of drive cycles.

215 The optimal power allocations over a single CSHVR drive cycle and 5 con-  
 216 catenated CSHVR drive cycles are described in Fig. 5, including the battery  
 217 power and the FC system power. According to Fig. 5-(b), every cycle's power  
 218 allocation is almost the same in the 5 CSHVR drive cycles.

219 The battery SOC trajectories are shown in Fig. 6. If the hydrogen price is  
 220 40 yuan/kg, the battery SOC decreases with the trip distance. If the trip length  
 221 is larger than the maximum battery-only driving distance, the end of SOC will  
 222 equal to the minimum SOC (20%). If the hydrogen price is 15 yuan/kg, the end  
 223 of battery SOC will equal to the maximum SOC (90%) with a charge sustenance,  
 224 regardless of driving mileage.

225 The battery cell power profiles and the associated SOH trajectories for a

226 single CSHVR drive cycle and 5 CSHVR drive cycles are shown in Fig. 7. For  
227 both hydrogen prices, it is evident that the majority of the battery charging  
228 occurs during vehicle deceleration, and discharging occurs during vehicle accel-  
229 eration. The change trend of the battery power is severe and overall similar  
230 to the vehicle's power demand. The change trend of the FC system power is  
231 relatively gentle, which is always located in high-efficiency regions. When the  
232 hydrogen price is 15 yuan/kg, the FC system power is higher than that in the  
233 case of 40 yuan/kg, in order to use more hydrogen.

## 234 5. Conclusions

235 This paper develops a CP framework for optimal energy management and  
236 component sizing of a PFCULV. The CP problem is mathematically formulated  
237 to optimize the power allocation between the FC system and the battery pack.  
238 At the same time, the CP strategy explicitly takes into account the optimization  
239 of the PFCULV power sources parameters. Different drive cycles and hydrogen  
240 prices are also considered in extensive simulation campaigns. Key findings are  
241 summarized below:

242 (1) Based on four different drive cycles, the optimal battery rated power is  
243 54.35 kW, and the optimal battery energy capacity is approximately 29 kWh.  
244 The varying drive cycles only affect the size of the FC system.

245 (2) The price of hydrogen affects the control strategy for the power distri-  
246 bution, which significantly influences the fuel economy of the PFCULV.

247 The proposed framework can be extended to incorporate thermal dynamics  
248 of the battery and FC systems in our future work.

## 249 References

- 250 [1] H. Chen, Z. Song, X. Zhao, T. Zhang, P. Pei, C. Liang, A review of dura-  
251 bility test protocols of the proton exchange membrane fuel cells for vehicle,  
252 Applied Energy 224 (2018) 289 – 299.

- 253 [2] T. Zhang, P. Wang, H. Chen, P. Pei, A review of automotive proton ex-  
254 change membrane fuel cell degradation under start-stop operating condi-  
255 tion, *Applied Energy* 223 (2018) 249 – 262.
- 256 [3] H. Li, A. Ravey, A. N'Diaye, A. Djerdir, A novel equivalent consumption  
257 minimization strategy for hybrid electric vehicle powered by fuel cell, bat-  
258 tery and supercapacitor, *Journal of Power Sources* 395 (2018) 262 – 270.
- 259 [4] M. G. Carignano, R. Costa-Castell, V. Roda, N. M. Nigro, S. Junco, D. Fer-  
260 oldi, Energy management strategy for fuel cell-supercapacitor hybrid vehi-  
261 cles based on prediction of energy demand, *Journal of Power Sources* 360  
262 (2017) 419 – 433.
- 263 [5] D. Y. Lee, A. Elgowainy, A. Kotz, R. Vijayagopal, J. Marcinkoski, Life-cycle  
264 implications of hydrogen fuel cell electric vehicle technology for medium-  
265 and heavy-duty trucks, *Journal of Power Sources* 393 (2018) 217 – 229.
- 266 [6] B. Lee, S. Kwon, P. Park, K. Kim, Active power management system for  
267 an unmanned aerial vehicle powered by solar cells, a fuel cell, and batteries,  
268 *IEEE Transactions on Aerospace and Electronic Systems* 50 (4) (2014) 3167  
269 – 3177.
- 270 [7] J. Chen, C. Xu, C. Wu, W. Xu, Adaptive fuzzy logic control of fuel-cell-  
271 battery hybrid systems for electric vehicles, *IEEE Transactions on Indus-  
272 trial Informatics* 14 (1) (2018) 292 – 300.
- 273 [8] P. Thounthong, V. Chunkag, P. Sethakul, B. Davat, M. Hinaje, Compara-  
274 tive study of fuel-cell vehicle hybridization with battery or supercapacitor  
275 storage device, *IEEE Transactions on Vehicular Technology* 58 (8) (2009)  
276 3892 – 3904.
- 277 [9] A. Ravey, B. Blunier, A. Miraoui, Control strategies for fuel-cell-based  
278 hybrid electric vehicles: From offline to online and experimental results,  
279 *IEEE Transactions on Vehicular Technology* 61 (6) (2012) 2452 – 2457.

- 280 [10] I. L. Sarioglu, O. P. Klein, H. Schroder, F. Kucukay, Energy management  
281 for fuel-cell hybrid vehicles based on specific fuel consumption due to load  
282 shifting, *IEEE Transactions on Intelligent Transportation Systems* 13 (4)  
283 (2012) 1772 – 1781.
- 284 [11] D. Zhou, A. Al-Durra, I. Matraji, A. Ravey, F. Gao, Online energy man-  
285 agement strategy of fuel cell hybrid electric vehicles: A fractional-order  
286 extremum seeking method, *IEEE Transactions on Industrial Electronics*  
287 65 (8) (2018) 6787 – 6799.
- 288 [12] U. R. Prasanna, P. Xuwei, A. K. Rathore, K. Rajashekara, Propulsion  
289 system architecture and power conditioning topologies for fuel cell vehicles,  
290 *IEEE Transactions on Industry Applications* 51 (1) (2015) 640 – 650.
- 291 [13] B. Geng, J. K. Mills, D. Sun, Two-stage energy management control of  
292 fuel cell plug-in hybrid electric vehicles considering fuel cell longevity, *IEEE*  
293 *Transactions on Vehicular Technology* 61 (2) (2012) 498 – 508.
- 294 [14] H. B. Jensen, E. Schaltz, P. S. Koustrup, S. J. Andreasen, S. K. Kaer,  
295 Evaluation of fuel-cell range extender impact on hybrid electrical vehicle  
296 performance, *IEEE Transactions on Vehicular Technology* 62 (1) (2013) 50  
297 – 60.
- 298 [15] Y. Zhang, Y. Mou, Z. Yang, An energy management study on hybrid power  
299 of electric vehicle based on aluminum air fuel cell, *IEEE Transactions on*  
300 *Applied Superconductivity* 26 (7) (2016) 1 – 6.
- 301 [16] A. Ravey, N. Watrin, B. Blunier, D. Bouquain, A. Miraoui, Energy-source-  
302 sizing methodology for hybrid fuel cell vehicles based on statistical descrip-  
303 tion of driving cycles, *IEEE Transactions on Vehicular Technology* 60 (9)  
304 (2011) 4164 – 4174.
- 305 [17] M. Pourabdollah, B. Egardt, N. Murgovski, A. Grauers, Convex optimiza-  
306 tion methods for powertrain sizing of electrified vehicles by using differ-

- 307 ent levels of modeling details, *IEEE Transactions on Vehicular Technology*  
308 67 (3) (2018) 1881 – 1893.
- 309 [18] X. Hu, N. Murgovski, L. Johannesson, B. Egardt, Optimal dimensioning  
310 and power management of a fuel cell/battery hybrid bus via convex pro-  
311 gramming, *IEEE/ASME Transactions on Mechatronics* 20 (1) (2015) 457  
312 – 468.
- 313 [19] S. Ahmadi, S. Bathaee, A. H. Hosseinpour, Improving fuel economy and  
314 performance of a fuel-cell hybrid electric vehicle (fuel-cell, battery, and  
315 ultra-capacitor) using optimized energy management strategy, *Energy Con-  
316 version and Management* 160 (2018) 74 – 84.
- 317 [20] X. Han, F. Li, T. Zhang, T. Zhang, K. Song, Economic energy management  
318 strategy design and simulation for a dual-stack fuel cell electric vehicle,  
319 *International Journal of Hydrogen Energy* 42 (16) (2017) 11584 – 11595.
- 320 [21] F. Martel, S. Kelouwani, Y. Dube, K. Agbossou, Optimal economy-based  
321 battery degradation management dynamics for fuel-cell plug-in hybrid elec-  
322 tric vehicles, *Journal of Power Sources* 274 (2015) 367 – 381.
- 323 [22] K. Song, F. Li, X. Hu, L. He, W. Niu, S. Lu, T. Zhang, Multi-mode energy  
324 management strategy for fuel cell electric vehicles based on driving pattern  
325 identification using learning vector quantization neural network algorithm,  
326 *Journal of Power Sources* 389 (2018) 230 – 239.
- 327 [23] R. A. Fernandez, S. C. Caraballo, F. B. Cilleruelo, J. A. Lozano, Fuel op-  
328 timization strategy for hydrogen fuel cell range extender vehicles applying  
329 genetic algorithms, *Renewable and Sustainable Energy Reviews* 81 (2018)  
330 655 – 668.
- 331 [24] C. Geng, X. Jin, X. Zhang, Simulation research on a novel control strategy  
332 for fuel cell extended-range vehicles, *International Journal of Hydrogen  
333 Energy*, Available online.

- 334 [25] M. Carignano, R. Costa-Castello, N. Nigro, S. Junco, A novel energy  
335 management strategy for fuel-cell/supercapacitor hybrid vehicles, IFAC-  
336 PapersOnLine 50 (1) (2017) 10052 – 10057, 20th IFAC World Congress.
- 337 [26] X. Hu, L. Johannesson, N. Murgovski, B. Egardt, Longevity-conscious di-  
338 mensioning and power management of the hybrid energy storage system in  
339 a fuel cell hybrid electric bus, Applied Energy 137 (2015) 913 – 924.
- 340 [27] Z. W. Wei Yueyuan, L. Yi, A study on the matching and optimization of  
341 fchev powertrain, Automotive Engineering 30 (10) (2008) 918 – 922 + 933,  
342 (in Chinese).
- 343 [28] A. Purvins, M. Sumner, Optimal management of stationary lithium-ion  
344 battery system in electricity distribution grids, Journal of Power Sources  
345 242 (2013) 742 – 755.
- 346 [29] X. Wu, X. Hu, X. Yin, S. J. Moura, Stochastic optimal energy management  
347 of smart home with pev energy storage, IEEE Transactions on Smart Grid  
348 9 (3) (2018) 2065 – 2075.
- 349 [30] S. Ebbesen, P. Elbert, L. Guzzella, Battery state-of-health perceptive en-  
350 ergy management for hybrid electric vehicles, IEEE Transactions on Vehic-  
351 ular Technology 61 (7) (2012) 2893 – 2900.
- 352 [31] L. Johannesson, N. Murgovski, S. Ebbesen, B. Egardt, E. Gelso, J. Hell-  
353 gren, Including a battery state of health model in the hev component sizing  
354 and optimal control problem, IFAC Proceedings Volumes 46 (21) (2013) 398  
355 – 403, 7th IFAC Symposium on Advances in Automotive Control.
- 356 [32] N. Murgovski, Cones: Matlab code for convex optimization in electromo-  
357 bility studies (<https://research.chalmers.se/en/publication/?id=192858>).
- 358 [33] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge, U.K.:  
359 Cambridge Univ. Press, 2004.



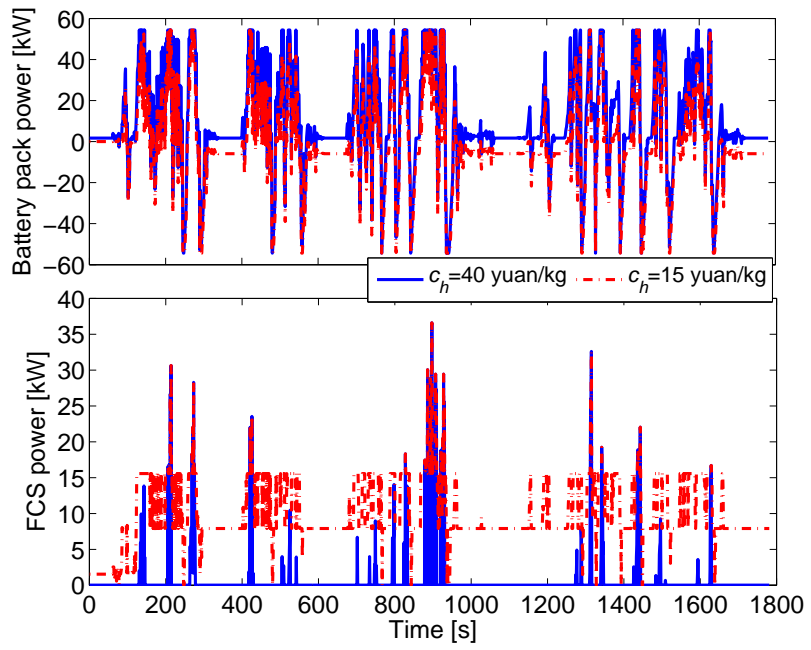
- 360 [34] X. Wu, X. Hu, Y. Teng, S. Qian, R. Cheng, Optimal integration of a hybrid  
361 solar-battery power source into smart home nanogrid with plug-in electric  
362 vehicle, *Journal of Power Sources* 363 (2017) 277 – 283.
- 363 [35] M. Chediak, The latest bull case for electric cars: The cheapest batteries  
364 ever, Tech. rep., Bloomberg New Energy Finance (BNEF) (2017).
- 365 [36] N. Zart, Batteries keep on getting cheaper, Tech. rep., Clean Technia  
366 (2017).
- 367 [37] M. Gaithersburg, U.s. department of energy hydrogen and fuel cells pro-  
368 gram, Tech. rep., U.S. Department of Energy (2016).

Table 7: Financial analysis of sub-optimal FC system parameters in different drive cycles.

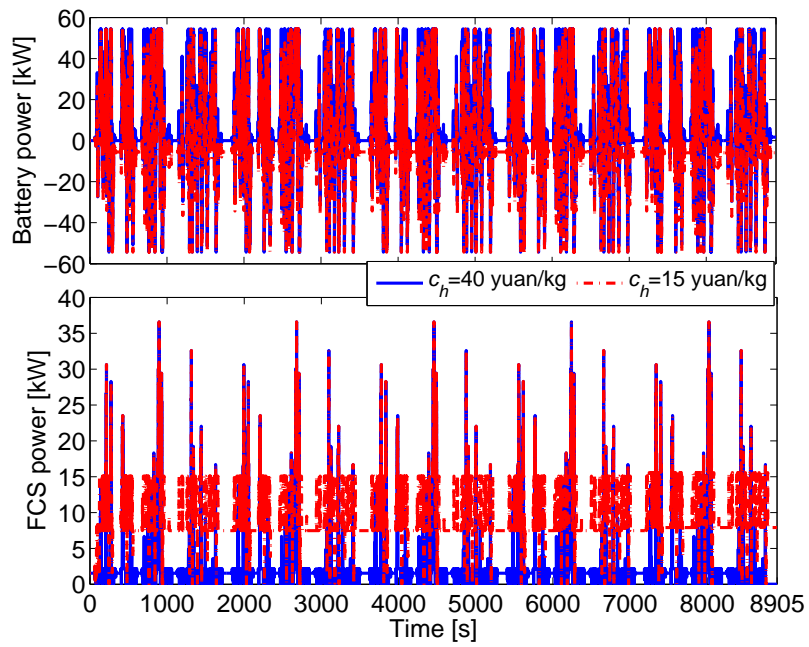
Drive cycles	$P_{FC}$ kW	$C_T$ yuan	$C_e$ yuan	$C_h$ yuan
WVUSUB	50	9.48	5.70	3.78
WVUSUB	45	9.17	5.69	3.48
WVUSUB	44.81 (optimal)	9.15	5.69	3.46
CSHVR	50	8.63	4.80	3.83
CSHVR	45	8.29	4.80	3.49
CSHVR	40	7.96	4.79	3.17
CSHVR	36.59 (optimal)	7.73	4.79	2.94
WVUCITY	50	5.32	2.63	2.69
WVUCITY	45	5.06	2.62	2.44
WVUCITY	40	4.80	2.62	2.18
WVUCITY	35	4.54	2.62	1.92
WVUCITY	30.07 (optimal)	4.28	2.61	1.67
CBDTRUCK	50	3.11	1.59	1.52
CBDTRUCK	45	2.96	1.59	1.37
CBDTRUCK	40	2.80	1.58	1.22
CBDTRUCK	35	2.64	1.58	1.06
CBDTRUCK	30	2.49	1.58	0.91
CBDTRUCK	25	2.33	1.57	0.76
CBDTRUCK	21.62 (optimal)	2.23	1.57	0.66

Table 8: Costs for concatenated CSHVR drive cycles based on different prices of hydrogen.

cycles	$L_a$	$c_h$	$C_T$	$C_e$	$C_h$
	km	yuan/kg	yuan	yuan	yuan
1*CSHVR	10.8	40	7.73	4.79	2.94
2*CSHVR	21.6	40	15.46	9.57	5.89
3*CSHVR	32.4	40	23.19	14.36	8.83
4*CSHVR	43.2	40	30.92	19.15	11.77
5*CSHVR	54.0	40	39.51	20.18	19.33
6*CSHVR	64.8	40	49.69	20.18	28.51
7*CSHVR	75.6	40	58.19	20.18	38.01
8*CSHVR	86.4	40	67.83	20.18	47.65
1*CSHVR	10.8	15	3.74	0	3.74
2*CSHVR	21.6	15	7.48	0	7.48
3*CSHVR	32.4	15	11.21	0	11.21
4*CSHVR	43.2	15	14.95	0	14.95
5*CSHVR	54.0	15	18.68	0	18.68
6*CSHVR	64.8	15	22.42	0	22.42
7*CSHVR	75.6	15	26.15	0	26.15
8*CSHVR	86.4	15	29.89	0	29.89



(a) Single CSHVR drive cycle.



(b) 5 CSHVR drive cycles

Figure 5: Power distribution based on different numbers of CSHVR drive cycles.

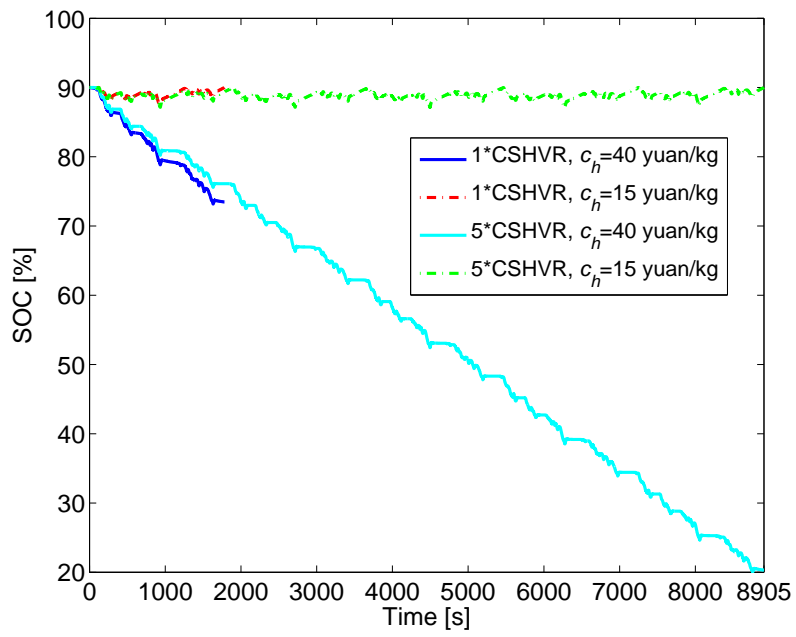
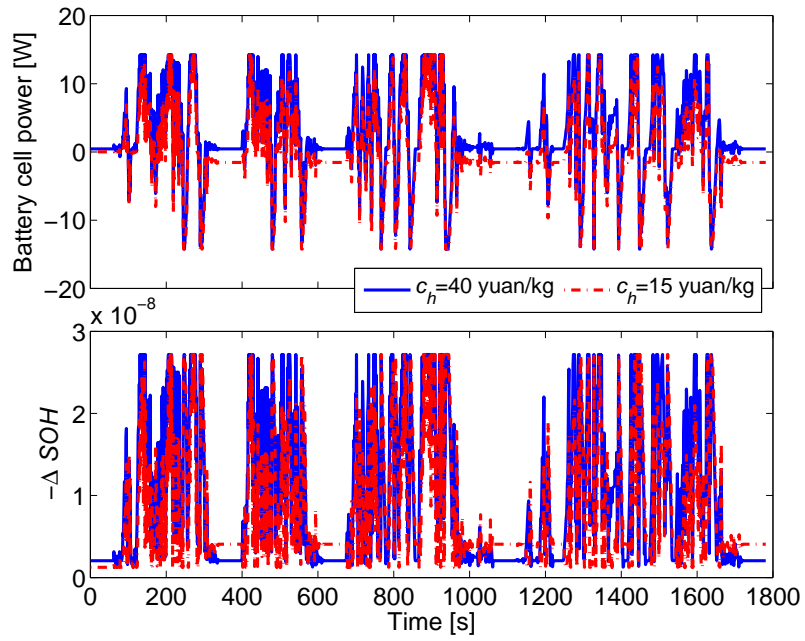
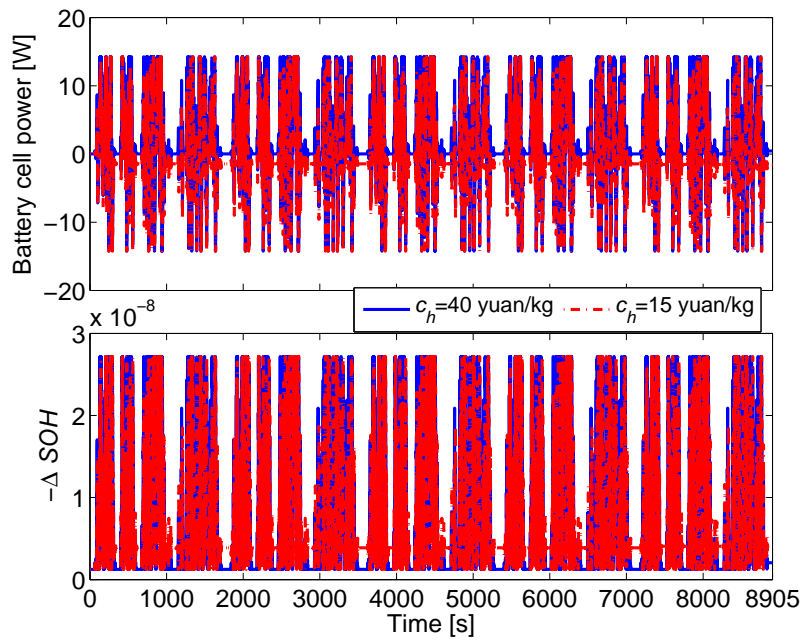


Figure 6: Battery SOC.



(a) Single CSHVR drive cycle.



(b) 5 CSHVR drive cycles

Figure 7: Battery cell power and SOH degradation.