Surface density function evolution in spherically expanding flames in globally stoichiometric droplet-laden mixtures

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ABSTRACT

The statistical behaviours of the magnitude of the reaction progress variable gradient (alternatively known as the Surface Density Function (SDF)) and the strain rates, which affect the SDF evolution, have been analysed using three-dimensional Direct Numerical Simulations (DNS) of spherically expanding flames in globally stoichiometric initially mono-sized droplet-laden mixtures for different initial turbulence intensities and droplet diameters. It has been found that gaseous phase combustion predominantly takes place under fuel-lean mode and this tendency strengthens for large droplets and high turbulence intensities. The mean values of flame displacement speed, dilatation rate and normal strain rate decrease with increasing turbulence intensity and droplet diameter. By contrast, the mean tangential strain rate increases with increasing turbulence intensity for all droplet diameters. The mean normal strain rate induced by flame propagation remains negative but its magnitude decreases with increasing droplet size and turbulence intensity. The mean tangential strain rate induced by flame propagation (alternatively curvature stretch rate) assumes negative values except for the laminar flame with small droplets. The mean effective normal strain rate has been found to assume predominantly positive values and increases with increasing turbulence intensity for the droplet cases considered here. The mean effective tangential strain rate (alternatively stretch rate) is found to assume mostly negative values except for the laminar case with small droplets. The mean effective tangential strain rate decreases with increasing droplet size, which leads to a smaller extent of flame surface area generation for flames with larger droplets.

Keywords: Droplet combustion, spherically expanding flame, Surface Density Function, Direct Numerical Simulations.
1. INTRODUCTION

The statistics of the reactive scalar gradient play a significant role in the analysis and modelling of flame-turbulence interaction. The magnitude of the reaction progress variable gradient $|\nabla c|$ is alternatively known as the Surface Density Function (SDF) (Vervisch et al., 1995 and Kollmann et al., 1998). The SDF and its evolution are of fundamental importance because of its close relation to the generalised Flame Surface Density (FSD) $\Sigma_{gen} = \overline{|\nabla c|}$ (Boger et al., 1998) (with overbar representing Reynolds averaging/filtering operation, as appropriate) and Scalar Dissipation Rate (SDR) $N_c = D|\nabla c|^2$ (Chakraborty et al., 2011) (where $D$ is the reaction progress variable diffusivity). Moreover, the SDF plays a key role in the analysis of pocket formation (Kollmann et al., 1998) and the inverse of the peak value of the SDF can be taken as a measure of flame thickness (Sankaran et al., 2007; Dopazo et al., 2018; Chakraborty et al., 2018). The statistical behaviour of the SDF and different terms of its transport equation have been extensively studied for turbulent premixed flames from various viewpoints (Chakraborty and Cant, 2005; Chakraborty and Swaminathan, 2007; Kim and Pitsch, 2007; Sankaran et al., 2007; Chakraborty and Klein, 2008; Chakraborty et al., 2008; Chakraborty and Klein, 2009; Dopazo et al., 2015a; Dopazo et al., 2015b; Dopazo and Cifuentes, 2016; Wang et al., 2017; Dopazo et al., 2018; Chakraborty et al., 2018; Klein et al., 2018; Sandeep et al., 2018). In comparison to purely gaseous premixed turbulent flames, relatively limited effort has been directed to the similar analyses for turbulent flame propagation in droplet-laden mixtures (Wacks and Chakraborty, 2016) in spite of its relevance in compression ignition and direct injection engines. The evolution of the SDF in droplet-laden mixtures is more complex than in turbulent premixed flames due to the simultaneous interactions of turbulence, flame and droplets. A recent analysis by Wacks and Chakraborty (2016) focussed on the SDF evolution for statistically planar flames propagating into droplet-laden mixtures for different turbulence intensities, droplet sizes and overall (liquid+gaseous phases) equivalence ratios and reported
important differences between gaseous premixed and droplet-laden cases. Moreover, most of the aforementioned analyses on the statistical behaviours of the SDF and is transport characteristics have been carried out for statistically planar flames but the SDF statistics in spherically expanding turbulent premixed flames can be different from those in statistically planar flames (Chakraborty and Klein, 2009; Chakraborty et al., 2007; Chakraborty et al., 2010). Mizutani and Nishimoto (1973) demonstrated that the burning characteristics of spherically expanding droplet-laden flames could be significantly different to the corresponding planar flames. However, the statistics of the SDF and the strain rates, which affect the SDF evolution, are yet to be analysed for turbulent spherically expanding flames in droplet-laden mixtures. The current analysis addresses the aforementioned gap in the existing literature by analysing the statistical behaviours of the SDF and different strain rates, which affect its evolution. For this purpose, a Direct Numerical Simulation (DNS) database (Ozel Erol et al., 2018,2019a,b) of spherically expanding flames propagating into mono-sized fuel-droplets for a range of different initial droplet diameters ($a_d/\delta_{st} = 0.04, 0.05, 0.06$ where $a_d$ is the droplet diameter and $\delta_{st}$ is the thermal flame thickness of the stoichiometric mixture) and turbulence intensities ($u'/S_{b(\phi_g=1)} = 0.0,4.0,8.0$ where $S_{b(\phi_g=1)}$ is the unstrained laminar burning velocity of the stoichiometric mixture and $u'$ is the root-mean-square velocity) at an overall equivalence ratio of unity (i.e. $\phi_{ov} = \phi_{lv} + \phi_g = 1.0$ considering fuel both in liquid and gaseous phases) has been considered. Thus, the main objectives of this paper are:

(a) To demonstrate and explain the effects of turbulence intensity and droplet diameter on the strain rates, which affect the evolution of the SDF in turbulent spherically expanding flames in droplet-mists.

(b) To indicate the implications of the above physical information in the context of modelling of turbulent combustion in droplet-laden mixtures.

The rest of this paper is organised as follows. The mathematical background and numerical
implementation pertaining to this analysis are presented next. Following that, the results are presented and subsequently discussed. Finally, the main findings are summarised and conclusions are drawn.

2. MATHEMATICAL BACKGROUND AND NUMERICAL IMPLEMENTATION

In the present analysis, a single-step irreversible reaction (i.e. \( \text{Fuel} + s \text{Oxidiser} \rightarrow (1 + s)\text{Products} \)) with the modified Arrhenius expression (Tarrazo et al., 2006) has been considered for computational economy so that a detailed parametric analysis can be conducted. In the context of this thermo-chemistry, the activation energy and the heat of combustion are taken to be functions of the gaseous equivalence ratio, \( \phi_g \) in order to capture a realistic equivalence ratio dependence of the unstrained laminar burning velocity \( S_b(\phi_g) \) in hydrocarbon-air flames (Tarrazo et al., 2006). The Lewis numbers of all species are assumed to be unity and all species in the gaseous phase are taken to be perfect gases. Standard values have been considered for the ratio of specific heats \( \gamma = C_p^g / C_v^g = 1.4 \), where \( C_p^g \) and \( C_v^g \) are the gaseous specific heats at constant pressure and volume, respectively) and Prandtl number \( Pr = \mu C_p^g / \lambda = 0.7 \) where \( \mu \) is the dynamic viscosity and \( \lambda \) is the thermal conductivity of the gaseous phase). Standard compressible Navier-Stokes equations for reactive flows for the carrier gas phase are solved in an Eulerian frame of reference, and these equations can be given by the following generic form (Wacks and Chakraborty, 2016; Reveillon and Vervisch, 2000; Wang and Rutland, 2005; Sreedhara and Huh, 2007; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou at al., 2010; Wacks et al., 2016):

\[
\frac{\partial (\rho \psi)}{\partial t} + \frac{\partial (\rho u_i \psi)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_{\psi} \frac{\partial \psi_1}{\partial x_j} \right) + \dot{w}_\psi + \dot{S}_g + \dot{S}_\psi \tag{1}
\]

where \( \rho \) is density, \( u_i \) is the \( i^{th} \) component of fluid velocity, and \( \psi = \{1, u_i, e, Y_F, Y_O\} \) and \( \psi_1 = \{1, u_i, T, Y_F, Y_O\} \) are considered for the conservation equations of mass, momentum, energy, and
mass fractions, respectively and $\Gamma_\psi = \rho v/\sigma_\psi$ for $\psi = \{1, u_l, Y_F, Y_O\}$ and $\Gamma_\psi = \lambda$ for $\psi = e$, respectively. Here, $e = C_\psi^g \left( T - T_{ref} \right) + u_k u_k/2$ represents the specific stagnation internal energy with $T$ being the dimensional temperature. The $\dot{S}_\psi$ term arises due to chemical reaction rate, $\dot{S}_g$ is an appropriate source/sink term in the gaseous phase and $\dot{S}_\psi$ is the appropriate source term due to droplet evaporation, which is tri-linearly interpolated from the droplet’s sub-grid position, $\tilde{x}_d$, to the eight surrounding nodes. Other variables are $\nu$, kinematic viscosity, and $\sigma_\psi$, an appropriate Schmidt number corresponding to $\psi$. The droplet source term for any variable $\psi$ is expressed as (Wacks and Chakraborty, 2016; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou at al., 2010; Wacks et al., 2016): $\dot{S}_\psi = -(1/V) \sum d(m_d \psi_d)/dt$ where, $V$ is the cell volume, $m_d = \rho_d(1/6)\pi a_d^3$ is the droplet mass. A Lagrangian approach is adopted to solve the quantities transported for each droplet, which are position, $\tilde{x}_d$, velocity, $\tilde{u}_d$, diameter, $a_d$ and temperature, $T_d$ of droplets. The transport equations for these quantities are (Wacks and Chakraborty, 2016; Reveillon and Vervisch, 2000; Wang and Rutland, 2005; Sreedhara and Huh, 2007; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou at al., 2010, 2012; Wacks et al., 2016):

\[
\frac{d\tilde{x}_d}{dt} = \tilde{u}_d; \quad \frac{d\tilde{u}_d}{dt} = \frac{\tilde{u}(\tilde{x}_d, t) - \tilde{u}_d}{\tau_u^d}; \quad \frac{d a_d^2}{dt} = -\frac{a_d^2}{\tau_a^d}; \quad \frac{dT_d}{dt} = \frac{T(\tilde{x}_d, t) - T_d - B_d L_v / C_p^g}{\tau_T^d}.
\]

(2)

where $L_v$ is the latent heat of vaporization, and $\tau_u^d, \tau_a^d$ and $\tau_T^d$ are relaxation timescales for velocity, droplet diameter and temperature respectively, which are defined as (Wacks and Chakraborty, 2016; Reveillon and Vervisch, 2000; Wang and Rutland, 2005; Sreedhara and Huh, 2007; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou at al., 2010, 2012; Wacks et al., 2016):

\[
\tau_u^d = \frac{\rho_d a_d^2}{18 C_a \mu}; \quad \tau_a^d = \frac{\rho_d a_d^2}{4 \mu} \frac{Sc}{Sh_c \ln(1 + B_d)}; \quad \tau_T^d = \frac{\rho_d a_d^2}{6 \mu} \frac{Pr}{N_c \ln(1 + B_d)} \frac{B_d}{C_p^g} \frac{C_p^L}{C_p^g}.
\]

(3)

where $\rho_d$ is the droplet density, $Sc$ is the Schmidt number, $C_p^L$ is the specific heat for the liquid...
phase, \( C_u = 1 + \frac{Re_d^{2/3}}{6} \) with the droplet Reynolds number \( Re_d \), \( B_d \) is the Spalding number, \( Sh_c \) is the corrected Sherwood number, \( Nu_c \) is the corrected Nusselt number and the aforementioned non-dimensional numbers are defined as (Wacks and Chakraborty, 2016; Reveillon and Vervisch, 2000; Wang and Rutland, 2005; Sreedhara and Huh, 2007; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou et al., 2010, 2012; Wacks et al., 2016):

\[
Re_d = \frac{\rho |\vec{u}(x_d, t) - \vec{u}_d|a_d}{\mu} \quad B_d = \frac{Y_F^s - Y_F(x_d, t)}{1 - Y_F^s};
\]

\[
Sh_c = Nu_c = 2 + \frac{0.555Re_dSc}{(1.232 + Re_dSc^{2/3})^{1/2}}
\]

Here, \( Y_F^s \) is the value of \( Y_F \) at the surface of the droplet and the partial pressure of the fuel vapour at the droplet surface \( p_F^s \) are given by (Reveillon and Vervisch, 2000; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou et al., 2010, 2012; Wacks and Chakraborty, 2016; Wacks et al., 2016):

\[
p_F^s = p_{ref} \exp \left( L_v \left( \frac{1}{RT_{ref}^s} - \frac{1}{RT_d^s} \right) \right); \quad Y_F^s = \left( 1 + \frac{W_{air}}{W_F} \left[ p(x_d, t) \right] - 1 \right)^{-1}
\]

In Eq. 5, \( T_{ref}^s \) is the boiling point of the fuel at pressure \( p_{ref} \), \( R \) is the gas constant and \( T_d^s \) is assumed to be \( T_d \), and \( W_{air} \) and \( W_F \) are the molecular weights of air and fuel, respectively.

A reaction progress variable, \( c \) based on oxygen mass fraction, \( Y_O \) and mixture fraction, \( \xi = (Y_F - Y_O/s + Y_{O∞}/s)/Y_{F∞} + Y_{O∞}/s \) can be defined as (Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Wacks and Chakraborty, 2016; Wacks et al., 2016):

\[
c = \frac{[(1 - \xi)Y_{O∞} - Y_O]}{[(1 - \xi)Y_{O∞} - \max(0, (\xi_{st} - \xi)/\xi_{st}) Y_{O∞}]} \quad (6)
\]

where \( Y_{O∞} = 0.233 \) is the oxygen mass fraction in air and \( Y_{F∞} = 1.0 \) is the fuel mass fraction in the pure fuel stream. Considering n-heptane, \( C_7H_{16} \) as the fuel in the analysis, \( s = 3.52 \) denotes the stoichiometric mass ratio of oxidiser to fuel and \( Y_{Fst} = \xi_{st} = 0.0621 \) represents the
corresponding fuel mass fraction and mixture fraction, respectively.

The transport equation of the reaction progress variable, $c$ can be obtained by using the transport equations for oxygen mass fraction, $Y_O$ and mixture fraction, $\xi$ in the following manner (Wacks and Chakraborty, 2016; Ozel erol et al., 2018; Wacks et al., 2016):

$$\rho \frac{\partial c}{\partial t} + \rho u_j \frac{\partial c}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \rho D \frac{\partial c}{\partial x_j} \right) + \dot{w}_c + \dot{S}_c + \dot{A}_c$$  \hspace{1cm} (7)

In Eq. 7, $\partial (\rho D \frac{\partial c}{\partial x_j}) / \partial x_j$ is the molecular diffusion term, $\dot{w}_c$ indicates the reaction rate, $\dot{S}_c$ represents the source/sink term arising due to droplet evaporation and $\dot{A}_c$ is the cross-scalar dissipation term arising due to mixture inhomogeneity. The molecular diffusion term can be rewritten by splitting into normal and tangential components as (Dopazo et al., 2015a,b; Dopazo and Cifuentes, 2016; Dopazo et al., 2018; Chakraborty et al., 2018; Sandeep et al., 2018; Klein et al., 2018; Wacks and Chakraborty, 2016): $\partial (\rho D \frac{\partial c}{\partial x_j}) / \partial x_j = \partial (\rho D \frac{\partial c}{\partial x_N}) / \partial x_N + 2 \rho D \kappa_m \frac{\partial c}{\partial x_N}$ where $\kappa_m = 0.5(\partial N_i / \partial x_i)$ is the local mean curvature and $N_i$ is the flame normal vector which can be defined as $N_i = -(\partial c / \partial x_i) / |\nabla c|$. The reaction rate of the reaction progress variable $\dot{w}_c$ can be expressed as (Wacks and Chakraborty, 2016; Wacks et al., 2016; Ozel Erol et al., 2018, 2019a,b):

$$\dot{w}_c = -\xi_{st} \dot{w}_o / [Y_{O\infty} \xi (1 - \xi_{st})] \text{ for } \xi \leq \xi_{st}; \text{ } \dot{w}_c = -\dot{w}_o / [Y_{O\infty} (1 - \xi)] \text{ for } \xi > \xi_{st}$$  \hspace{1cm} (8)

The terms due to droplet evaporation $\dot{S}_c$ and mixture inhomogeneity $\dot{A}_c$ are given as (Wacks and Chakraborty, 2016; Wacks et al., 2016; Ozel Erol et al., 2018, 2019a,b):

$$\dot{S}_c = -[\xi_{st} / Y_{O\infty} \xi^2 (1 - \xi_{st})] \cdot \left[ \xi \dot{S}_o + (Y_{O\infty} - Y_O) \dot{S}_\xi \right] \text{ for } \xi \leq \xi_{st};$$

$$\dot{S}_c = -[1 / Y_{O\infty} (1 - \xi)^2] \cdot \left[ (1 - \xi) \dot{S}_o + Y_O \dot{S}_\xi \right] \text{ for } \xi > \xi_{st}$$  \hspace{1cm} (9)

$$\dot{A}_c = 2 \rho D \nabla \xi \cdot \nabla c / \xi \text{ for } \xi \leq \xi_{st}; \text{ } \dot{A}_c = -2 \rho D \nabla \xi \cdot \nabla c / (1 - \xi) \text{ for } \xi > \xi_{st}$$  \hspace{1cm} (10)

where $\dot{w}_o$ is the reaction rate of oxidiser, $\dot{S}_\xi = (\dot{S}_F - \dot{S}_o / s) / (Y_{F\infty} + Y_{O\infty} / s)$ is the droplet
source/sink term in the mixture fraction transport equation, $\dot{S}_F = \Gamma_m (1 - Y_F)$ and $\dot{S}_O = -\Gamma_m Y_O$ are the droplet source/sink terms in the species transport equation for fuel and oxygen, respectively and $\Gamma_m$ is the source term in the mass conservation equation due to evaporation.

Alternatively, Eq. 7 can be rewritten in the kinetic form as (Peters et al., 1998; Echekki and Chen, 1999; Dopazo et al., 2015a,b; Dopazo and Cifuentes, 2016; Wacks and Chakraborty, 2016; Wacks et al., 2016; Chakraborty et al., 2018; Dopazo et al., 2018; Sandeep et al., 2018; Klein et al., 2018):

$$\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = S_d |\nabla c|$$

where $S_d$ is the displacement speed, which is given as (Peters et al., 1998; Echekki and Chen, 1999; Wacks and Chakraborty, 2016; Wacks et al., 2016):

$$S_d = \frac{\nabla \cdot (\rho D \nabla c) + \dot{\omega}_c + \dot{A}_c + \dot{S}_c}{\rho |\nabla c|} = S_n + S_t + S_r + S_z + S_s$$

where $S_n, S_t, S_r$ are the normal diffusion, tangential diffusion, reaction components and $S_z$ and $S_s$ are the contributions due to cross-scalar dissipation term and droplet evaporation, respectively, which are given as (Wacks and Chakraborty, 2016 and Wacks et al., 2016):

$$S_n = \frac{\vec{N} \cdot \nabla (\rho D \vec{N} \cdot \nabla c)}{\rho |\nabla c|}; S_t = -2D \kappa_m; S_r = \frac{\dot{\omega}_c}{\rho |\nabla c|}; S_z = \frac{\dot{A}_c}{\rho |\nabla c|}; S_s = \frac{\dot{S}_c}{\rho |\nabla c|}$$

The magnitude of the gradient $c$, is often referred to as the Surface Density Function (SDF), and its transport equation takes the following form (Pope, 1988; Candel and Poinson, 1990; Chakraborty and Cant, 2005; Dopazo et al., 2015a,b; Dopazo and Cifuentes, 2016; Dopazo et al., 2018; Chakraborty et al., 2018; Sandeep et al., 2018; Klein et al., 2018; Wacks and Chakraborty, 2016):

$$\frac{\partial |\nabla c|}{\partial t} + v_j \frac{\partial |\nabla c|}{\partial x_j} = - \left( a_N + N_j \frac{\partial S_d}{\partial x_j} \right) |\nabla c|$$
\[
\frac{\partial |\nabla c|}{\partial t} + \frac{\partial (v_j |\nabla c|)}{\partial x_j} = (a_T + 2S_d\kappa_m)|\nabla c| \tag{15}
\]

where the \(j^{th}\) component of the flame propagation velocity is given by \(v_j = u_j + S_dN_j\) and the flame normal strain rate is stated as \(a_N = N_iN_j \partial u_i/\partial x_j\). The fractional temporal changes of the infinitesimal normal distance between two neighbouring \(c\) isosurfaces \((1/\Delta N)(d\Delta x_N/dt)\) and the surface area element \((1/A)(dA/dt)\) are related to the flame normal strain rate, \(a_N\) and tangential strain rate, \(a_T = \nabla \cdot \vec{u} - a_N = (\delta_{ij} - N_iN_j) \partial u_i/\partial x_j\) as (Pope, 1988; Candel and Poinsot, 1990; Dopazo et al., 2015a,b):

\[
(1/\Delta N)(d\Delta x_N/dt) = a_N + \partial S_d/\partial x_N = a_N^{\text{eff}} \tag{16}
\]

\[
(1/A)(dA/dt) = a_T + 2\kappa_mS_d = a_T^{\text{eff}} \tag{17}
\]

where \(\nabla \cdot \vec{u}\) is dilatation rate, and \(d(\ )/dt = \partial(\ )/\partial t + v_j \partial(\ )/\partial x_j\) is the total derivative based on \(v_j = u_j + S_dN_j\). In Eqs. 16-17, \(a_T^{\text{eff}}\) and \(a_N^{\text{eff}}\) represent the effective tangential strain rate (alternatively stretch rate (Pope, 1988; Candel and Poinsot, 1990)) and normal strain rate, respectively (Dopazo et al., 2015a,b; Dopazo and Cifuentes, 2016; Dopazo et al., 2018; Chakraborty et al., 2018; Sandeep et al., 2018; Klein et al., 2018; Wacks and Chakraborty, 2016).

The DNS database used for this analysis is generated using a three-dimensional compressible code SENGa+ (Chakraborty and Cant, 2005; Chakraborty and Swaminathan, 2007; Chakraborty and Klein, 2008; Chakraborty et al., 2008; Chakraborty and Klein, 2009; Dopazo et al., 2018; Klein et al., 2018; Wacks and Chakraborty, 2016; Chakraborty et al., 2010; Ozel Erol et al., 2018, 2019a,b; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou et al., 2010, 2012; Wacks et al., 2016), which adopts high order finite-difference (10\textsuperscript{th} order central difference scheme for internal grid points and the order of
differentiation drops gradually to one-sided 2nd order scheme at non-periodic boundaries) and explicit third-order Runge-Kutta schemes for spatial discretisation and temporal advancement, respectively. All these boundaries are assumed to be partially non-reflecting of the cubic domain of size $84.49\delta_z \times 84.49\delta_z \times 84.49\delta_z$ where $\delta_z = \alpha T_0 / S_b(\phi_g=1)$ is the Zel’dovich flame thickness with $\alpha T_0$ being the thermal diffusivity in the unburned gas. The simulation domain is discretised by a uniform Cartesian grid of $(512)^3$, which ensures about 10 grid points within $\delta_{st} = (T_{ad}(\phi_g=1) - T_0) / \max|\nabla T|_L$. The reacting flow field is initialised using COSILAB (Rotexo-Softpredict-Cosilab) for three different initial values of droplet diameter $a_d$ (i.e. $a_d / \delta_{st} = 0.04, 0.05$ and 0.06) and for an overall equivalence ratio $\phi_{ov}$ of unity (i.e. $\phi_{ov} = 1.0$) as described elsewhere (Ozel Erol et al., 2018, 2019a,b) following the analysis of Neophytou and Mastorakos (2009). A perfectly spherical flame is placed at the centre of the domain and an incompressible homogeneous isotropic velocity field is superimposed on the laminar spherical flames when the radius of the fully burned gas region (i.e. the region corresponding to reaction progress variable $c$ values greater than 0.99) $r_0$ reaches $2\delta_{st}$ (i.e. $r_0 / \delta_{st} = 2.0$). The unburned gas temperature $T_0$ is assumed to be 300 K and this implies a heat release parameter $\tau = (T_{ad}(\phi_g=1) - T_0) / T_0$ of 6.54. The turbulent flame simulations are carried out for initial values of normalised root-mean-square turbulent velocities $u' / S_b(\phi_g=1) = 4.0$ and 8.0 with a non-dimensional longitudinal integral length-scale of $L_{11} / \delta_{st} = 2.5$, and have been continued for at least 2.1 chemical time scale (i.e. $2.1 t_{chem} = 2.1 \alpha T_0 / S_b^2(\phi_g=1)$), which corresponds to about 2.0 and 4.0 initial eddy turnover times (i.e. $L_{11} / \sqrt{k}$ where $k$ is the kinetic energy evaluated over the whole domain) for $u' / S_b(\phi_g=1) = 4.0$ and 8.0 cases, respectively. By that time, the turbulent kinetic energy and its dissipation rate evaluated over the whole domain were not varying rapidly with time, and this simulation time remains comparable to several previous studies (Reveillon and Vervisch, 2000; Wang and Rutland, 2005; Sreedhara
The initial droplet number density $\rho_N$ varies between $1.38 \leq (\rho_N)^{1/3} \delta_{th} \leq 2.07$ in the unburned gas, and the liquid volume fraction remains well below 0.01. The ratio of initial droplet diameter to the Kolmogorov length scale is $a_d/\eta = 0.25, 0.32, 0.38$ for the cases of $a_d/\delta_{st} = 0.04, 0.05, 0.06$ respectively for the initial $u'/S_{b(\phi_g=1)} = 8.0$ case. The mean normalised inter-droplet distance $s_d/\eta$ ranges between 4.17 and 6.25 for the highest turbulence intensity (i.e. the initial $u'/S_{b(\phi_g=1)} = 8.0$) case considered here. The ratio of the initial droplet diameter to grid spacing used here remains comparable to several previous analyses (Wang and Rutland, 2005; Sreedhara and Huh, 2007; Wandel et al., 2009; Wandel, 2014; Schroll et al., 2009; Neophytou et al., 2010; Wacks et al., 2016). Moreover, the evaporation of droplets leads to a significant decrease in droplet diameter (by around 50%) when they reach the reaction zone. Thus, the droplets, which interact with the flame front, are much smaller than the initial size and as a result the assumption of sub-grid evaporation is not expected to affect the statistics of the SDF, which is the focal point of this analysis. Further information on the numerical implementation of this DNS database can be found elsewhere (Ozel Erol et al., 2018; 2019a,b) and thus is not repeated here for the sake of conciseness.

3. RESULTS AND DISCUSSION

Flame morphology

The distributions of $c$ in the central mid-plane for initial $u'/S_{b(\phi_g=1)} = 4.0$ and 8.0 cases at $2.1t_{chem}$ are shown in Fig. 1 for initial droplet diameters $a_d/\delta_{st} = 0.04, 0.05$ and 0.06. The droplets, which reside on this plane, are shown by grey dots. Figure 1 indicates that the droplets
evaporate and thereby shrink in size as they approach the flame. As the droplets evaporate, they release fuel in the gaseous phase, which eventually burns within the flame front. In turbulent cases with large $a_d/\delta_{st}$, some droplets escape through the flame and evaporate within the burned gas region and some of this evaporated fuel diffuses back to the flame and eventually burns. The probability density functions (PDFs) of equivalence ratio of the gaseous phase $\phi_g$ within the region corresponding to $0.01 \leq c \leq 0.99$ are shown in Fig. 2, which indicates that the evaporation of fuel is not sufficient to produce stoichiometric mixture in most locations even though the overall equivalence ratio $\phi_{ov}$ remains unity (i.e. $\phi_{ov} = 1.0$) for all cases. Thus, combustion predominantly takes place under fuel-lean mode in all cases and this tendency strengthens further with increasing droplet diameter. Figure 2 further shows that the probability of finding fuel-lean mixture increases with increasing droplet diameter due to reduced evaporation rate for large droplets. The evaporated fuel is more readily dispersed from the droplet locations into the surrounding air for higher values of $u'/S_b(\phi_g=1)$ and this gives rise to an increase in the probability of finding fuel-lean mixture in the initial $u'/S_b(\phi_g=1) = 8.0$ cases in comparison to the corresponding initial $u'/S_b(\phi_g=1) = 4.0$ cases (Ozel Erol et al., 2018, 2019a,b). This predominantly fuel-lean combustion affects the statistical behaviour of $|\nabla c|$ and the strain rates affecting its evolution.
Figure 1. Distribution of $c$ on the central $x$-$y$ mid-plane for laminar, initial $u'/S_b(\phi_g=1) = 4.0$ and 8.0 cases for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06. Grey dots show the droplets residing on the plane (not to the scale). All figures correspond to $t = 2.1t_{chem}$.

Figure 2. PDFs of $\phi_g$ in the region corresponding to $0.01 \leq c \leq 0.99$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar (---), initial $u'/S_b(\phi_g=1) = 4.0$ (---) and 8.0 (---) cases.
Mean behaviour of the SDF and displacement speed

The profiles of the mean values of $|\nabla c| \times \delta_{st}$ conditional upon $c$ are shown in Fig. 3, which shows that the peak value of the profile of $|\nabla c| \times \delta_{st}$ decreases with increasing $u'/S_b(\phi_g=1)$. Moreover, the peak mean value of $|\nabla c| \times \delta_{st}$ decreases with increasing droplet diameter. The inverse of the peak value of $|\nabla c|$ can be taken to be a measure of flame thickness (i.e. $\delta \sim 1/\max |\nabla c|$) (Sankaran et al., 2007; Dopazo et al., 2018; Chakraborty et al., 2018), and thus Fig. 3 indicates that the flame thickens in the mean sense with increasing turbulence intensity and droplet diameter. The probability of fuel-lean combustion increases with increasing $u'/S_b(\phi_g=1)$ and $a_d$ (see Fig. 2) and this contributes to the thickening of the flame. It is worth noting that the mean radii of the flames are not the same when the statistics are extracted (Ozel Erol et al., 2018). In the spherically expanding flames $|\nabla c|$ and flame curvature $\kappa_m$ remain correlated (Chakraborty and Klein, 2009; Chakraborty et al., 2007), and thus the variations of the mean values of $|\nabla c| \times \delta_{st}$ are also affected by the differences in mean flame radii of these cases.

![Figure 3](image)

**Figure 3.** Profiles of mean values of $|\nabla c| \times \delta_{st}$ conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Equations 13-17 indicate that the flame curvature affects the displacement speed behaviour, which in turn influences the evolution of $|\nabla c|$. The variations of the mean values of normalised
displacement speed and its components $S_i/S_b(\phi_g=1)$ (where $i = d, r, n, t, z, s$) conditional upon $c$ are shown in Fig. 4 and Fig 5, respectively, which reveal that $S_d/S_b(\phi_g=1)$ increases from the unburned to the burned gas side due to density drop as a result of thermal expansion. Figure 4 further shows that the mean value of the reaction component $S_r/S_b(\phi_g=1)$ remains positive but its magnitude increases from the unburned to the burned gas side of the flame, whereas the mean normal diffusion component $S_n/S_b(\phi_g=1)$ assumes small positive and large negative values towards the unburned and burned gas sides of the flame front respectively. The mean contributions of $S_x$ and $S_s$ remain smaller in magnitude than those of $S_r$ and $S_n$. The mean contribution of tangential diffusion component of displacement speed $S_t/S_b(\phi_g=1) = -2D\kappa_m/S_b(\phi_g=1)$ remains negative throughout the flame due to predominantly positive values of curvature $\kappa_m$ and the magnitude of the mean negative value increases with decreasing mean radius of the flame kernel. The magnitude of negative mean value of $S_t/S_b(\phi_g=1)$ for the initial $a_d/\delta_{st} = 0.06$ case remains considerably greater than in the corresponding cases with initial $a_d/\delta_{st} = 0.04$ and 0.05 cases because of the smaller mean flame radius (Ozel Erol et al., 2018, 2019a,b). The statistics of $S_d$ and its components influence the statistical behaviours of $N_j \partial S_d/\partial x_j$ and $2S_d\kappa_m$. 
Figure 4. Profiles of mean values of $S_i/S_b(\phi_g=1)$ (where $i = d, r, n$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04, 0.05$ and 0.06 (1st - 3rd row). for laminar (---), initial $u'/S_b(\phi_g=1) = 4.0$ (---) and 8.0 (---) cases
Figure 5. Profiles of mean values of $S_i/S_{b(\phi_g=1)}$ (where $i = t, z, s$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04, 0.05$ and $0.06$ (1st -3rd row). for laminar (---), initial $u'/S_{b(\phi_g=1)} = 4.0$ (---) and $8.0$ (---) cases

Mean behaviour of the fluid dynamic strain rates

The mean values of $\{\nabla \cdot \vec{u}, a_N, a_T = \nabla \cdot \vec{u} - a_N\} \times \delta_{st}/S_{b(\phi_g=1)}$ conditional upon $c$ are shown in Fig. 6, which shows that the mean value of $\nabla \cdot \vec{u}$ remains positive and the mean value decreases with increasing $a_d$ and $u'/S_{b(\phi_g=1)}$. It has already been shown in Fig. 2 that the probability of fuel-lean combustion increases with increasing $a_d$ and $u'/S_{b(\phi_g=1)}$. This reduces the strength of thermal expansion in the large droplet cases, and this effect is further augmented for high values of $u'/S_{b(\phi_g=1)}$, and this is reflected in the reduced magnitude of dilatation rate $\nabla \cdot \vec{u}$. The normal strain rate $a_N$ can be expressed as: $a_N = (e_\alpha \cos^2 \theta_\alpha + e_\beta \cos^2 \theta_\beta + e_\gamma \cos^2 \theta_\gamma)$ where $e_\alpha, e_\beta$ and $e_\gamma$ are the most extensive, intermediate and most compressive principal strain rates, and $\theta_\alpha, \theta_\beta$ and $\theta_\gamma$ are the angles between $\nabla c$ and the eigenvectors.
corresponding to $e_\alpha$, $e_\beta$ and $e_\gamma$, respectively (Chakraborty and Swaminathan, 2007; Dopazo et al., 2018; Chakraborty et al., 2018; Klein et al., 2018; Sandeep et al., 2018). The reactive scalar gradient $\nabla c$ aligns with the eigenvector associated with $e_\alpha$ (i.e. $\cos^2 \theta_\alpha \approx 1$) when the strain rate induced by flame normal acceleration dominates over turbulent straining (Chakraborty and Swaminathan, 2007; Kim and Pitsch, 2007; Chakraborty et al., 2010). By contrast, $\nabla c$ aligns with the eigenvector associated with $e_\gamma$ (i.e. $\cos^2 \theta_\gamma \approx 1$) when turbulent straining overwhelms the strain rate due to flame normal acceleration (Chakraborty and Swaminathan, 2007; Kim and Pitsch, 2007; Chakraborty et al., 2010). As combustion takes place increasingly in fuel-lean mode with increasing $a_d$, the flame normal acceleration weakens with increasing droplet size, which also decreases the extent of collinear alignment between $\nabla c$ with the eigenvector associated with $e_\alpha$. However, in all cases, $\nabla c$ aligns with the eigenvector associated with $e_\alpha$ in the reaction zone where the effects of heat release are strong and thus $a_N$ assumes positive values in this region. However, the extent of collinear alignment with the eigenvector associated with $e_\alpha$ decreases with increasing droplet size and turbulence intensity. Thus, $\nabla c$ aligns with the eigenvector associated with $e_\gamma$ on both unburned and burned gas sides of the flame for cases with high turbulence intensities and accordingly $a_N$ assumes negative values in these regions. The relative magnitudes of $\nabla \cdot \bar{u}$ and $a_N$ determine the variation of $a_T = \nabla \cdot \bar{u} - a_N$ across the flame. In the laminar flames $a_T$ decreases from the unburned to the burned gas side of the flame, whereas this behaviour is less prominent for the turbulent cases and the mean value of $a_T$ increases with increasing turbulence intensity because of small mean values of $a_N$ for large $u'/S_b(\phi_g=1)$ values.
Figure 6. Profiles of mean values of \{\nabla \cdot \bar{u}, a_N, a_T = \nabla \cdot \bar{u} - a_N\} \times \delta_{st}/S_{b(\phi_g=1)} conditional upon \(c\) for initial \(a_d/\delta_{st} = 0.04, 0.05\) and 0.06. for laminar (---), initial \(u'/S_{b(\phi_g=1)} = 4.0\) (----) and 8.0 (-----) cases.

Mean behaviour of the strain rates induced by flame propagation

The profiles of the normalised mean values of the normal strain rate induced by displacement speed and its components \(\partial S^i_t/\partial n = N_j \partial S_t/\partial x_j \times \delta_{st}/S_{b(\phi_g=1)}\) (where \(i = d, r, n, t + z + s\)) conditional upon \(c\) are shown in Fig. 7. The mean value \(N_j \partial S_r/\partial x_j \times \delta_{st}/S_{b(\phi_g=1)}\) assumes negative values throughout the flame because \(S_r\) predominantly increases from the unburned to the burned gas side of the flame. By contrast, \(N_j \partial S_n/\partial x_j \times \delta_{st}/S_{b(\phi_g=1)}\) assumes positive values towards the burned gas side, which is consistent with the behaviour of \(S_n\) across the flame (see Fig. 4). The magnitudes of mean values of \(N_j \partial S_r/\partial x_j\) and \(N_j \partial S_n/\partial x_j\) remain much greater than the mean contribution of \(N_j \partial S_{t+z+s}/\partial x_j\). For the laminar case negative contributions of \(N_j \partial S_r/\partial x_j\) dominate over positive contributions of \(N_j \partial S_n/\partial x_j\) to yield
predominantly negative mean values of $N_j \partial S_d / \partial x_j$ within the reaction zone but positive values can be discerned on both unburned and burned gas sides. However, an opposite behaviour is observed for turbulent cases and the mean contribution of $N_j \partial S_d / \partial x_j$ assumes higher positive values for greater values of $u'/S_{b(\phi_g=1)}$.

![Profiles of mean values of $N_j \partial S_i / \partial x_j \times \delta_{st}/S_{b(\phi_g=1)}$](image.png)

**Figure 7.** Profiles of mean values of $N_j \partial S_i / \partial x_j \times \delta_{st}/S_{b(\phi_g=1)}$ (where $i = d, r, n, t + z + s$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06. for laminar ( ), initial $u'/S_{b(\phi_g=1)} = 4.0$ ( ), and 8.0 ( ) cases.

The variations of the normalised mean values of the tangential strain rate induced by displacement speed (or curvature stretch rate) $(2S_t \kappa_m)^+ = 2S_t \kappa_m \times \delta_{st}/S_{b(\phi_g=1)}$ (where $i = d, r + n + z + s, t$) conditioned upon $c$ are shown in Fig. 8. It has been found that the mean value of $2S_d \kappa_m \times \delta_{st}/S_{b(\phi_g=1)}$ assumes mostly negative values throughout the flame except for the laminar case with initial droplet size of $a_d/\delta_{st} = 0.04$. Figure 8 further shows that the mean values of $2S_t \kappa_m \times \delta_{st}/S_{b(\phi_g=1)} = -4D \kappa_m^2 \times \delta_{st}/S_{b(\phi_g=1)}$ dominates over the mean
values of $2S_{r+n+z}s\kappa_m \times \delta_{st}/S_b(\phi_g=1)$ to yield negative mean value of $2S_d\kappa_m \times \delta_{st}/S_b(\phi_g=1)$ for all cases except for the laminar case with initial droplet size of $a_d/\delta_{st} = 0.04$. Small droplets in the initial $a_d/\delta_{st} = 0.04$ laminar case evaporate rapidly and do not impart much influence on flame wrinkling. The droplets in the initial $a_d/\delta_{st} = 0.05$ and $0.06$ cases induce considerable flame wrinkling due to flame droplet interaction under laminar conditions (Ozel Erol et al., 2018, 2019a,b). Furthermore, it has been shown elsewhere (Ozel Erol et al., 2018, 2019a,b) that the flame radius decreases with increasing droplet size for the cases considered here. For the above reasons, the contribution of $2S_t\kappa_m = -4D\kappa_m^2$ remains small for the droplet cases with initial droplet size of $a_d/\delta_{st} = 0.04$, whereas this contribution plays a more significant role in the laminar droplet cases with initial $a_d/\delta_{st} = 0.05$ and $0.06$ because of the smaller flame radii and larger extent of flame wrinkling in these cases. Thus, the mean curvature stretch remains predominantly positive due to spherical positive curvature in the laminar case with initial $a_d/\delta_{st} = 0.04$, whereas it assumes negative values for the laminar cases with initial $a_d/\delta_{st} = 0.05$ and $0.06$. The extent of flame wrinkling increases with increasing $u'/S_b(\phi_g=1)$, which widens the range of curvature $\kappa_m$ values and increases the mean values of $\kappa_m^2$ and $2S_t\kappa_m = -4D\kappa_m^2$.

Mean behaviours of the effective strain rates

The mean values of normalised effective normal and tangential strain rates (i.e. $\{a^\text{eff}_N, a^\text{eff}_T\} \times \delta_{st}/S_b(\phi_g=1))$ conditioned on $c$ are shown in Fig. 9, which shows that the mean values of $a^\text{eff}_N$ show an increasing trend with turbulence intensities for all initial values of $a_d$ considered here. Moreover, the mean value of $a^\text{eff}_N$ remains positive in turbulent cases, which suggests that $a^\text{eff}_N$ promotes thickening of the flame in a mean sense, which is consistent with the observations made from Fig. 3. The mean value of $a^\text{eff}_T$ remains mostly negative for all the cases except for
the laminar case corresponding to initial $a_d/\delta_{st} = 0.04$. In the initial $a_d/\delta_{st} = 0.04$ case, predominantly positive mean values of $2S_d\kappa_m$ along with positive mean values of $a_T$ give rise to the net positive mean value of $a_T^{\text{eff}}$ but for other cases the negative mean values of $2S_d\kappa_m$ dominate over positive mean values of $a_T$ to yield negative mean values of $a_T^{\text{eff}}$. The negative mean values of $a_T^{\text{eff}}$ due to predominantly negative mean values of $2S_d\kappa_m$ indicate smoothening of the wrinkled flame front due to flame propagation. The decreasing trend of $a_T^{\text{eff}}$ with increasing droplet size is consistent with the evolution of the normalised flame surface area $A = \int_0^L |\nabla c|dV/(4\pi r_0^2)$ (where $r_0 = 2.0\delta_{st}$) shown in Fig.10, which suggests that the normalised flame area $A/(4\pi r_0^2)$ drops with an increase in $a_d$. Further discussion of the effects of droplet size and turbulence intensity on flame surface area and overall burning rate for the cases considered here can be found elsewhere (Ozel Erol et al., 2018, 2019a,b) and thus is not repeated here.

**Modelling implications**

The transport equation of $|\nabla c|$ can alternatively be written by expanding eq. 15:

\[
\frac{\partial |\nabla c|}{\partial t} + \frac{\partial (u_j |\nabla c|)}{\partial x_j} = a_T |\nabla c| + 2S_d\kappa_m |\nabla c| - \frac{\partial (S_dN_j |\nabla c|)}{\partial x_j} \tag{18i}
\]

\[
\frac{\partial |\nabla c|}{\partial t} + \frac{\partial (u_j |\nabla c|)}{\partial x_j} = a_T^{\text{eff}} |\nabla c| - \frac{\partial (S_dN_j |\nabla c|)}{\partial x_j} \tag{18ii}
\]

The transport equation of the generalised FSD (i.e. $\Sigma_{gen} = |\nabla c|$) (Boger et al., 1998; Klein et al., 2018) can be obtained on Reynolds averaging/LES filtering eqs. 18i and 18ii.

Multiplying eq. 18i by $2|\nabla c|$ yields:

\[
\frac{\partial |\nabla c|^2}{\partial t} + u_j \frac{\partial |\nabla c|^2}{\partial x_j} = -2a_N |\nabla c|^2 - 2N_j \frac{\partial S_d}{\partial x_j} |\nabla c|^2 - S_dN_j \frac{\partial |\nabla c|^2}{\partial x_j} \tag{19i}
\]
Figure 8. Profiles of mean values of \(2S_i \kappa_m \times \delta_{st}/S_p(\phi_g=1)\) (where \(i = d, r + n + z + s, t\)) conditional upon \(c\) for initial \(a_d/\delta_{st} = 0.04, 0.05\) and \(0.06\). for laminar ( ), initial \(u'/S_p(\phi_g=1) = 4.0\) ( ) and 8.0 ( ) cases.

Further, algebraic manipulation of eq. 19 provides the transport equation of SDR \(N_c = D|\nabla c|^2\) as (Chakraborty et al., 2011; Klein et al., 2018):

\[
\rho \frac{\partial N_c}{\partial t} + \rho u_j \frac{\partial N_c}{\partial x_j} = -2\rho a_N N_c - 2\rho N_j \frac{\partial S_d}{\partial x_j} N_c - \rho S_d N_j \frac{\partial N_c}{\partial x_j} + \rho S_d N_j N_c \frac{1}{D} \frac{\partial D}{\partial x_j} + \rho N_c \frac{\partial D}{\partial t} + u_j \frac{\partial D}{\partial x_j}
\]

(19ii)

\[
\rho \frac{\partial N_c}{\partial t} + \rho u_j \frac{\partial N_c}{\partial x_j} = -2\rho a_N^{eff} N_c - \rho S_d N_j \frac{\partial N_c}{\partial x_j} + \rho S_d N_j N_c \frac{1}{D} \frac{\partial D}{\partial x_j} + \rho N_c \frac{\partial D}{\partial t} + u_j \frac{\partial D}{\partial x_j}
\]

(19iii)

Equations 18 and 19 indicate that the statistics of the strain rates \(a_N, a_T, N_j \partial S_d/\partial x_j, 2S_d \kappa_m, a_N^{eff}\) and \(a_T^{eff}\) are expected to play important roles in the evolution
of the SDF. Therefore the statistical behaviours of $a_N, a_T, N_j \partial S_d / \partial x_j, 2S_d \kappa_m, a_N^{\text{eff}}$ and $a_T^{\text{eff}}$ in response to droplet size and turbulence intensity need to be adequately addressed for the purpose of modelling FSD and SDR in flames propagating into droplet-laden mixtures. The mean behaviours of $|\nabla c|, a_N, a_T, N_j \partial S_d / \partial x_j, 2S_d \kappa_m, a_N^{\text{eff}}$ and $a_T^{\text{eff}}$ for spherically expanding flames in droplet-laden mixture have been found to be qualitatively consistent with previous findings by Wacks and Chakraborty (2016) for statistically planar flames propagating into droplet mists. This further suggests that the modelling methodologies for statistically planar turbulent spray flames are likely to be valid for spherically expanding spray flames without any need of major modifications.

![Figure 9. Profiles of mean values of $\{a_N^{\text{eff}}, a_T^{\text{eff}}\} \times \delta_{st} / S_b(\phi_g=1)$ conditional upon $c$ for initial $a_d / \delta_{st} = 0.04, 0.05$ and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.](image)
Figure 10. Temporal evolution of normalised flame surface area $A/(4\pi r_0^2)$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

4. CONCLUSIONS

The statistical behaviours of the Surface Density Function (SDF) and the strain rates, which affect its evolution, have been analysed using DNS data for different turbulence intensities and droplet diameters for spherically expanding flames in droplet-laden mixtures with an overall (gaseous+liquid) equivalence ratio of unity. It has been found that the combustion in the gaseous phase takes place predominantly under fuel-lean mode even though the overall equivalence ratio remains unity and this tendency strengthens for large droplets and high turbulence intensities. The predominance of fuel-lean combustion for large droplets and/or at high turbulence intensities acts to promote flame thickening. The increasing extent of fuel-lean combustion for the cases with large droplets and high turbulence intensities leads to a decreasing trend in the mean values of displacement speed, dilatation rate and normal strain rate. The relative magnitudes of dilatation rate and normal strain rate gives rise to positive mean values of tangential strain rate $a_T$, and the mean value of $a_T$ increases with increasing droplet size and turbulence intensity. The mean normal strain rate induced by flame propagation assumes predominantly negative values but the magnitude decreases with increasing turbulence intensity for all cases considered here. The mean tangential strain rate induced by flame propagation (alternatively curvature stretch rate) assumes negative values for the turbulent
cases considered here. However, for the laminar case with small droplets the curvature stretch rate assumes predominantly positive values due to the reduced extent of flame deformation and relatively larger flame radius in comparison to the laminar cases with larger droplet sizes. The mean effective normal strain rate remains predominantly positive and increases with increasing turbulence intensity. By contrast, mean effective tangential strain rate (alternatively stretch rate) remains mostly negative as the predominantly negative curvature stretch rate dominates over the mean positive fluid-dynamic tangential strain rate except for the laminar case with small droplets where mean values of both fluid-dynamic tangential strain rate and curvature stretch rate are positive. The mean effective tangential strain rate (alternatively stretch rate) shows a decreasing trend with increasing droplet size, which gives rise to a smaller extent of flame surface area generation due to turbulence for larger droplets. The aforementioned statistics of the SDF and strain rates, which affect the SDF evolution, are of fundamental importance for the purpose of FSD and SDR based closures. Thus, the effects of droplet diameter and turbulence intensity should be explicitly accounted for in the FSD and SDR modelling of turbulent flames propagating into droplet-laden mixtures. Although several previous analyses suggested that the SDF statistics for premixed flames obtained from simple chemistry DNS data (Chakraborty and Cant., 2005; Chakraborty and Klein., 2008; Chakraborty and Klein., 2009; Dopazo et al., 2015a; Dopazo et al., 2015b; Dopazo and Cifuentes, 2016; Dopazo et al., 2018; Klein et al., 2018) remain in good agreement with the findings from tabulated chemistry (Sandeep et al., 2018) and detailed chemistry (Chakraborty et al., 2008; Chakraborty et al., 2018; Sankaran et al., 2007; Wang et al., 2017) DNS, the present findings for flames propagating into droplet-laden mixtures based on modified single-step chemistry will need to be confirmed using detailed chemistry DNS in the future for the sake of completeness.

ETHICS STATEMENT
This work did not involve any active collection of human data.

COMPETING INTERESTS STATEMENT

We have no competing interests.

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FIGURE CAPTIONS

Figure 1. Distribution of $c$ on the central x-y mid-plane for laminar, initial $u'/S_b(\phi_g=1) = 4.0$ and 8.0 cases for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06. Grey dots show the droplets residing on the plane (not to the scale). All figures correspond to $t = 2.1t_{chem}$.

Figure 2. PDFs of $\phi_g$ in the region corresponding to $0.01 \leq c \leq 0.99$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 3. Profiles of mean values of $|\nabla c| \times \delta_{st}$ conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 4. Profiles of mean values of $S_i/S_b(\phi_g=1)$ (where $i = d, r, n$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 (1st - 3rd rows) for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 5. Profiles of mean values of $S_i/S_b(\phi_g=1)$ (where $i = t, z, s$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 (1st - 3rd rows) for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 6. Profiles of mean values of $\{\nabla \cdot \vec{u}, a_N, a_T = \nabla \cdot \vec{u} - a_N\} \times \delta_{st}/S_b(\phi_g=1)$ conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 7. Profiles of mean values of $N_j \partial S_i/\partial x_j \times \delta_{st}/S_b(\phi_g=1)$ (where $i = d, r, n, t + z + s$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.

Figure 8. Profiles of mean values of $2S_i \kappa_m \times \delta_{st}/S_b(\phi_g=1)$ (where $i = d, r + n + z + s, t$) conditional upon $c$ for initial $a_d/\delta_{st} = 0.04$, 0.05 and 0.06 for laminar ( ), initial $u'/S_b(\phi_g=1) = 4.0$ ( ) and 8.0 ( ) cases.
Figure 9. Profiles of mean values of $\{a_{N}^{eff}, a_{T}^{eff}\} \times \delta_{st}/S_{b(\phi_g=1)}$ conditional upon $c$ for initial $a_d/\delta_{st} = 0.04, 0.05$ and 0.06 for laminar ( ), initial $u'/S_{b(\phi_g=1)} = 4.0$ ( ) and 8.0 ( ) cases.

Figure 10: Temporal evolution of normalised flame surface area $A/(4\pi r_0^2)$ for initial $a_d/\delta_{st} = 0.04, 0.05$ and 0.06 for laminar ( ), initial $u'/S_{b(\phi_g=1)} = 4.0$ ( ) and 8.0 ( ) cases.