Critical Transport and Vortex Dynamics in a Thin Atomic Josephson Junction


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We study the onset of dissipation in an atomic Josephson junction between Fermi superfluids in the molecular Bose-Einstein condensation limit of strong attraction. Our simulations identify the critical population imbalance and the maximum Josephson current delimiting dissipationless and dissipative transport, in quantitative agreement with recent experiments. We unambiguously link dissipation to vortex ring nucleation and dynamics, demonstrating that quantum phase slips are responsible for the observed resistive current. Our work directly connects microscopic features with macroscopic dissipative transport, providing a comprehensive description of vortex ring dynamics in three-dimensional inhomogeneous constricted superfluids at zero and finite temperatures.

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Interest is growing in model systems that allow for investigating the interplay between resistive and dissipationless quantum transport phenomena. In this context, ultracold gases in tailored optical potentials represent an ideal framework, owing to the real-time control over the relevant parameters in experiments [1,2], combined with the ability for ab initio modeling [3,4]. A paradigmatic example is the study of the dynamics between two atomic superfluids weakly coupled through a thin tunneling barrier. This realizes a Josephson junction [5,6], which represents a minimal platform to observe both coherent quantum transport [6,7], and its breakdown driven by dissipative microscopic mechanisms [8,9].

The coherent dynamics of atomic Josephson junctions (JJs) [10–20] is governed by the competition between the charging energy $E_C$ and the Josephson tunneling energy $E_J$ [10,11]. $E_C$ relates the chemical potential difference across the tunneling barrier to the relative population imbalance between the reservoirs, and depends on interparticle interactions. $E_J$ promotes the delocalization of the superfluid across the two reservoirs and sets the maximum coherent flow through the weak link. When $E_J$ dominates, superfluid current and relative phase oscillate in quadrature at the Josephson plasma frequency. In the opposite regime, and in the absence of dissipation [11,12], the system may enter the Macroscopic Quantum Self-Trapping (MQST) regime. This is characterized by high-frequency coherent oscillations of the population imbalance around a nonzero value, driven by a monotonically increasing relative phase [10,13,15–18]. Even without thermally induced decay of the population imbalance [12,17,21], the stability of MQST depends on whether vortices nucleated inside the barrier annihilate therein [22,23], or penetrate into the superfluid reservoirs. Recent experiments with inhomogeneous three-dimensional Fermi superfluids [24,25] revealed the intimate connection between phase slippage and dissipation arising from vortices created within the barrier and shed into the superfluid. Similar effects have been studied in ring-shaped bosonic condensates [26–29], mesoscopic structures [30,31], and lower-dimensional geometries [32–34]. While vortices crossing the weak link are known to yield a finite resistance [25,27,30], the relation between microscopic vortex nucleation, dynamics, and macroscopic dissipative flow is still poorly understood.

In this work we demonstrate the connection between resistive superfluid currents and vortex ring (VR) dynamics in an atomic JJ of fermionic superfluids. We obtain the critical population imbalance and the maximum coherent current delimiting the boundary between dissipationless and dissipative transport even at finite temperatures. We find excellent agreement with recent measurements [24,25], thus clarifying their interpretation. Trap asymmetry is shown to foster the emergence of elliptical VRs exhibiting Kelvin-wave excitations, while thermal fluctuations reduce the VR lifetime.

Methodology.—Our numerical simulations are based on the experimental parameters of Ref. [25]. We consider two
molecular Bose-Einstein condensates (BECs) of about $10^5$ atom pairs of $^6$Li, weakly coupled through a thin optical barrier, at $1/k_F a \sim 4.6$ (where $k_F$ is the Fermi wave vector and $a$ the interatomic scattering length). The harmonic trapping potential is asymmetric with approximately (1:12:10) ratio along the $x$, $y$, and $z$ axes, respectively. The Gaussian barrier bisects the gas along the weakest ($x$) direction (with $\xi = 15$ Hz), with a $1/e^2$ waist, $w \sim 4\xi$, where $\xi$ is the superfluid coherence length [25]. The superfluid transport through the barrier is triggered by an initial nonzero population imbalance $\zeta_0 = \zeta_{\text{BEC}}(0)$ between the two reservoirs. Here, $\zeta_{\text{BEC}}(t) = [N_R(t) - N_L(t)]/N_{\text{BEC}}$, with $N_L(N_R)$ the BEC number in the left (right) reservoir, and $N_{\text{BEC}} = N_L + N_R$ the total condensate number. The imbalance corresponds to a chemical potential difference $\Delta \mu = \mu_L - \mu_R = E_C \zeta_0 N_{\text{BEC}}/2$. Dynamics in the $T = 0$ limit are simulated via the time-dependent Gross-Pitaevskii equation (GPE), extended to nonzero temperatures $T \lesssim 0.4 T_c$ (where $T_c$ is the BEC critical temperature), via its coupling to a collisionless Boltzmann equation [3,35,36]. We stress that the standard two-mode model [10,13] that captures both Josephson and MQST dynamics of previous experiments [16,18] is out of its validity range due to the considered values of the ratio $V_0/\mu$ and to the thinness of the junctions [36]. Although dissipative effects can be phenomenologically modeled by damped two-mode [21,57,58] and RSJ-circuitual models [7], such approaches provide limited insight into the microscopic dissipative processes.

**Dynamical regimes and phase diagram.**—We study $\zeta_{\text{BEC}}(t)$, varying both the initial population imbalance $\zeta_0$ and the barrier height $V_0$. At each value of $V_0$ we observe two distinct dynamical regimes. For $\zeta_0$ smaller than a critical value $\zeta_0 < \zeta_{\text{cr}}$, $\zeta_{\text{BEC}}(t)$ exhibits sinusoidal plasma oscillations (Josephson regime). For $\zeta_0 \geq \zeta_{\text{cr}}$, we instead observe an initial rapid decay of $\zeta_{\text{BEC}}(t)$ (dissipative regime), followed by plasma oscillations with amplitude smaller than $\zeta_{\text{cr}}$. We validate our numerics by comparing $\zeta_{\text{BEC}}(t)$ with experiments under the same conditions, finding excellent agreement [Fig. 1(a), insets]. Combining calculated and newly extracted experimental $\zeta_{\text{cr}}$ values, we map out the phase diagram delimiting Josephson and dissipative regimes as a function of the normalized barrier height $V_0/\mu(T)$ [Fig. 1(a)], where $\mu(T)$ is the chemical potential including the thermal mean-field contribution [35,36]. Increasing $V_0/\mu(T)$, the onset of dissipation appears at smaller $\zeta_{\text{cr}}$. This reproduces the observed boundary within experimental uncertainty up to $T \approx 0.3 T_c$ upon keeping the condensate number equal to the $T = 0$ case. Our findings can also be interpreted in terms of the critical current $I_{\text{max}}$ across the junction, defined as the maximum value of $I = \zeta_{\text{BEC}} N_{\text{BEC}}/2$ at $\zeta_0 = \zeta_{\text{cr}}$ [Fig. 1(b)]. Numerically, $|I|_{\text{max}}$ is well approximated by $\zeta_{\text{cr}} \omega_f N_{\text{BEC}}/2$, where $\omega_f$ is the Josephson plasma frequency. The corresponding $|I|_{\text{max}}$ from the experimentally determined $\zeta_{\text{cr}}$ and $\omega_f$ is in excellent agreement with the theoretical prediction.

The overall trend of $|I|_{\text{max}}$ against $V_0/\mu(T)$ is also quantitatively captured by extending to inhomogeneous systems an analytical model, originally developed for two homogeneous BECs weakly coupled through a rectangular barrier [20,36,59].

**Vortex ring nucleation and evolution.**—The onset of the dissipative regime for $\zeta_0 \geq \zeta_{\text{cr}}$ in [24,25] has been linked to the appearance of topological defects in the superfluid. Here, we fully characterize such dynamical features at $T = 0$, by computing the superfluid velocity $v = (\hbar/M) \nabla \phi$, where $M$ is the atom pair mass and $\phi$ the condensate phase (see later for thermal effects). Given the symmetry of our junction, we consider the $x$ component of the superfluid velocity, weighted over the transverse density in the $x = 0$ plane, $\langle v_x \rangle$ [36]; we identify three distinct dynamical stages (I, II, and III, see Fig. 2) in the nucleation process of the first VR (the emerging pattern applies to subsequent VRs). In stage I, following the Josephson-Anderson relation $Mv = -\nabla \mu$ [60–62], the
comparable to the transversal TF radius of the BEC, and radial density inhomogeneity in the barrier region, until it is accelerated superfluid flow across the junction. When \( \nabla \mu \) [Fig. 2(a)], until some time later \( \Delta \mu_{\text{slip}} \approx h/\Delta \mu \), when it has already traveled a considerable distance from the barrier edge, another VR is nucleated at the trap center [see, e.g., Fig. 2(e)]. Note that early on in stage III, before the VR exits the barrier region (i.e., before reaching the point of maximum transversal TF radius), \( R_{VR} \) continues decreasing due to the strong background density gradient.

For a deeper insight into the underlying superfluid dynamics, we decompose at \( x = 0 \) the total axial superfluid velocity \( v_x = v_f + v_w \), where \( v_f \) is the main flow velocity (which is slowly varying compared to the timescale of the early vortex dynamics) and \( v_w \) is the VR-generated swirling velocity [36]; we also initially neglect compressibility effects (addressed in the next paragraph). By the end of stage II, the shrinking VR has just left the trap center, and so the vortex contribution \( v_w \) evaluated at \( x = 0 \) (where the local superfluid velocity \( \langle v_x \rangle \) shown in Fig. 2(a) is calculated) tends to 0. This leads to a drop of \( \langle v_x \rangle \) with amplitude \( \Delta \langle v_x \rangle \sim \kappa/R_{VR} \), corresponding to the change in the axial superfluid velocity at the trap center due to the lost vortex contribution, where \( \kappa \) is the quantum of circulation [36]. This sawtooth-like profile of \( \langle v_x \rangle \) [Fig. 2(a)] is typical of phase slippage phenomena seen in superfluid helium [60–63], with the less abrupt drop found here stemming from the initial persistence of the VR within the barrier region.

The drop \( \Delta \langle v_x \rangle \) can even overcome the generating flow velocity, leading to flow reversal (i.e., backflow) in the postnucleation dynamics, in agreement with Biot-Savart calculations [36]. The amplitude of each subsequent velocity drop is reduced due to the overall decay of \( z_{\text{BEC}}(t) \).

To connect the dissipation with the microscopic VR nucleation and dynamics, and phonon emission, we decompose the temporal evolution of the BEC total kinetic energy in its incompressible \( E_{k}^{i} \), compressible \( E_{k}^{c} \), and quantum pressure \( E_{q} \) contributions [36,64]. \( E_{k}^{i} \) and \( E_{k}^{c} \) correspond respectively to the kinetic energy of the flow (both potential flow driven by \( \nabla \mu \) and vortex generated swirls) and to the sound wave energy in the superflow. \( E_{q} \) accounts for the energy arising from density inhomogeneities due to the trapping potential [36]. When each VR enters the TF surface (end of stage II), and while still propagating within the barrier’s region of increasing density, sound waves are emitted and \( E_{k}^{c} \) increases at the expenses of \( E_{k}^{i} \) [Fig. 2(d)] [36]. The dissipation of Josephson oscillations [25,60–62] thus stems from two effects: the incompressible kinetic energy transferred from the axial flow to the vortex swirling flow and the phonon emission occurring during vortex nucleation and propagation within the barrier region.

We further quantify both those effects by considering the effect of \( z_0 \) on the velocity \( v_{\text{VR}} \) and the incompressible
VR activation occurs (rarefaction pulse [22].

Increasing $\rho_{VR}$ profiles with oscillating aspect ratio, corresponding to a nucleated vortices $N_{VR}$ of VRs penetrating the bulk (left axis, circles) and vortex induced dissipations $\epsilon_i$ and $\epsilon_v$ (right axis, green and yellow squares). Blue line connects $N_{VR}$ estimates from the time-averaged phase-slippage rate $\Delta \mu(t)/\hbar$ [60]. Inset: Lifetimes, $\tau$, of first nucleated VRs. Each subplot shows $T = 0$ (black symbols) and $T \approx 0.4T_c$ (red symbols) results.

kinetic energy $E_{k,VR}$ of the first VR nucleated; this is shown in Fig. 3 for $z_0 \in [0.13, 0.37]$ and $V_{0}/\mu = 0.8$. We find that increasing $z_0$ leads to a decreasing $v_{VR}$ and to a monotonic increase of $E_{k,VR}$ [Fig. 3(a)] [36]. Calculating the fraction of the total kinetic energy flowing through the junction until the nucleation of the first VR which is dissipated in sound ($\epsilon_s$) or transferred to the first VR ($\epsilon_v$) [36], we find that both sources of dissipation increase as $z_0$ gets larger [see Fig. 3(b)], and can cumulatively account for a significant fraction of the total energy. Surprisingly, the acoustic dissipation $\epsilon_s$ is always larger than the incompressible contribution $\epsilon_i$. Increasing $z_0$ leads to more nucleated vortices $N_{VR}$ [Fig. 3(b)], due to the larger time-averaged chemical potential difference [60], consistent with Ref. [25] and with earlier studies of controlled vortex generation [65,66]. Similarly, the VR lifetime increases by increasing $z_0$ [Fig. 3(b) (inset)]. The VR survival during its propagation in the superfluid bulk is thus determined by two competing effects: On the one hand, the VR tends to expand [67] to conserve its incompressible kinetic energy as it moves towards lower-density regions with decreasing transverse size. On the other hand, the radial trapping asymmetry ($\omega_r \neq \omega_z$) leads to elliptical VR profiles with oscillating aspect ratio, corresponding to a $m = 2$ Kelvin-wave excitation on a circular VR [68]. This wobbling motion induces dissipation of the VR incompressible kinetic energy via the emission of phononlike excitations, reducing the VR radius [69,70]: When $R_{VR} \sim \xi$, the VR loses its circulation and annihilates in a rarefaction pulse [22].

This picture remains qualitatively correct for the probed $T \lesssim 0.4T_c$ (red symbols in Figs. 2–3), for which no thermal VR activation occurs ($k_BT < 0.8V_0$). For a fixed BEC number, superfluid flow, VR generation, and early dynamics are not noticeably affected by the thermal cloud, whose main effect is to add an extra potential to the BEC [35,36,71–73]. Over longer timescales, dissipation due to relative BEC-thermal motion becomes relevant, decreasing the VR lifetime [Fig. 3(b)].

To connect with Refs. [24,25], we implement in our simulations the same protocol by which vortices were observed in time of flight after gradually removing the barrier with a 40 ms linear ramp. The dynamics of the fourth VR generated in the same conditions as in Fig. 2 [right VR in Fig. 2(c)] is shown in Fig. 4, including or excluding the barrier removal procedure. Upon removing the barrier (orange curve), the VR propagates for longer time and longer distance [Fig. 4(a)]. This facilitates the direct observation of Kelvin-wave oscillations [visible in Fig. 4(b)], whose period is consistent with the dispersion relation $\omega(k) \sim k^2/(4\pi) \ln(2)/(\xi^2) = 0.5772$ [36,68]. The longer lifetime can be attributed to the larger kinetic energy

FIG. 3. Role of initial population imbalance $z_0$ (for fixed $V_0/\mu = 0.8$ for which $z_{cr} = 0.11$) on: (a) velocity $v_{VR}$ (left axis, red and black circles) and incompressible kinetic energy $E_{k,VR}$ (right axis, grey squares) of first nucleated VRs. Pink triangles indicate the VR energy calculated with the analytical formula for homogeneous unbounded BECs [36]; (b) total number $N_{VR}$ of VRs penetrating the bulk (left axis, circles) and vortex induced dissipations $\epsilon_i$ and $\epsilon_v$ (right axis, green and yellow squares). Blue line connects $N_{VR}$ estimates from the time-averaged phase-slippage rate $\Delta \mu(t)/\hbar$ [60]. Inset: Lifetimes, $\tau$, of first nucleated VRs. Each subplot shows $T = 0$ (black symbols) and $T \approx 0.4T_c$ (red symbols) results.

FIG. 4. Vortex ring evolution under different conditions: (a) Evolution of the semiaxes and mean radius of the fourth VR ($z_0 = 0.25$, $V_0/\mu = 0.8$), with barrier kept on (blue line) or removed during $[13,53]$ ms (orange line). Shadowed areas mark limiting values of the two semiaxes. Dashed blue and orange lines on top: Transverse TF radius at the instantaneous VR location. (b) Dynamical 2D VR profiles with barrier on (blue) and removed (orange) plotted alongside the corresponding transverse TF surface (dash-dotted lines). Displayed profiles correspond to evolution times marked by vertical solid lines in (a), with the VR surviving only until $t \approx 23$ ms with barrier on. (c) Typical evolution of a 2D VR profile in the case of barrier removal at $T \approx 0.4T_c$. The VR moves off axis, generating a single vortex handle at the boundary.
of VRs nucleated during the gradual barrier removal process. As the VR approaches the edge of the condensate, it breaks up into two antiparallel vortex lines [Fig. 4(b), final snapshot] [22,67,74]. Critically, thermal fluctuations destabilize the VR, causing it to drift off axis, and reach the transversal boundary asymmetrically [Fig. 4(c)] (see also Ref. [75]). There, it reconnects with its image and forms a “vortex handle” [76–78]. This could explain why a single vortex line is typically detected in each experimental run after barrier removal [24,25].

Conclusions.—We have studied the complex interplay between coherent and dissipative dynamics in a thin atomic Josephson junction. We have shown that resistive currents are directly connected with nucleations of vortex rings and their propagation into the superfluid bulk. In particular, dissipation originates from two irreversible effects: phonon emission when vortex rings are nucleated, and incompressible kinetic energy transfer from the superfluid flow to the swirling one of the nucleated vortex rings. The detailed understanding of the connection between vortex-ring dynamics and dissipation is valuable for advancing our comprehension of the complex superfluid dynamics in emerging atomtronic devices [79].

Data supporting this publication is openly available under an Open Data Commons Open Database License [80].

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See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.124.045301, which includes Refs. [37–56], for details on methodology, analysis and extraction of vortex ring properties at zero and finite temperatures.


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