The relation between flame surface area and turbulent burning velocity in statistically planar turbulent stratified flames

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The relation between turbulent burning velocity and flame surface area for turbulent stratified flames has been analysed using a direct numerical simulation database. The simulations have been carried out for different initial turbulence intensities and initial integral length scales of equivalence ratio fluctuation for a given root-mean-square value in a globally stoichiometric mixture. Additionally, statistically similar turbulent premixed flames have been considered for comparison. The turbulent burning velocity and flame surface area for stratified flames are found to be significantly smaller than in premixed flames for slow mixing cases (large scalar integral length scale, low turbulence intensity), though this trend weakens in fast mixing cases (low scalar integral length scale, high turbulence intensity). In slow mixing cases, the increase of burning rate occurs at a smaller proportion than the increase of flame surface area. The contributions of the components arising from tangential diffusion of displacement speed and cross-scalar dissipation rate to the turbulent burning velocity are found to be negligible in comparison to that arising from the combined reaction and normal diffusion component for all cases. The increased probability of finding fuel-lean and fuel-rich mixture affects the contribution of turbulent burning velocity arising from the combined reaction and normal diffusion component of displacement speed for slow mixing cases. Front-supported flame elements dominate over back-supported flame elements for large scalar length scale cases, but this behaviour reverses for small scalar length scale cases. These findings suggest that increases of burning rate and flame surface area do not take place in the same proportions for turbulent stratified combustion.

Keywords: Stratified mixture, mixture inhomogeneity, turbulent burning velocity, flame surface area, direct numerical simulations

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I. INTRODUCTION

In many engineering combustion devices, the fuel and oxidiser are inhomogeneously mixed either by design (e.g. Direct Injection (DI) engines and Lean Premixed Prevaporised (LPP) gas turbine combustors), or due to insufficient mixing time. This type of combustion marks a departure from the homogeneous mixture combustion, and is commonly referred to as stratified mixture combustion. It has been reported that the mixture stratification may lead to an augmentation of burning rate under some conditions^{1,2}, whereas the opposite was also reported in other studies^{3,4}. Furthermore, stratified mixture combustion has the potential to reduce the pollutant emission⁵, and therefore has been a subject of several experimental^{1,6–9} and computational^{2–4,10–31} studies and interested readers are referred to the review paper by Lipatnikov⁵ for a detailed account of the current state-of-art of stratified mixture combustion. A lot of effort has also been directed to the modelling of the mean reaction rate^{12,14}, scalar variance and co-variance^{17,19,21,27}, scalar dissipation rate^{17,19,22,23}, flame surface density²⁶ and scalar flux²⁵ for turbulent stratified mixture combustion. The orientation of mixture fraction gradient with respect to the direction of flame propagation has also been shown to have influences on the flame propagation rate²⁰ and scalar gradient statistics²⁴ in turbulent stratified flames. The effects of mean and root-mean-square (rms) values of equivalence ratio on the overall burning rate in turbulent stratified flames have been analysed in detail by Malkeson and Chakraborty^{19–27}. It has been demonstrated by Swaminathan et al.¹⁶ and Patel and Chakraborty^{32,33} that the localised ignition of turbulent stratified mixture is affected not only by the mean and rms values of equivalence ratio but also by the length scale of mixture inhomogeneity in addition to turbulence intensity and the integral length scale of turbulence. However, to date, there has not been any analysis which focused on the effects of integral length of mixture inhomogeneity on the flame propagation and burning characteristics in turbulent stratified mixtures.

The modelling methodologies for turbulent stratified flames often use concepts which are well-established for turbulent premixed flames^{12,14,17,19,20,22–26,34}. Many modelling methodologies of mean reaction rate closure (e.g. FSD methodology) implicitly assume that Damköhler’s first hypothesis remains valid. Thus, it is necessary to assess the validity of Damköhler hypothesis-like relations in the case of stratified combustion so that new more robust modelling techniques for stratified flames can be developed. For example, several

computational analyses^{35–37} demonstrated that burning rate for statistically planar turbulent premixed flames increases in proportion to the flame surface area generation according to Damköhler’s first hypothesis³⁸. Although Damköhler’s first hypothesis was originally proposed for turbulent premixed flames within the flamelet regime for turbulent length scales much greater than the flame thickness, this relation remains valid for statistically planar turbulent premixed flames even in the thin reaction zones regime^{35–37}. The explanation for this behaviour for statistically planar premixed flames with unity Lewis number was provided by Chakraborty et al.³⁹ In comparison to the vast body of literature on turbulent premixed combustion, limited attention^{6,13,16,18} has been directed to the analysis of flame surface area and burning rate statistics for turbulent flames propagating into stratified mixtures. Swaminathan et al.¹⁶ analysed the effects of stratification length scale on the flame wrinkling based on Direct Numerical Simulation (DNS) data, whereas Grout et al.¹⁸ used DNS data to analyse flame wrinkling statistics for stratified mixture conditions under low turbulence intensity. Garrido-López and Sarkar¹³ used two-dimensional DNS data to study the effects of turbulence intensity and mixture composition fluctuation on the burning rate statistics in globally fuel-lean mixtures. The experimental study by Anselmo-Filho et al.⁶ also analysed the flame surface area generation arising from the flame front curvature distribution caused by differential propagation speeds in lean and rich mixture pockets for a weakly turbulent stratified methane-air V-flame. However, it is yet to be known if the relation between turbulent burning velocity and flame surface area in statistically planar turbulent stratified flames can be approximated using a relation similar to Damköhler’s first hypothesis, and the possible range of validity for such a relation. In order to understand the effects of the integral length scale of mixture inhomogeneity on the burning rate and flame surface area, and to assess the interrelation between turbulent burning rate and flame surface area in turbulent stratified flames, a three-dimensional DNS database of statistically planar turbulent stratified flames with a modified single-step chemistry⁴⁰ representing methane-air combustion has been considered. For this database, the global mean equivalence ratio $\langle\phi\rangle$ is taken to be stoichiometric (i.e. $\langle\phi\rangle = 1.0$) following Hlie and Trouv³ and Malkeson and Chakraborty^{19–27}. A range of different values of normalised integral length scales of mixture inhomogeneity (i.e. $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 where ℓ_ϕ and ℓ are the integral length scales of equivalence ratio fluctuations and turbulent velocity fluctuations, respectively) has been considered for the initial rms of equivalence ratio fluctuation $\phi' = 0.35$. The DNS database

has been interrogated to address the following objectives:

- (a) To investigate the effects of the length scale of mixture inhomogeneity on turbulent burning velocity and flame surface area in statistically planar turbulent stratified flames.
- (b) To identify the relation between turbulent burning velocity and flame surface area in statistically planar turbulent stratified flames and contrast them with turbulent premixed flames under statistically similar turbulent flow conditions.

The rest of the paper will be organised as follows. The information related to mathematical background and numerical implementation will be provided in Sections II and III. This is followed by the presentation and discussion of the results in section IV. The main findings are summarised, and conclusions are drawn in the final section of this paper.

II. MATHEMATICAL BACKGROUND

In this analysis, the chemistry is simplified in the form of single-step chemistry (i.e. 1 unit of mass of fuel + s units of mass of oxidiser $\rightarrow (1 + s)$ units of mass of products, where s is the mass stoichiometric ratio) in the interest of reducing the computational cost required for a detailed parametric analysis in terms of ℓ_ϕ/ℓ for a range of different turbulence intensities. The modified one-step chemistry by Tarrazo et al.⁴⁰ has been considered where both the activation energy and heat of combustion are taken to be functions of equivalence ratio. These equivalence ratio dependences of activation energy and heat of combustion enable the accurate variation of the laminar burning velocity $S_{b(\phi)}$ with equivalence ratio ϕ , as shown in the past⁴⁰. To simplify the problem, the Lewis number of all the species are taken to be unity and specific heats of all the species are taken to be identical in the current analysis.

In the context of stratified mixture combustion, it is possible to define a reaction progress variable c in the following manner^{3,19–27,34}:

$$c = \frac{\xi Y_{F\infty} - Y_F}{\xi Y_{F\infty} - \max\left\{0, \frac{\xi - \xi_{st}}{1 - \xi_{st}}\right\} Y_{F\infty}} \quad (1)$$

where Y_F is the fuel mass fraction, $Y_{F\infty} = 1.0$ is the fuel mass fraction in pure fuel and ξ is the mixture fraction, which is defined as⁴¹:

$$\xi = \frac{Y_F - Y_O/s + Y_{O\infty}/s}{Y_{F\infty} + Y_{O\infty}/s} \quad (2)$$

where Y_O is the mass fraction of oxygen and $Y_{O\infty} = 0.233$ is the mass fraction of oxygen in pure air. For methane-air mixtures $s = 4.0$, which leads to the stoichiometric mixture fraction ξ_{st} as:

$$\xi_{st} = \frac{Y_{O\infty}/s}{Y_{F\infty} + Y_{O\infty}/s} = 0.055 \quad (3)$$

Equation 1 ensures that the reaction progress variable increases monotonically from 0 in the unburned reactants to 1.0 in fully burned products. The transport equation of the reaction progress variable c takes the following form for stratified mixtures²⁰:

$$\rho \frac{\partial c}{\partial t} + \rho u_j \frac{\partial c}{\partial x_j} = \dot{\omega}_c + \frac{\partial}{\partial x_j} \left(\rho D \frac{\partial c}{\partial x_j} \right) + A_\xi \quad (4)$$

where ρ is the gas density, u_j is the j^{th} component of velocity, D is the reaction progress variable diffusivity, $\dot{\omega}_c$ is the reaction rate of the reaction progress variable and A_ξ is the term arising from the mixture inhomogeneity. The expressions for $\dot{\omega}_c$ and A_ξ are given by^{20,34}:

$$\dot{\omega}_c = \begin{cases} -\frac{\dot{\omega}_F}{\xi Y_{F\infty}} & \text{for } \xi \leq \xi_{st} \\ -\frac{\dot{\omega}_F(1-\xi_{st})}{\xi_{st}(1-\xi)Y_{F\infty}} & \text{for } \xi > \xi_{st} \end{cases} \quad (5)$$

$$A_\xi = \begin{cases} 2\rho D(\nabla c \cdot \nabla \xi)/\xi & \text{for } \xi \leq \xi_{st} \\ -2\rho D(\nabla c \cdot \nabla \xi)/(1-\xi) & \text{for } \xi > \xi_{st} \end{cases} \quad (6)$$

where $\dot{\omega}_F$ is the fuel reaction rate and A_ξ contains the contribution of the cross-scalar dissipation rate $2D(\nabla c \cdot \nabla \xi)$, which vanishes in the case of premixed flame (i.e. for homogeneous mixtures), where $\nabla \xi$ is identically equal to zero.

Equation 4 can be written in kinematic form for a given c -isosurface in the following manner^{20,41–43}:

$$\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = S_d |\nabla c| \quad (7)$$

A comparison between Eqs. 4 and 7 reveals that the displacement speed S_d of that c -isosurface is given by^{20,41–43}:

$$S_d = \frac{\dot{\omega}_c + \nabla \cdot (\rho D \nabla c) + A_\xi}{\rho |\nabla c|} \quad (8)$$

The displacement speed S_d represents the speed with which the given c -isosurface moves normal to itself with respect to an initially coincident material surface. Thus, the volume integrated burning rate $\Omega_c = \int_V \dot{\omega}_c dV$ can be expressed using Eq. 8 as:

$$\Omega_c = \int_V \dot{\omega}_c dV = \int_V \rho S_d |\nabla c| dV - \int_V A_\xi dV \quad (9)$$

because $\int_V \nabla \cdot (\rho D \nabla c) dV$ vanishes (i.e. $\int_V \nabla \cdot (\rho D \nabla c) dV = 0.0$) due to the divergence theorem. It is possible to utilise Ω_c to define turbulent burning velocity S_T as⁴⁴:

$$S_T = \frac{\Omega_c}{\rho_0 A_L} \quad (10)$$

where A_L is the projected flame area in the direction of the mean flame propagation. In the case of a premixed flame with an equivalence ratio corresponding to $\phi = \langle \phi \rangle$, the unstrained laminar burning velocity S_L can be evaluated according to Eq. 10 for a one-dimensional freely propagating unstretched flame with $\phi = \langle \phi \rangle$. The second term on the right-hand side of Eq. 9 vanishes for a turbulent premixed flame and the approximation $\int_V \rho S_d |\nabla c| dV \approx \rho_0 S_L A_T$ leads to $\Omega_c = \rho_0 S_L A_T$ where ρ_0 is the unburned gas density and $A_T = \int_V |\nabla c| dV$ is the turbulent flame surface area^{39,45}. The expression in Eq. 10 leads to:

$$S_T/S_L = A_T/A_L \quad (11)$$

The expression in Eq. 11 is commonly referred to as Damköhler's first hypothesis³⁸. It was demonstrated by Chakraborty et al.³⁹ that Eq. 11 remains reasonably valid for statistically planar flames where the mean flame curvature vanishes and the correlation between $|\nabla c|$ and local flame curvature $\kappa_m = 0.5 \nabla \cdot \vec{N}$ is negligible, where $\vec{N} = -\nabla c/|\nabla c|$ is the local flame normal vector which points towards the unburned gas side. To explain this behaviour, it is worthwhile to consider different components of displacement speed $S_d = S_r + S_n + S_t + S_\xi$, which are given by^{20,41–43}:

$$S_r = \frac{\dot{\omega}_c}{\rho |\nabla c|}; \quad S_n = \frac{\vec{N} \cdot \nabla (\rho D \vec{N} \cdot \nabla c)}{\rho |\nabla c|}; \quad S_t = -2D\kappa_m; \quad S_\xi = \frac{A_\xi}{\rho |\nabla c|} \quad (12)$$

where S_r , S_n , S_t and S_ξ are the reaction, normal diffusion, tangential diffusion and cross-scalar dissipation rate of displacement speed, respectively. For a premixed flame, the cross-scalar dissipation rate component S_ξ disappears. Thus, in the case of a premixed flame $\Omega_c = \int_V \dot{\omega}_c dV = \int_V \rho S_d |\nabla c| dV$ takes the following form³⁹:

$$\Omega_c = \int_V \dot{\omega}_c dV = \int_V \rho S_d |\nabla c| dV = \int_V \rho (S_r + S_n) |\nabla c| dV - 2 \int_V \rho D \kappa_m |\nabla c| dV \quad (13)$$

It has been argued by Chakraborty et al.³⁹ and demonstrated by Ahmed et al.³⁷ that the second term on the right-hand side of Eq. 13 becomes negligible for statistically planar premixed flames due to weak correlation between κ_m and $|\nabla c|$ (leading to $\int_V \rho D \kappa_m |\nabla c| dV \approx$

$\overbrace{(\kappa_m)} \int_V \rho D |\nabla c| dV$ where $\overbrace{(\kappa_m)} = \int_V \kappa_m dV / V$) due to negligible magnitudes of the volume-averaged flame curvature (i.e. $|\overbrace{(\kappa_m)}| \approx 0$). Moreover, $\int_V \rho (S_r + S_n) |\nabla c| dV$ can be approximated by $\int_V \rho (S_r + S_n) |\nabla c| dV \approx \rho_0 S_L A_T$ for statistically planar premixed flames with unity Lewis number³⁹, which leads to $\Omega_c \approx \rho_0 S_L A_T$. Therefore, one can obtain Eq. 11. Equation 13 takes the following form for turbulent stratified flames:

$$\Omega_c = \underbrace{\int_V \rho (S_r + S_n) |\nabla c| dV}_{T_1} - 2 \underbrace{\int_V \rho D \kappa_m |\nabla c| dV}_{T_2} = \int_V \rho S_d |\nabla c| dV - \underbrace{\int_V \rho S_\xi |\nabla c| dV}_{T_3} \quad (14)$$

It can be appreciated from Eq. 14 that the applicability of Eq. 11 in statistically planar turbulent stratified flames depends on the statistical behaviours of T_1 , T_2 and T_3 . This aspect along with the responses of T_1, T_2 and T_3 to the variations in turbulence intensity u'/S_L (where u' is the rms velocity) and ℓ_ϕ/ℓ and will be discussed in Section IV of this paper.

III. NUMERICAL IMPLEMENTATION

The simulations for statistically planar turbulent stratified flames have been conducted using a well-known DNS code SENG+^{19–27,46} where the conservation equations of mass, momentum, energy and species mass fractions are solved in a non-dimensional form. The simulations considered statistically planar turbulent stratified methane-air flames propagating into inhomogeneous mixtures. The computational domain is taken to be $70.2\delta_{st} \times 30.1\delta_{st} \times 30.1\delta_{st}$, which is discretised using a Cartesian grid of $800 \times 400 \times 400$ with uniform grid spacing. This grid spacing ensures at least 10 grid points within $\delta_{st} = (T_{ad(\phi=1)} - T_0) / \max |\nabla \hat{T}|_L$, which is the thermal flame thickness of the stoichiometric mixture where $T_{ad(\phi=1)}$, T_0 and $\hat{T}$ are the adiabatic flame temperature of the stoichiometric mixture, unburned gas temperature and the instantaneous dimensional temperature, respectively and the subscript ℓ refers to the values in the 1D unstretched laminar premixed stoichiometric mixture flame. The boundaries in the direction of mean flame propagation are taken to be partially non-reflecting which are specified using the Navier-Stokes Characteristic Boundary Conditions (NSCBC) technique⁴⁷. The spatial discretisation is accounted for by a 10th order central difference scheme for the internal grid points and the order of accuracy drops gradually to one sided 2nd order scheme at the non-periodic boundaries. The time advancement has been

carried out using a low storage 3rd order Runge-Kutta scheme⁴⁸. The turbulent velocity fluctuations have been initialised by a homogeneous isotropic incompressible field using a well-known pseudo-spectral method⁴⁹ for prescribed values of rms velocity u' and the longitudinal integral length scale ℓ of turbulent velocity fluctuations following Batchelor-Townsend spectrum⁵⁰. The mixture inhomogeneity in the unburned gas is initialised by a bi-modal distribution of $\phi = [\xi(1 - \xi_{st})]/[\xi_{st}(1 - \xi)]$ following the methodology proposed by Eswaran and Pope⁵¹ for the prescribed values of mean equivalence ratio $\langle\phi\rangle$, rms equivalence ratio fluctuation ϕ' and integral length scale ℓ_ϕ of equivalence ratio fluctuations, where ℓ_ϕ is evaluated in terms of the integral of the scalar two-point correlation. All the gaseous species are considered to be perfect gases and the Lewis number value of all the species is taken to be unity. The heat release parameter $\tau = (T_{ad(\phi=1)} - T_0)/T_0$ is taken to be 4.5 for all cases considered here, which corresponds to preheating of unburned gas mixture to 415 K for stoichiometric methane-air mixtures. Standard values are considered for Prandtl num-

TABLE I. Simulation parameters considered in this analysis.

Case	u'/S_L	ℓ/δ_{st}	ℓ_ϕ/ℓ	ϕ'	Da	Ka	Re_t	Domain	Grid
A(i)	4.0	3.0	premixed	0.0	0.750	4.62	42.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
A(ii)	4.0	3.0	0.5	0.35	0.750	4.62	42.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
A(iii)	4.0	3.0	1.0	0.35	0.750	4.62	42.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
A(iv)	4.0	3.0	2.0	0.35	0.750	4.62	42.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
A(v)	4.0	3.0	3.0	0.35	0.750	4.62	42.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
B(i)	8.0	3.0	premixed	0.0	0.375	13.1	84.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
B(ii)	8.0	3.0	0.5	0.35	0.375	13.1	84.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
B(iii)	8.0	3.0	1.0	0.35	0.375	13.1	84.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
B(iv)	8.0	3.0	2.0	0.35	0.375	13.1	84.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
B(v)	8.0	3.0	3.0	0.35	0.375	13.1	84.0	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
C(i)	10	3.0	premixed	0.0	0.300	18.3	105	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
C(ii)	10	3.0	0.5	0.35	0.300	18.3	105	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
C(iii)	10	3.0	1.0	0.35	0.300	18.3	105	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
C(iv)	10	3.0	2.0	0.35	0.300	18.3	105	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2
C(v)	10	3.0	3.0	0.35	0.300	18.3	105	$70.2\delta_{st} \times (30.1\delta_{st})^2$	800×400^2

ber and the ratio of specific heats (i.e. $Pr = 0.7$ and $\gamma = 1.4$) and globally stoichiometric mixture (i.e. $\langle\phi\rangle = 1.0$ and $S_L = S_{b(\phi=1.0)}$ where $S_{b(\phi)}$ is the unstretched laminar burning velocity for equivalence ratio ϕ) is considered following previous analyses^{3,19–27}. It is worth noting that the choice of $\langle\phi\rangle$ does not affect the physical explanations regarding flame surface area and turbulent burning velocity variations presented later in this paper. The reacting scalar and temperature fields within the flame are initialised by 1D unstretched laminar premixed flame solution corresponding to $\phi = 1.0$ and $\tau = 4.5$. The initial values of u'/S_L , ℓ/δ_{st} , ℓ_ϕ/ℓ and ϕ' are listed in Table 1 along with the nominal values of Damköhler number $Da = \ell S_L/u'\delta_{st}$, Karlovitz number $Ka = (u'/S_L)^{1.5}(\ell/\delta_{st})^{-0.5}$ and turbulent Reynolds number $Re_t = \rho_0\sqrt{k}\ell/\mu_0$, where $k = 3u'^2/2$ is the turbulent kinetic energy evaluated over the whole domain. Under these Da and Ka values, combustion in these cases takes place in the thin reaction zones regime⁵². At least one grid point resides within the Kolmogorov length scale η for all cases at the initial condition and a greater number of grid points reside within the Kolmogorov length scale with progress in time as η grows with time due to the decay of turbulence level. Using the definition $\eta \approx \ell Re_t^{-3/4}$ gives an initial η resolution of 2.06, 1.22 and 1.04 grid points for the initial conditions corresponding to $u'/S_L = 4.0$, 8.0 and 10 cases respectively. Since the flame-turbulence interaction takes place under decaying turbulence, the simulations should be conducted for $t_{sim} \geq \max(t_e, t_{chem})$ where $t_e = \ell/u'$ is the initial eddy turn over time and $t_{chem} = \delta_{st}/S_L$ is the chemical time scale. In all the cases t_{chem} remains greater than t_e and all cases considered here have been run for $t = 2.0t_{chem}$, which is about $2.67t_e$, $5.33t_e$ and $6.67t_e$ for initial $u'/S_L = 4.0$, 8.0 and 10.0 cases, respectively. This simulation time remains comparable to several previous analyses^{2–4,19–27}.

IV. RESULTS AND DISCUSSION

The distributions of fuel mass fraction Y_F in the central mid-plane for initial $u'/S_L = 10$ cases are exemplarily shown in Fig. 1 for different initial values of ℓ_ϕ/ℓ at $t = 2t_{chem}$. The contours of $c = 0.1$, 0.5 and 0.9 (from left to right) are superimposed on the Y_F field. Figure 1 also shows the fuel mass fraction field for the corresponding premixed (i.e. homogeneous mixture) case. It can be seen from Fig. 1 that there is a significant amount of inhomogeneity in fuel mass fraction distribution in the unburned gas for the stratified flames. A comparison with the homogeneous mixture case reveals that the extent of mixture inhomogeneity in-

creases with increasing ℓ_ϕ/ℓ . In order to explain this behaviour, it is worthwhile to consider the time evolution of the probability density functions (PDFs) of ϕ in the unburned gas (i.e. $c < 0.01$) for the cases shown in Fig. 1 (see Fig. 2). It is evident from Fig. 2 that although the equivalence ratio was initialised following an initial bi-modal distribution, the PDFs take a quasi-Gaussian shape as a result of mixing over time. This behaviour is consistent with previous findings by Hlie and Trouv³ and Malkeson and Chakraborty²². In the case of turbulent scalar mixing, the PDF of the scalar field reaches a quasi-Gaussian state as the mixing process is driven by turbulent fluid motion, where the velocity fluctuations follow approximately Gaussian distributions. This has been demonstrated by Meyer⁵³ and Jenny and Meyer⁵⁴, and has been discussed in detail by Pope⁵⁵ and Cant and Mastorakos⁵⁶. However, there are differences in the behaviour of the PDFs in response to the changes in ℓ_ϕ/ℓ . It can be seen that the PDFs become narrower and the peak at the mean value $\langle\phi\rangle = 1.0$ becomes sharper for smaller initial values of ℓ_ϕ/ℓ . The volume-averaged value of scalar dissipation rate of mixture fraction $\langle N_\xi \rangle = \langle D \nabla \xi \cdot \nabla \xi \rangle$ (where $\langle q \rangle$ is the volume-averaged value of q in the unburned gas and can be scaled as $\langle N_\xi \rangle \sim D \langle \xi'^2 \rangle / \lambda_\xi^2$ where λ_ξ is the Taylor micro-scale of mixture fraction variation and $\langle \xi'^2 \rangle$ is the mixture fraction variance. The

FIG. 1. Distributions of the fuel mass fraction Y_F in the central mid-plane at $t = 2.0t_{chem}$. The reaction progress variable $c = 0.1, 0.5$ and 0.9 contours (left to right) are shown with white lines for cases with initial $u'/S_L = 10.0$.

cases considered here have identical initial values of ϕ' and therefore the initial values of $\langle \xi'^2 \rangle$ are also the same. Because of the moderate values of Reynolds number for the cases considered here, the length scales λ_ξ and ℓ_ϕ remain comparable (e.g. the initial values of λ_ξ/ℓ_ϕ are 0.82, 0.76, 0.61 and 0.61 for initial $\ell_\phi/\ell=0.5, 1.0, 2.0$ and 3.0 , respectively). This suggests that the cases with small values of ℓ_ϕ/ℓ are subjected to high values of scalar dissipation rate, which acts to increase the rate of micro-mixing. Thus, the extent of mixing increases and PDFs of ϕ become increasingly narrow with sharp peaks at $\phi = \langle \phi \rangle$ with a decrease in ℓ_ϕ/ℓ . This behaviour is further augmented by the increase in u'/S_L because the

FIG. 2. Time evolution of the probability density functions of ϕ in the unburned gas (i.e. $c < 0.01$) for the stratified mixture cases with initial $u'/S_L = 10.0$.

scalar dissipation rates of passive scalars, such as mixture fraction ξ , increase with increasing turbulence intensity under the action of turbulent straining⁵⁷. This can be substantiated from the temporal evolutions of ϕ'/ϕ'_0 (where ϕ'_0 is the initial rms value of equivalence ratio fluctuation) in the unburned gas (i.e. $c \leq 0.01$) as shown in Fig. 3. It can be seen from Fig. 3 that the decay rate of ϕ' in comparison to its initial value is faster for smaller values of ℓ_ϕ/ℓ and higher values of u'/S_L . The temporal evolutions of ℓ_ϕ/ℓ_0 in the unburned gas (i.e. $c \leq 0.01$) for these cases are shown in Fig. 4, where ℓ_0 is the initial value of the longitudinal integral length scale of velocity fluctuations. The scalar and velocity length scales are evaluated using the integral of the scalar and longitudinal two-point correlation functions, respectively. Figure 4 shows that ℓ_ϕ/ℓ_0 evolves over time and tends to converge towards $\ell_\phi/\ell_0 \approx 2.0$ for all cases. The changes in ℓ_ϕ occur as a result of turbulent mixing. In the case of small initial values of ℓ_ϕ , the small-scale inhomogeneities are rapidly mixed due to high values of scalar dissipation rate N_ξ , and thus the resulting scalar inhomogeneity with the progression of time leads to a mixture distribution with an integral length scale of mixture inhomogeneity ℓ_ϕ , which is comparable to the integral length scale of turbulence ℓ . In the cases with large initial values of ℓ_ϕ where $\ell_\phi > \ell$, the scalar dissipation rate N_ξ remains small so initial mixing rate remains small, but background fluid turbulence breaks the large-scale islands of mixture inhomogeneities to smaller pockets, and therefore the integral length scale of mixture inhomogeneity approaches to a value comparable to the integral length scale of turbulence ℓ as time progresses. The rms value of velocity fluctuation u' also decays with time under decaying turbulence and the value of u'/u'_0 (where u'_0 is the

FIG. 3. Temporal evolutions of ϕ'/ϕ'_0 in the unburned gas (i.e. $c < 0.01$) for all the stratified cases with (a–c) initial $u'/S_L = 4.0, 8.0$ and 10.0 for different initial values of ℓ_ϕ/ℓ .

initial value of rms velocity fluctuation) remains 2.15, 4.47 and 6.49 in the unburned gas (i.e. $c \leq 0.01$) for initial $u'/S_L = 4.0, 8.0$ and 10.0 , respectively. The temporal evolutions of u'/u'_0 for this configuration were presented earlier²² and the current cases show qualitatively similar behaviour and thus are not repeated here.

FIG. 4. Temporal evolutions of ℓ_ϕ/ℓ_0 in the unburned gas (i.e. $c < 0.01$) for all the stratified cases with (a–c) initial $u'/S_L = 4.0, 8.0$ and 10.0 for different initial values of ℓ_ϕ/ℓ .

The temporal evolutions of S_T/S_L and A_T/A_L for different initial values of ℓ_ϕ/ℓ and u'/S_L for all the cases considered here are shown in Fig. 5. The corresponding premixed flame case is also shown in Fig. 5 for the sake of comparison. It can be seen that both S_T/S_L and A_T/A_L increase with increasing initial u'/S_L due to an increase in the flame wrinkling caused by the turbulent fluid motion with an increase in turbulence intensity. Once the scalar stratification starts to interact with the flame, the flame area tends to change depending on the combination of the initial values of ℓ_ϕ/ℓ and u'/S_L . The transient responses of S_T/S_L and A_T/A_L , which lead to overshoots during $0.5 < t/t_{chem} < 1.0$ for large values of turbulence intensity, i.e. $u'/S_L = 10$ originate due to temporary over-wrinkling of the flame caused by the relatively strong interaction of bi-modal PDF of ϕ with locally intense turbulent mixing, before the PDF converges to a quasi-Gaussian state (see Fig. 2). The ratio of S_T/S_L and A_T/A_L (i.e. $(S_T/S_L)/(A_T/A_L)$) for all the cases with different initial values of u'/S_L and ℓ_ϕ/ℓ are shown in Fig. 6. It can be seen that the agreement between S_T/S_L and A_T/A_L is reasonably maintained for premixed flames throughout the temporal evolution of these quantities irrespective of the initial value of u'/S_L . This suggests that Damköhler's first hypothesis remains valid for the premixed flames considered here, which is consistent with previous findings^{35–37,39}. Figure 6 also shows that the agreement between S_T/S_L and A_T/A_L is maintained throughout their evolution for the initial $\ell_\phi/\ell = 0.5$ case and these

FIG. 5. Temporal evolutions of normalised turbulent burning velocity S_T/S_L , normalised turbulent flame surface area A_T/A_L , different components of S_T/S_L : $T_1^+ = T_1/\rho_0 A_L S_L$, $T_2^+ = T_2/\rho_0 A_L S_L$ and $T_3^+ = T_3/\rho_0 A_L S_L$ and $T_{1L}^+ = \int_V \rho_0 S_{b(\phi)} |\nabla c| dV / \rho_0 S_L A_L$ for the premixed (1st row) and stratified flame cases with initial $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 (2nd–5th row) for initial $u'/S_L = 4.0, 8.0$ and 10.0 (1st–3rd column).

FIG. 6. Temporal evolutions of the ratio of normalised turbulent burning velocity to the normalised turbulent flame surface area $(S_T/S_L)/(A_T/A_L)$ for all cases with (a–c) initial $u'/S_L = 4.0, 8.0$ and 10 respectively.

values remain close to the values obtained for the corresponding turbulent premixed flames. However, a departure can be seen from the equality between S_T/S_L and A_T/A_L during their temporal evolutions for larger values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 2.0$ and 3.0 cases) and this tendency strengthens with increasing ℓ_ϕ/ℓ . Moreover, even for cases with large initial values of ℓ_ϕ/ℓ , the differences between S_T/S_L and A_T/A_L decrease with increasing u'/S_L . Furthermore, it can be discerned from Fig. 5 that S_T/S_L decreases with increasing ℓ_ϕ/ℓ in comparison to the value obtained from the corresponding premixed flame case for small values of u'/S_L (e.g. initial $u'/S_L = 4.0$ case), which is consistent with previous findings by Hlie and Trouv³ and Haworth et al.⁴. However, the values of S_T/S_L for the stratified cases remain comparable to the corresponding value obtained in the premixed flame case for large values of u'/S_L (e.g. initial $u'/S_L = 8.0$ and 10.0 cases).

In order to explain the aforementioned behaviour, it is worthwhile to consider the temporal evolutions of the terms $T_1^+ = T_1/\rho_0 A_L S_L$, $T_2^+ = T_2/\rho_0 A_L S_L$ and $T_3^+ = T_3/\rho_0 A_L S_L$, which are the combined reaction and normal diffusion components of displacement speed, tangential diffusion component of displacement speed and mixture inhomogeneity contribution of displacement speed to the normalised turbulent burning velocity S_T/S_L (i.e. $S_T/S_L = T_1^+ + T_2^+ = \int_V \rho S_d |\nabla c| dV / [\rho_0 A_L S_L] - T_3^+$). The temporal evolutions of T_1^+, T_2^+ and T_3^+ are also shown in Fig. 5, which reveals that the contributions of T_2^+ and T_3^+ remain negligible in comparison to the magnitude of T_1^+ . This is consistent with previous findings²⁰, suggesting that the magnitude of S_ξ remains small in comparison to the magnitudes of

FIG. 7. Temporal evolutions of correlation coefficient p between $|\nabla c|$ and κ_m with the reaction progress variable range $0.05 \leq c \leq 0.95$ (red) and the normalised surface-weighted curvature $\langle \kappa_m \rangle_s \delta_{st}$ for the premixed (1st row) and stratified flame cases with initial $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 (2nd -5th row) for initial $u'/S_L = 4.0, 8.0$ and 10.0 (1st -3rd column).

$(S_r + S_n)$. The magnitude of T_2^+ remains small in all cases and this behaviour originates

from the statistically planar flame and weak correlation between $|\nabla c|$ and κ_m . This can be verified from the temporal evolutions of the correlation coefficient p between $|\nabla c|$ and κ_m with the reaction progress variable range $0.05 \leq c \leq 0.95$, which are shown in Fig. 7. This leads to $\langle \kappa_m \rangle_s = \int_V \kappa_m |\nabla c| dV / A_T \approx \overbrace{(\kappa_m)}$ where the global mean curvature $\overbrace{(\kappa_m)}$ vanishes for statistically planar flames. It can also be verified from the temporal evolutions of $\langle \kappa_m \rangle_s \delta_{st}$ shown in Fig. 7. The joint PDFs between $|\nabla c|$ and κ_m for turbulent stratified flames have been presented earlier²⁰ along with the reasons for their weak correlation and the same qualitative behaviours have been observed for the current database.

Although the contribution of T_3^+ remains negligible, the mixture inhomogeneity affects the statistical behaviour of the combined reaction and normal diffusion component of displacement speed $(S_r + S_n)$, which is reflected in the magnitude of T_1^+ . This behaviour can be illustrated by comparing T_1^+ with $T_{1L}^+ = \int_V \rho_0 S_{b(\phi)} |\nabla c| dV / \rho_0 S_L A_L$ because $\rho(S_r + S_n) = \rho_0 S_{b(\phi)}$ in steady unstretched 1D laminar flame. Note that T_2^+ and T_3^+ are individually zero in 1D unstretched premixed flames. It can be seen from the temporal evolutions of T_{1L}^+ shown in Fig. 5 that T_{1L}^+ matches closely with both S_T/S_L and A_T/A_L for turbulent stratified flames with small values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 0.5$ and 1.0 values). It is worth noting that T_{1L}^+ becomes identical to A_T/A_L irrespective of turbulence intensity for turbulent premixed flames because $S_{b(\phi)} = S_L$ in these cases. For larger values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 2.0$ and 3.0 values), T_{1L}^+ remains smaller than A_T/A_L but greater than S_T/S_L at early stages of flame-turbulence interaction but this difference decreases with time, as the mixing progresses. This behaviour is particularly prominent in the case of small turbulence intensity (i.e. initial $u'/S_L = 4.0$ case) but the difference between T_{1L}^+ and S_T/S_L diminishes with increasing u'/S_L (e.g. these values are close to each other for initial $u'/S_L = 8.0$ and 10.0 cases). This is due to the fact that the PDFs of ϕ approach a Gaussian distribution with a peak at $\phi = \langle \phi \rangle$ as shown in Fig. 2. Moreover, ϕ' in the unburned gas decreases with time due to mixing (see Fig. 3) and the mixing rate is faster for smaller values of ℓ_ϕ/ℓ and/or greater values of u'/S_L .

It has been discussed in the context of Eqs. 10 and 11 that the equality between S_T/S_L and A_T/A_L necessitates $\int_V \rho S_d |\nabla c| dV \approx \rho_0 S_L A_T$, which suggests that T_1^+ needs to be equal to A_T/A_L in order to ensure Damköhler's first hypothesis to be valid. The laminar burning velocity $S_{b(\phi)}$ for $\phi < 1.0$ and $\phi > 1.1$ is expected to be smaller than S_L for this thermochemistry^{19,40} and therefore T_1^+ and T_{1L}^+ remain smaller than A_T/A_L for turbulent stratified

FIG. 8. Fractional shares of front- and back-supported flames on S_T/S_L and A_T/A_L for stratified flame cases with initial $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 (1st -4th row) for initial $u'/S_L = 4.0, 8.0$ and 10.0 (1st -3rd column).

flames with large values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 2.0$ and 3.0 cases) and small values of u'/S_L (e.g. initial $u'/S_L = 4.0$ case) where the mixing rate is relatively weak and thus there is significant probability of obtaining $\phi < 1.0$ and $\phi > 1.1$ (see Fig. 2). However, the probability of finding $\phi < 1.0$ and $\phi > 1.1$ decreases and probability of $\phi = \langle \phi \rangle \approx 1.0$

increases with time (see Fig. 2). As a result, the difference between A_T/A_L , T_1^+ and T_{1L}^+ decreases with time.

FIG. 9. Shares of the ratio T_{1L}^+/T_1^+ for front- and back-supported flame elements for stratified flame cases with initial $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 (1st -4th row) for initial $u'/S_L = 4.0, 8.0$ and 10.0 (1st -3rd column).

The inequality between T_{1L}^+ and T_1^+ for large values of ℓ_ϕ/ℓ and small values of u'/S_L suggests that $\int_V \rho(S_r + S_n)|\nabla c|dV$ cannot be approximated using $\int_V \rho S_{b(\phi)}|\nabla c|dV$ in tur-

bulent stratified flames and this can be generalised further for statistically planar flames as: $\int_V \rho S_d |\nabla c| dV \neq \int_V \rho S_{b(\phi)} |\nabla c| dV$ due to negligible magnitudes of T_2^+ and T_3^+ in statistically planar flames. In order to explain this behaviour, it is worthwhile to consider the flame normal vector alignment with the local mixture fraction gradient, which can be quantified by $\cos \theta_{c\xi}$ given as²⁸:

$$\cos \theta_{c\xi} = \frac{\nabla c \cdot \nabla \xi}{|\nabla c| |\nabla \xi|} \quad (15)$$

where $\cos \theta_{c\xi} > 0.0$ in the case when the flame propagates towards the leaner mixture and commonly referred to as the back-supported flame^{28,58}. By contrast, the flame propagates towards the richer mixture for $\cos \theta_{c\xi} < 0.0$ and is known as the front supported flame. The histograms of fractional shares of front- and back-supported flames on S_T/S_L and A_T/A_L at different stages of the flame evolution are shown in Fig. 8. Likewise, the ratio T_{1L}^+/T_1^+ for both front- and back-supported flame elements at different stages of flame evolution is shown in Fig. 9. Figure 8 reveals that the fractional share of front-supported flame elements towards S_T/S_L and A_T/A_L remains higher than that of the back-supported flame for large values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 2.0$ and 3.0 cases) and only the opposite behaviour is obtained for the initial $\ell_\phi/\ell = 0.5$ case. Furthermore, T_{1L}^+/T_1^+ assumes a value considerably greater than unity for the back-supported flames but this ratio remains close to unity for the front-supported flame elements. The fuel mass fraction gradient magnitude $|\nabla Y_F|$ can be expressed as:

for $\xi \leq \xi_{st}$:

$$|\nabla Y_F| = \xi Y_{F\infty} \sqrt{(1-c)^2 |\nabla \xi|^2 + |\nabla c|^2 - 2(1-c) |\nabla \xi| |\nabla c| \cos \theta_{c\xi}} \quad (16i)$$

for $\xi > \xi_{st}$:

$$|\nabla Y_F| = \frac{Y_{F\infty}}{1 - \xi_{st}} \sqrt{[1 - \xi_{st}(1-c)]^2 |\nabla \xi|^2 + \xi_{st}^2 (1-\xi)^2 |\nabla c|^2 - 2\xi_{st}[1 - \xi_{st}(1-c)](1-\xi) |\nabla \xi| |\nabla c| \cos \theta_{c\xi}} \quad (16ii)$$

Equations 16i and 16ii suggest that for a given set of values of $|\nabla \xi|$ and $|\nabla c|$, the fuel mass fraction gradient magnitude $|\nabla Y_F|$ results in higher magnitudes for front-supported flames than in back-supported flames. Within the flame, the large magnitudes of $|\nabla Y_F|$ are indicative of high fuel reaction rate magnitude $|\dot{\omega}_F|$, which can indeed be substantiated from the positive correlations between $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ observed from the joint PDFs shown in Fig. 10 in the region given by $0.5 \leq c \leq 0.9$. The same qualitative

behaviour is obtained for the joint PDFs between $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $\dot{\omega}_c \times \delta_{st}/\rho_0 S_L Y_{Fst}$ and thus are not explicitly shown here. The possibility of obtaining high (small) values of $\dot{\omega}_c$ for front-supported (back-supported) flame elements leads to higher contribution of front-supported flame elements to S_T/S_L than the back-supported flame elements in the cases for large values of ℓ_ϕ/ℓ where a significant probability of obtaining $\phi < 1.0$ and $\phi > 1.10$ exists (see Fig. 2). Moreover, the high values of $\dot{\omega}_c$ in the front-supported regions in these flames acts to promote high values of $|\nabla c|$, which also leads to a larger share of A_T/A_L from the front-supported flame elements to these cases for large values of ℓ_ϕ/ℓ . For small values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 0.5$ cases), the PDF of ϕ remains narrow around the mean value of $\langle \phi \rangle = 1.0$ (see Fig. 2) and the correlation between $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ remains weak in this case. Therefore, $|\dot{\omega}_F|$ variation for small values of ℓ_ϕ/ℓ is principally determined by the local equivalence ratio ϕ . In these cases, any tendency of back-supported flame propagation from $\phi > 1.10$ to $1.0 < \phi < 1.10$ produces greater magnitudes of $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ than the front-supported flame propagation from $1.0 < \phi < 1.10$ mixture to $\phi > 1.10$ mixture. This behaviour contributes to a larger percentage share of S_T/S_L and A_T/A_L from the back-supported flame elements in comparison to that of the front-supported flame elements in the initial $\ell_\phi/\ell = 0.5$ case.

Although narrowing of the PDFs of ϕ is relatively weak for large values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 2.0$ and 3.0 cases), the mixing rate increases with time due to combined actions of turbulent mixing and a decrease in ℓ_ϕ with time (see Fig. 4). This increases the probability of highly reactive mixtures around $\langle \phi \rangle = 1.0$ where any tendency of front-supported flame propagation from $\phi < 1.0$ to $1.0 < \phi < 1.10$ produces greater magnitudes of $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ than in case of back-supported flame elements. By contrast, the integral length scale of scalar field increases with time for small initial values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 0.5$ and 1.0 cases in Fig. 4) and thus it acts to decrease the mixing rate, and thereby increases the probability of obtaining back-supported flame propagation from $\phi > 1.10$ to $1.0 < \phi < 1.10$ leading to greater magnitudes of $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ than the front-supported flame propagation from $1.0 < \phi < 1.10$ mixture to $\phi > 1.10$ mixture. Therefore, in the cases with small initial values of ℓ_ϕ/ℓ (e.g. initial $\ell_\phi/\ell = 0.5$ and 1.0 cases) the percentage contribution to S_T/S_L and A_T/A_L arising from the back-supported flame elements increases with time.

It has already been discussed that high values of reaction rate magnitudes of Y_F are

FIG. 10. Contours of joint PDFs between $|\nabla Y_F| \times \delta_{st}/Y_{Fst}$ and $|\dot{\omega}_F| \times \delta_{st}/\rho_0 S_L Y_{Fst}$ in the region given by $0.74 \leq c \leq 0.76$ for stratified flame cases with initial $\ell_\phi/\ell = 0.5, 1.0, 2.0$ and 3.0 (1st -4th row) for initial $u'/S_L = 4.0, 8.0$ and 10.0 (1st -3rd column).

obtained for front-supported flame elements (see Fig. 10) and thereby suggesting that the chemical time scales are relatively smaller in the case of front-supported elements than in the case of back-supported elements. This, in turn, suggests that the characteristic Damköhler number (i.e. ratio of turbulent timescale to chemical timescale) is relatively

higher for the front-supported flame elements than in the case of back-supported flame elements because turbulent flow timescale remains unchanged for both front- and back-supported flame elements. By the same token, the characteristic Karlovitz number (i.e. ratio of chemical timescale to the Kolmogorov timescale) is smaller for the front-supported flame elements than in the case of back-supported flame elements because turbulent flow timescale remains unchanged for both front- and back-supported flame elements. A unity value of T_{1L}^+/T_1^+ is expected when the flamelet assumption is valid which allows for $T_{1L}^+ = \int_V \rho_0 S_{b(\phi)} |\nabla c| dV / \rho_0 S_L A_L = \int_V \rho (S_r + S_n) |\nabla c| dV / \rho_0 S_L A_L = T_1^+ \approx T^+$ in the case of statistically planar turbulent stratified flames. As the flamelet assumption is valid for high values of Damköhler number and small values of Karlovitz number, the assumption $T_{1L}^+ = T_1^+ \approx T^+$ is expected to hold more closely for the front-supported flame elements than in the case of back-supported flame elements.

The foregoing discussion implies that the equality between S_T/S_L and A_T/A_L cannot be considered for turbulent stratified mixtures even for statistically planar flames especially for small turbulence intensities. This is in contrast to statistically planar premixed turbulent flames despite the fact that the stratified mixture combustion takes place predominantly under premixed mode. However, the discrepancy between S_T/S_L and A_T/A_L decreases with decreasing integral length scale of mixture inhomogeneity and increasing turbulence intensity. The evolutions of S_T/S_L and A_T/A_L depend on percentage shares of front-supported and back-supported flame elements and therefore accurate estimations of the proportion of front-supported and back-supported flame elements will be necessary for the modelling of turbulent stratified flames.

V. CONCLUSIONS

A three-dimensional modified simple chemistry DNS database of statistically planar turbulent stratified flames with a mean equivalence ratio of unity (i.e. $\langle \phi \rangle = 1.0$) subjected to different initial values of the integral length scale of mixture inhomogeneity and turbulence intensity has been utilised to analyse the relation between turbulent burning velocity normalised by the unstretched stoichiometric laminar burning velocity S_T/S_L and normalised flame surface area A_T/A_L . In addition, statistically planar turbulent premixed flames under similar statistically unburned gas turbulence have been considered for the purpose of

comparison. The turbulent burning velocity and flame surface area have been found to be smaller than the values obtained in the corresponding premixed flame for large values of ℓ_ϕ/ℓ , but this trend weakens with increasing turbulence intensity. For large turbulence intensities, S_T/S_L and A_T/A_L values are found to be comparable for turbulent premixed and stratified flames irrespective of the value of ℓ_ϕ/ℓ . For the combination of large ℓ_ϕ/ℓ and small u'/S_L , the value of A_T/A_L remains greater than S_T/S_L but this difference decreases with increasing turbulence intensity and small integral length of equivalence ratio variation. The statistical behaviour of S_T/S_L has been analysed based on contributions arising from different components of displacement speed and it has been found that the components arising from tangential diffusion component of displacement speed, and cross-scalar dissipation rate remain negligible in comparison to the contribution originating from the combined reaction and normal diffusion components of displacement speed for all cases considered here. The mixture inhomogeneity and the increased probability of finding $\phi < 1.0$ and $\phi > 1.10$ where the rate of burning is smaller than the stoichiometric (i.e. $\phi = 1.0$) mixture affect the contribution of S_T/S_L arising from the combined reaction and normal diffusion components of displacement speed for small values of u'/S_L and/or large values of ℓ_ϕ/ℓ , where the rate of mixing remains relatively slow. This leads to smaller values of S_T/S_L and A_T/A_L in turbulent stratified flames for small values of u'/S_L and/or large values of ℓ_ϕ/ℓ in comparison to the corresponding value obtained for turbulent premixed flames subjected to statistically similar unburned gas turbulence. It has been found that front-supported flame elements dominate over back-supported flame elements in terms of their contributions to S_T/S_L and A_T/A_L for large values of ℓ_ϕ/ℓ but this behaviour reverses for small values of ℓ_ϕ/ℓ . The findings of this analysis suggest that the equality between S_T/S_L and A_T/A_L cannot necessarily be assumed for turbulent stratified combustion even for statistically planar flames. Furthermore, the modelling of turbulent stratified combustion needs to account for the differences in the behaviours of front- and back-supported flame elements to accurately predict S_T/S_L and A_T/A_L values in turbulent stratified mixture combustion. The current analysis focuses on only the interrelation between flame surface area and turbulent burning velocity for turbulent stratified flames which is related to the Damköhler's first hypothesis³⁸. However, further analysis will be needed to assess the validity of Damköhler's second hypothesis³⁸ for small-scale turbulence which relates turbulent burning velocity to the ratio of turbulent diffusivity and molecular diffusivity. Moreover, the present analysis

has been conducted using a single step chemistry and DNS of the selected cases using a two-step chemical mechanism^{59,60} yielded same qualitative findings, which are not shown here for the sake of brevity. Although the displacement speed statistics between simple^{20,61,62} and detailed chemistry^{41–43} DNS are found to be qualitatively similar and the physics analysed in this paper are not likely to be affected by the choice of chemical mechanism, a similar analysis needs to be conducted using detailed chemistry and transport for validation of current findings, and for a more comprehensive understanding of the influences of differential diffusion induced by mixture stratification.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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(a)

(b)

(c)

(a)

(b)

(c)

(a)

(b)

(c)

$\ell_\phi/\ell = 0.5$ $u'/S_L = 4.0$ $u'/S_L = 8.0$ $u'/S_L = 10$ $\ell_\phi/\ell = 1.0$ $\ell_\phi/\ell = 2.0$ $\ell_\phi/\ell = 3.0$