1 Contrary to basic kinematic modelling, during tight turns, wheelchairs
2 can slide sideways by non-negligible magnitudes: collision risks may
3 be increased

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Abstract

Small spaces make collision-free turns difficult for wheelchair users. Contrary to basic kinematic wheelchair modelling and not considered previously by engineers; when wheelchairs turn in small spaces, could lateral drift (sideways slipping) increase the risk of collisions with surrounding structures such as walls and doorways? An improved kinematic understanding can inform building design and therapeutic involvement with wheelchair use.

Wheelchair lateral drift is investigated. Two experiments with assisted wheelchair use are reported. Weights represented wheelchair occupants. First, lateral drift was measured by motion capture (VICON): turns ($n=225$) made by the experimenter, for six radii of curvature (0 - 613mm) and four total-masses (81 - 142kg). Second, lateral drift was measured by ruler: turns ($n=105$) made by experienced wheelchair assistants ($n=22$), for three radii of curvature (0, 306, and 800 mm), and self-selected maximum comfortable weights.

Lateral drift *away* from centre of curvature occurred for all radii of curvature greater than 0mm, and a maximum median lateral drift of 27mm/rad occurred for 142kg total mass with 459mm radius of curvature. Lateral drift increased with mass ($r^2=0.68$ for 358mm radius of curvature). During tight turns, lateral drift magnitude and direction is such that it can increase risks of collisions with surrounding structures.

Introduction

Wheelchair users place great importance on collision-free experiences [1]. It should not be difficult for non-wheelchair users to appreciate the frustration occasioned by, for example, colliding with a door frame when turning into a room and having to reverse and try again. Ensuring that building layouts offer collision-free experiences necessitates a satisfactory understanding of the space wheelchair users occupy when turning. One aspect of that satisfactory understanding comes from modelling the displacements wheelchairs can and cannot make when turning, that is wheelchair kinematics.

Some aspects of wheelchair kinematics are fully understood and variously described [2], [3], [4], but this article will argue, one aspect potentially important to collision-free wheelchair user experience requires further engineering attention. The fully understood aspect is this basic kinematic description, conventional wheelchairs can turn with *any* radius of curvature. This kinematic has led to numerous developments [5], [6] and [7]. Nevertheless, an additional, distinct kinematic feature denoted here as “lateral drift” requires engineering consideration.

Lateral drift away from the centre of curvature is illustrated in Figure 1 where the wheelchair can be seen to turn with lateral displacement as shown by the path of the right wheel hub (solid line) rather than a pure rotation (dashed line). Thus, lateral drift would occur if the *fixed* wheels do *not* provide an ideal lateral constraint during turns, i.e. the tyres can slip on the ground, and so the wheelchair can move sideways.

The following recent evidence suggests that lateral drift occurs. Wheelchair lateral dynamics investigations, [8], [9], [10] have demonstrated that for the *same* dynamic input, wheelchair simulations with different tyre parameters required *different* lateral spaces to turn. Therefore, wheelchair lateral kinematics are more complex than the basic kinematic description. More directly, this author reported to occupational therapists that lateral drift occurred during assisted wheelchair user turns but for this audience no demonstration of engineering coherence was made [11]. Lateral drift importance also increases as wheelchair turning space decreases.

Indeed, while buildings designed to facilitate wheelchair use [12] may have space-sizes that ensure collision-free wheelchair user experiences (despite lateral drift), not all buildings are, or necessarily will be, designed to such standards. Wheelchair user difficulties continue to be reported [13] and anecdotally some of these difficulties are collisions. Indeed, some collisions are so severe, building modifications are necessitated [14]. Furthermore, lateral drift related collision risks may be increasing given a possible trend of heavier wheelchair users [15], larger wheelchairs [16] and some domestic dwellings [17] getting smaller. Therefore, where minimal space necessitates tight turns, but for lateral drift; it is conceivable that wheelchair users would not have collided with, for example, the door frame. Despite the potential importance of lateral drift to wheelchair users' collision-free experience, no engineering publications address this.

The research aim was first, to determine if lateral drift during wheelchair tight turns could increase collision risks, and if yes, second, to inform building design and therapeutic involvement with wheelchair use. This investigation considers *assisted* wheelchair users so follows [11]. However, assisted wheelchair user turning investigations are difficult to design [18], and in contrast to other assisted wheelchair topics [19], [20], no prior engineering publications were found. Therefore, as the novel experimental design requires considerable article-space, this article fully addresses the first aim by reporting lateral drift measurements and presenting a coherent engineering-based qualitative description of the relationship between lateral drift, tightness and direction of turn and wheelchair user weight. A single illustrative example demonstrates an increased collision risk, but a further publication will fully consider the implications for building design and those who are therapeutically involved with wheelchair use.

Method

Experimental Motivation

In pursuance of an understanding of the difficulties assistants had when making tight wheelchair turns, and following the strategy of simplifying the complexity of assisted wheeled chair turning investigation by first investigating turning with approximately constant radii [21], a pilot study found that when assistants attempted tight turns, visually guided by keeping a pointer-end over, but not touching, a marked wheelchair-fixed horizontal bar (Figure 2), a noteworthy phenomenon occurred. As the turn progressed, the pointer remained over bar-width centre line, but, viewed from bar, the pointer visibly moved along bar *length*, for example, 20mm over a quarter turn. Lateral drift appeared to occur. The experimental method used this setup.

Variables of Experimental Interest

Seven independent variables (Table 1) were selected to investigate lateral drift. Five of these were prompted by pilot study observations of the relative lateral drift magnitude differences for factors of mechanical importance, namely: radius of curvature, total mass, mass distribution, turn direction and what is being denoted a *contrived* (explanation to follow) turn. The rationale and details for these and the sixth variable, floor type, follows the consideration of the turn operator variable, the assistant, which is now made.

Turn operator

Turns by experimenter *and* participants were necessary. The experimenter could make multiple repetitions of turns, so lateral drift variation was intra-subject. In contrast, the pilot study indicated

that suitable participants (description to follow) could only make five turns in available participation-time and that substantial inter-subject lateral drift variation existed. Therefore, experimenter data provided a clearer interpretative framework of tight turn lateral drift and participant data supported the representativeness of experimenter data. Further details of the participant experiment is found in [22].

Radius of curvature

The pilot study observations of lateral drift and curvature radii are usefully expressed in terms of the three fixed wheel rolling direction combinations hereinafter “rolling combinations” as can be ascertained from Figure 3: i) wheels rolling in opposite directions, ii) one static wheel and one rolling wheel, and iii) wheels rolling in common directions, hereinafter “contra-rolling”, “single rolling” and “co-rolling” respectively. Radii of curvature locations can therefore be classified as singular (considered ideally), ranged between fixed wheels, and ranged towards infinity: for single-rolling, contra-rolling and co-rolling respectively. Lateral drift for contra-rolling was comparatively small *and* varied in direction whereas for co-rolling it was comparatively large and was always away from curvature centre.

Representative radii (Table 1) were therefore selected as follows. The tightest contra-rolling radii (0mm) was included as it was at an extremity. Also, as co-rolling lateral drift appeared to peak with a 460mm curvature radii, co-rolling curvature radii at and on either side of a 460mm curvature radius were chosen. A further contra-rolling curvature radii between the tightest (0mm) and single rolling was also chosen. Single-rolling was selected as it constituted one of the rolling combinations.

Participant time-availability only permitted three curvature radii to be investigated. Single-rolling the tightest contra-rolling turn (0mm) were chosen as explained for the experimenter. For the co-rolling representation, as only a qualitative comparison of experimenter–participant lateral drift was intended, the opportunity was taken to investigate lateral drift at an additional curvature radius rather than duplicating an experimenter co-rolling curvature radii, namely 800mm. As the motivation was turns in tight spaces increasingly longer curvature radii were of increasingly less interest.

Turn direction

The pilot study observations also suggested that lateral drift was larger for co-rolling push turns, $\dot{\theta} > 0$ (Figure 3), compared with pull turns, $\dot{\theta} < 0$ and that for contra-rolling, turning direction and lateral drift direction were related. Thus turn direction was investigated.

Total mass and distribution

The pilot study observations also indicated that total mass, i.e. combined wheelchair and load, affected lateral drift magnitude. Experimenter turns were therefore made with four different total masses (Table 1). An approximately 20kg step increase was chosen as this was judged sufficient to maintain the four distinct total mass despite handle-force loading.

The Table 1 total masses approximate to 60kg, 80kg, 100kg, and 120kg equivalent wheelchair users – assuming a typical 20kg wheelchair weight (the experimental wheelchair mass is detailed in a later section) - and so includes the 100kg ISO test dummy weight.

Weights gave an approximately 60%-40% rear-front weight distribution, and approximately equal left-right distribution. Participants had self-selected total masses as additional experimental objectives existed [22].

Dynamic theory (see Appendix) indicates that a caster wheel rolling reaction component acts in the direction of lateral drift. Additionally, this reaction would be affected by load. Therefore, as the caster could theoretically contribute to lateral drift, this effect was investigated with experiment turns with a non-standard 40%-60% rear-front mass distribution.

Weight distribution was determined by placing the four wheels on four domestic scales.

Floor

The experimenter turns were mostly (215/225) made on untreated hard flooring, 5/8th inch thickness medium density fibreboard which was laid on an undamaged, debris-free short-pile carpet floor. However, in order to investigate if lateral drift persisted on a different flooring, 10 turns were made directly on the carpet. As persistence of lateral drift on a further floor type was of interest, and as only qualitative comparison was intended and other experimental objectives existed [22], participant turns were made on a vinyl covering soundly adhering to a 5/8th inch thickness medium

density fibreboard placed on a short pile debris-free carpet as other experimental objectives existed. The experimental floors were of buildings constructed within 60 years and considered sufficiently level without need for measurement.

Contrived turns and turning instruction

The operators, experimenter and participant, were asked to turn the wheelchair *and* second, to keep the pointer-end over the horizontal bar-width centre *without* controlling pointer displacement, from bar-view, along bar *length*. This produced approximately constant curvature radii.

However, a pilot observation was that some control of the pointer displacement, from bar-view, along bar *length* was possible. It was therefore necessary to have some measure of the effect this control had on lateral drift. One of the experimenter turns therefore included the instruction, oppose pointer displacement, from bar-view, along bar *length*. This was denoted the *contrived* turn set. As the motivation for the contrived turns was to see if operator actions could intentionally affect lateral drift, the 0mm target radius was chosen as the pilot study observation was that lateral drift was relatively small and that direction varied and hence any effect would be relatively large. As manual handling risks existed [21], [22] for the *contrived* turn and anecdotally were experienced by the experimenter, only the smallest total mass (81kg) was chosen.

The wheelchair was turned as slowly as required to meet the intentions.

Turns number

The independent variable combinations generated twenty-seven experimenter turn-sets. These were carried out in an ad hoc order but were not randomized as no reason for interference between turn-sets was identified. Variability expectations guided the turn-set trial number and 225 turns were made. Participants made a single turn per radii with radii in randomised order.

Experimenter Turning Process

For each turn: i) wheelchair was loaded, ii) wheelchair front was raised, caster forks were set to trail orientation and then lowered, iii) pointer-end was positioned millimetres above chosen horizontal-bar target mark, iv) wheelchair was turned approximately 90 degrees whilst recording positional and force/torque measurements. (The wheelchair rear was also raised prior to 'ii)' for 175/225 turns but this is not discriminated in the results: the motivation was that it was observed in the pilot study that if the wheelchair was initially unloaded, then as the turn progressed, fixed wheel ground-contact displaced laterally from the initial wheel-equator position (Figure 3).)

Turn direction	Rolling directions	Curvature radii mm	Experimenter				Participants
			Total mass (kg)/equivalent occupant mass for standard shower chair				Comfortable maximum load
			81/60	101/80	122/100	142/120	
Push	Contra-	0	20+20 ³	25+5 ¹ +5 ²	10	10	22
		153			5		
	Single	306			5		22
	Co-	358	5	5	10	10	
		459	5	5	5		
		613		5 ²	5		
		800					22
Pull	Contra-	0		5	5		
		153	5				
	Single	306			5		22
	Co-	358		10	15+10 ¹		
		459			5		
		613		5 ²			
		800					22

Table 1: number of turns for independent variables: operator, turn direction, radii (grouped by rolling combinations), total mass,

¹ Non-standard loadings 40%-60% rear-front on hard floor,

² On carpet

³ *Contrived* turn-set: lateral drift opposed.

Experimenter Wheelchair and Instrumentation

The experimental wheelchair was a “shower wheelchairs” [23] type. These permit showering, toileting and dressing to be carried out in one wheelchair [24]. The rigid wheelchair frame (Prism Medical UK T40) had three modifications.

First, a solid axle was added to minimise camber and toe-in/out (confirmed to be negligible by measurement with plumb line, set square and Vernier calliper), and maximise roll axis commonality. This was fitted with injection moulded 600mm diameter wheels with solid tyres (Urethane 25mm height, 37mm width). DLF Data [25] shows nineteen of twenty shower chair models had injection moulded wheels and rigid frames. Casters had solid 125mm diameter wheels and caster fork trail orientation position (see Theory Section) was determined and marked.

Second, the assistant push handles were replaced with a modified push handle system comprised of: aluminium box sections and plates with a six-axis, pre-calibrated force/torque sensor (AMTI MC3-A-6-500) and handles were connected to the force/torque sensor via a Hooke’s joint (see [21] for details) to minimise handle moment effects on front-back or right-left loading.

Third, a low mass, 25mm wide horizontal bar was fixed with long-axis coincident with fixed wheel common roll axis. An adjustable floor stand terminated in a pointer that could be located millimetres above the horizontal bar. This horizontal bar was marked to correspond to the six target radii (Table 1), measured from point B (Figure 3).

Infra-red reflective ultra-light weight 15mm diameter marker spheres were attached to: wheelchair frame ($n=6$) at front and rear of the wheelchair frame and on the non-revolving static fixed wheels centres, and to caster forks ($n=4$) located at the fork front-rear extremities, non-revolving caster wheel centre and near the fork-wheelchair spindle, all left and right. Six 2 Mp infra-red detecting cameras (T20) with related hardware (MX Giganet box and Patch Panel) and software (Nexus 1.8.5) (all, VICON, Oxford, UK) collected positional data and synchronised force/torque sensor data at 100 Hz. The cameras were mounted on suitable tripods (Manfrotto, Italy). The photogrammetric method required post processing but does not require any alteration to the wheelchair. VICON is considered a gold standard in human study [26], but it can also measure mechanical displacements. Global 3D data positions were imported to Maple 2019, data was levelled using the two markers at the wheelchair frame front which were separately measured to have negligible vertical motion. Wheelchair-frame and caster fork marker displacements were error reduced [27]. Caster fork orientations with respect to wheelchair-frame and lateral drift were determined by standard transformation methods [28] and theoretical errors in lateral drift were also considered see Appendix.

Participant Wheelchair and Instrumentation

Participant turns were made by twenty-two people experienced in assisting assisted wheelchair users. Each participant for their comfortable maximum load followed the same twofold intention as stated for the experimenter. The following differences from experimenter turns existed: i) wheelchair was not lifted, ii) lateral drift was measured with a chrome faced metal ruler, rounded to mm divisions, iii) wheelchair angular displacement was approximately 30 degrees, iv) there was no Hooke's joint.

Limitations

Air flow, non-level ground and wheelchair occupant movement are disregarded.

It was also observed that for the approximately first 15 degrees of wheelchair turning, the pointer wobbled, viewed from the bar, but stopped doing so once caster forks reached, in accordance with theory (Appendix) an approximately fixed orientation with respect to wheelchair, the steady state. The investigation considers turning once pointer-wobbling stops.

Statistics

The goal was to describe the relative magnitude effects for the selected variables and indicate an order of magnitude for lateral drift, i.e. to make a qualitative description in order to make a first evaluation of collision risks. To that end only basic statistical testing of the qualitative differences and individual regression of two load related trends were required and employed. The non-parametric Mann Whitney U test was used as not all samples were normally distributed, identically distributed or of equal sample numbers, and for all statistical tests sample numbers satisfied $n1>3$, $n2>4$, $n1+n2>8$. Alpha inflation was not controlled as none of the significant results were ad hoc searches for significance. Rather, all statistically significant comparisons had an a priori expectation as described in the variable selection and with the further expectation of a coherent physical explanation relating the differences.

RESULTS

Preliminaries

Experimenter data was successfully processed for 220/225 turns and results for 218 turns are

- 1 presented: two exclusions are considered at section end. Experimenter results are for steady state
- 2 (see Appendix). Participant data were successfully collected for 105/110 turns.

Figure 4 shows three related graphs.

Bottom shows the general trend for lateral drift (normalised against wheelchair angular displacement magnitude) increasing with increasing target radii to 459mm and then decreasing. This is for push (squares) and pull (circles) turns for all 60% rear loading turn-sets, but without the *contrived* (see Method Section) turn set.

Top shows the same data as *bottom*, but with target radii results presented in detail, i.e. by turn-set.

Middle shows summed lateral force divided by time duration (normalised against wheelchair angular displacement magnitude and total mass).

For *top* and *middle* there are, 27 turn sets with numbers of turns as Table 1, with large symbols indicating median for turns sets and small symbols indicating individual turns, and median symbol

size indicating relative total mass size. Directions, load distribution, and floor type are as shown by legend graphics. White on black numerals are statistical comparisons: all but ⑨ and ①① are statistically significantly different.

Lateral Drift

Lateral drift is unidirectional for *non-0mm* targets, i.e. non-0mm curvature radii: away from curvature centre, that is negative (Figure 4, top). Again, for *non-0mm* targets, comparing the lateral drift and force results (Figure 4 top and middle), push turns (squares) have codirected (negative) lateral handle force, whereas pull turns (circles) have a contra-directed (positive) lateral force, i.e. *towards* curvature centre.

Rear-front load distribution affects lateral drift. Lateral drift magnitude was statistically significantly smaller ($p=0.00$, $n1=8$, $n2=15$) with 40% compared with 60% rear loading: Figure 4 top, comparison ① for co-rolling 122kg total mass, pull turn sets.

Lateral drift is load related. Lateral drift magnitude increase is associated with total mass increase as shown by three groups (Figure 4 top) of co-rolling turn-sets of similar curvature radii and identical turn direction. Using regression analysis for the first two groups the total mass, lateral drift relationship is strong ($r^2 = 0.68$) for comparison ② for 81,101,122,142 kg push turns for 358mm targets, and strong ($r^2 = 0.62$) for comparison ③ for 81, 101, 122 kg push turns for 459mm targets. Finally, for comparison ④ lateral drift magnitude is statistically significantly larger ($p=.008$, $n1=10$, $n2=15$) for the 122kg than the 101kg pull turn (358mm target).

Turn direction affects lateral drift. Push turns have statistically significantly larger lateral drift magnitudes than pull turns for all four comparisons for co-rolling turns with 40% front loading, equal total mass and similar curvature radii: ⑤ for 358mm and 101kg, ($p=0.001$, $n1=10$, $n2=5$), ⑥ for 459mm and 122kg ($p=0.016$, $n1=4$, $n2=5$), ⑦ for 613mm (on carpet) and 101kg ($p=0.016$, $n1=4$, $n2=5$), and ⑧ for 358mm and 122kg ($p=0.004$, $n1=15$, $n2=5$), target and total mass, respectively.

Contra-rolling and co-rolling lateral drift have one overlap but otherwise are different. The *largest* contra-rolling and the *smallest* co-rolling magnitudes were not statistically significantly different ($p=.315$, $n1=10$, $n2=5$) (Figure 4 top) : comparison ⑨, both standard loading (40% total mass at front), 142kg, 0mm target push turn with co-rolling, 101kg, 358mm target pull turn. Therefore, contra-rolling lateral drift magnitude are never statistically significantly greater than co-rolling whereas for all but ⑨, co-rolling are statistically significantly greater than contra-rolling.

Single rolling lateral drift is different. Single rolling lateral drift magnitude is statistically significantly larger ($p=.015$, $n1=10$, $n2=5$) than the largest contra-rolling magnitude: comparison ①①, 122kg push and 101kg pull turns 0mm target with 142kg 0mm target push turn. There is no statistically significant difference ($p=0.31$, $n1=10$, $n2=5$) for lateral drift between single roll 122kg push turn and the 101kg pull turns: comparison ①①.

For 0mm targets (Figure 4, top), with two exceptions, pull turns (circles) have leftward (positive), lateral drift whereas push turns (squares) have rightward lateral drift (negative). The exceptions are the “contrived” (Method Section) turn set and two (of twenty) of one push turn set (101 kg).

Maximum displacement magnitude medians (Figure 4 top) for the three rolling combinations are approximately 7, 13, 27 mm/ rad for contra-rolling, single wheel and co-rolling respectively.

Median lateral force magnitudes tend to be greater (Figure 4 middle) for contra-rolling turns.

Participant Lateral drift

Six points relate to Figure 5 for participant turns.

First, seventy-seven of the eighty-four single wheel or co-rolling turns by participants have lateral drift away from curvature centre whereas towards occurs for one turn by a participant.

Second, participant push turns had statistically significantly larger ($p=0.0001$, $n1=20$, $n2=22$, $p=0.0000$, $n1=20$, $n2=22$) magnitude lateral drift than pull turns for both single rolling and co-rolling, respectively.

Third, comparing lateral drift magnitudes, the smallest of the single rolling and co-rolling, which is single rolling, is statistically significantly larger ($p=0.035$, $n1=20$, $n2=22$) than the contra-rolling turn.

Fourth, maximum median lateral drift magnitudes/rad was 23.8mm/|rad|.

Fifth, two of twenty-four turns by participants produced leftward (positive) lateral drift for 0mm curvature radius (push turns).

Sixth, for single rolling, push turns have a statistically, significantly greater ($p=0.0001$, $n1=$, $n2=$) lateral drift magnitude than pull turns.

Figure 5 shows the lateral drift for participants for three target radii, with pull and push medians indicated by circles and squares, respectively and duplicate lateral drift results spread horizontally to visually separate individual rounded measures, and directions as shown by legend graphic.

Numbers of turns as per Table 1.

Wheelchair angular-displacement and speed

The minimum, median and maximum average wheelchair angular-speed, for all results, were 0.02, 0.03, 0.05 rad/s, respectively. Therefore, approximate maximum speeds for B point (Figure 3) and fixed wheel centres were 0.03m/s and 0.05m/s respectively.

Curvature radius

The steady state experimental curvature radii, represent six distinct samples (Figure 6) which approximate the targets. There are no overlaps, including between 306mm and 358mm where inter-target difference is smallest.

Errors

Data examination showed lateral drift magnitudes were magnitudes of order larger than errors (Equation 7, Appendix). Visual inspection showed caster fork orientation data conformed to Figure 10 right, see Appendix.

Figure 6 shows by Bland-Altman plot, the extent to which the target curvature radii were achieved in the experiment with: experimental steady state curvature radii indicated by median, data points for minimum and maximum and each *target* indicated by a vertical dash line.

Numbers of turns as per Table 1, and approximate target locations are shown by the legend graphic.

Excluded Turns

Fixed wheel ground-contact displacement laterally from the initial wheel-equator position had negligible effect on results after excluding two turns.

DISCUSSION

This investigation addresses the question, during assisted wheelchair user tight turns, could lateral-drift increase collision risks.

Measurement Validity

The lateral drift measurements represent wheelchair behaviour for assisted wheelchair users with typical shower chairs during tight turns with very slow wheelchair angular speeds. Therefore, the application is to environments with minimal free space where collision-avoiding slow turns are required. The results are applicable to three indoor floor types: hard floor, firm carpet and vinyl.

While six *fixed* target radii were set, and experimental curvature radii *varied* (Figure 6), the median experimental curvature radii results could have served as targets. The only requirement was that inter-target differences were large enough to provide distinct curvature radii turn-sets. Therefore, results provide a basis for formulating a qualitative description.

Qualitative statements for lateral drift direction, and magnitude	Contra-rolling		Single rolling	Co-rolling
	0mm	non-0mm (153mm)	(306mm)	(358, 459, 613 mm)
Q1: mostly away from curvature centre	direction varied	YES	YES	YES
Q2: increased by fixed wheel loading	not examined	not investigated	not investigated	YES
Q3: greater for push turns than pull turns	not examined	not examined	NO	YES
Q4: smallest for contra-rolling				
Maximum experimenter median magnitude mm per rad	8	7	13	27

Table 3 details the fixed wheel rolling direction combinations against qualitative statements Q1-Q4, and lateral drift magnitudes.

Qualitative Description of Lateral Drift

Four qualitative statements, summarised in Table 3, describe lateral drift during tight turning as follows.

First, lateral drift was mostly *away* from curvature centre for non-0mm target radii: *all* experimenter-controlled turns (Figure 4) and seventy-seven of eighty-four (Figure 5) turns by participants showed this. Lateral drift away from curvature centre is not peculiar to the experimenter. For the 0 mm target, lateral drift direction was more variable.

Second, lateral drift away from curvature centre increased with fixed wheel loading for co-rolling turns. Two different sets of result support this: experimenter turn sets where total mass varied and when fixed wheel loading changed from 60% to 40%. Also, the next discussion section concludes that lateral drift is a tyre effect and tyre effects are reasonably concluded to be load related.

Third, push turns had larger lateral drift magnitudes away from curvature centre than pull turns for co-rolling turns, for equivalent target radii and equal total mass. The results demonstrated this for participant turn sets (Figure 5), and for four (experimenter) comparable turn-sets (Figure 4).

Fourth, contra-rolling turns had smaller lateral drift magnitudes than co-rolling and single rolling for both experimenter and participants.

These qualitative statements and the following dynamic analysis forms a coherent account of lateral drift.

Coherent View of Lateral Drift

The four qualitative statements cohere with both the following dynamical analysis and recent wheelchair tyre property investigations [8], [9], [10]. A preliminary observation for assisted wheelchairs is that all dynamic *actions* are handle forces. A first analytical step is noting that *lateral handle force* is not a coherent explanation of pull turn, non-0 mm target, lateral drift. For pull turns, the lateral handle force and lateral caster wheel rolling resistance components are directed *towards* curvature centre, Figure 7, left. So lateral drift *away* from curvature centre cannot be a result of lateral handle force or caster wheel motion resistance. Given that, [8], [9], [10] have demonstrated that wheelchair simulations with different tyre parameters results in different wheelchair path curvatures, it is reasonably concluded that during pull turns, tyre properties play a critical part in lateral drift away from curvature centre.

Figure 7 shows with a simple model of the steady state, for co-rolling turns, for pull turns (left), the towards centre lateral force (Figure 4 directions) and caster wheel rolling reactions (Table 3 values) cannot produce away from centre (rightward) lateral drift, whereas for push turns (left) they can.

Push turns are dynamically different from pull turns. For push turns, lateral handle force and lateral caster wheel rolling resistance components are directed *away* from curvature centre, Figure 7 right. It is therefore concluded that during push turns, lateral handle force and caster wheel motion resistance contributed to the resulting lateral drift *away* from curvature centre. Nevertheless, it is also reasonably concluded that the tyre property effects [8], [9] and [10] present for pull turns, were present for push turns. Push and pull turn lateral drift magnitudes therefore differed.

During push turns, lateral handle force and caster wheel motion resistance sum with lateral drift related tyre property effects whereas during pull turns they subtract from those effects. A similar line of argument illuminates the distinctiveness of contra-rolling lateral drift.

Contra-rolling lateral drift was of smaller magnitude and more variably directed. As the wheels are rolling in opposite directions a reasonable interpretation is that tyre-property lateral-drift effects cancelled. Therefore, for contra-rolling, lateral handle forces had a greater impact on lateral drift direction. Therefore, contra-rolling lateral drift direction could vary and could be *contrived*. (As leftward lateral drift was so infrequent for the participant 0 mm push turns, the term *contrived* is justified.)

Single rolling is distinct. The rolling wheel of single rolling is reasonably expected to function as one of the co-rolling wheels, but the non-rolling wheel may function differently. As minimally rolling wheels have a distinct explanation, turn-slip [29], and wheel rolling turning resistance is a function of curvature radius [30], lateral drift may function differently for single rolling compared with co-rolling and contra-rolling. This may explain why, unlike co-rolling, there was no significant lateral drift difference between experimenter single rolling push turn and pull turn sets. Nevertheless, lateral drift was still away from curvature centre.

While the lateral drift results do not permit the relationship with tyre parameters to be determined some agreements and contrasts with [8], [10] are possible. For co-rolling turns, Figure-graphics of simulations indicate that, decreasing tyre stiffness [8], [10] decreases path curvature. This would suggest that different tyres might reduce lateral drift. However, [8], [10], appear to show that after π rad, the wheelchair-path curves towards the original curvature centre whereas, the steady state results of this investigation suggest that wheelchair path continues to drift away from curvature centre. However, occupant-controlled wheelchairs generate action forces at ground-wheel contact and unlike assistants, wheelchair occupants cannot apply external lateral forces so these dynamic differences may explain result differences from [8], [10].

Other Curvature Radii

Results are for discrete curvature radii, but it is reasonably concluded that lateral drift is a non-linear continuum tending to zero as curvature tends to zero. Results indicate lateral drift magnitude peaked between 358-613mm curvature radii.

Collision Risks

The analysis which follows claims that it is reasonable to expect that for some assistants, lateral drift contributes to collision risks. However, before considering the realistic but much more complex case of real-world turning, some helpful initial observations are facilitated by first considering a *much simpler* situation by defining an imaginary assistant who only has the ability to turn with a specified *fixed* curvature radius and centre location. This will prove useful to the more complex real-world turning analysis which follows. However, while the consideration of assistant turning is initially simplified, the turning is still set in a real-world layout type (Figure 8). Anecdotally the author has encountered assistants with this layout who have had to lift the wheelchair handles and swivel on the casters to turn due to the proximity of wall 'A'. For the imaginary assistant in this layout type, it is straightforward and will prove useful to the observations that follow to select a wall 'A' – lift dimension which means that while it is possible to turn with zero lateral drift, with non-zero lateral drift the intended quarter turn is interrupted by a collision. Figure 8 illustrates this for an assistant (not shown for visual clarity) pull turn for an exit from a through-floor¹ lift [31].

Three observations are evident from the simplified turning illustrated in the Figure 8 layout, and these remain pertinent to the greater real-world complexity where the assistant is able to turn with a range of curvature radii. First, for the illustrated layout and turn, lateral drift is directed into wall 'A'. Second, while the hypothetical assistant's limited abilities (only turning with a specific fixed curvature radius and centre location) have been chosen to make a simple introduction to the implications of lateral drift, the idea that real-world assistants who turn in a Figure 8 layout type might find some paths are easier than others is unremarkable. Given this, it will be useful to the

¹ Through-floor lifts are usually retrofits rather than an original building design element. The exit is a pull turn on the assumption that the wheelchair-occupant was pushed into the lift on another floor. The standard internal-width dimension for the referenced lift is shown.

analysis which follows to view the illustrated collision as an example of an assistant's limitations combining with the wall 'A' location and to see this as an example of turning in minimal space or, since there is a specific point of likely collision, turning in a difficult pinch point. Third, while the illustrated turn is theoretically possible with zero lateral drift, as the turn requires the almost simultaneous avoidance of wheel and foot-plate collisions with wall 'A' and 'B' respectively, it is intuitive that if a real-world assistant was to turn with the specified curvature radius and centre, some off-path wobbling would occur and therefore a real-world assistant would require some additional clearance from wall 'A' and 'B' to avoid a collision. These three observations will assist the analysis of real-world turning which is to follow.

As the analysis which follows assumes that real-world assistants might turn in the Figure 8 layout by any of the theoretically² possible continuous quarter turn paths³, the claim that lateral drift contributes to collision risks for some assistants is *not* based on the idea that lateral drift makes it impossible to turn without a collision. Thus, the claim is not undermined by the observation that even if a real-world assistant began the turn as per the Figure 8 layout, in theory, the turn could be completed by tightening curvature during turning to ensure that the right rear-wheel clears wall 'A'. Indeed, it is also expected that for the Figure 8 dimensions there will be other possible turn paths for different combinations of start position and initial curvature radius, and that this process can be repeated for this layout type using alternative dimensions.

The claim that lateral drift increases collision risks for some assistants is based on the following observation which allows for all possible turning paths. Consider those assistants with Figure 8 layout types for whom this *is* a difficult pinch point turn. For some assistants, wall 'A' may only have to be moved (away from the lift) by a *small* dimension for those assistants to find that turning difficulties are removed. (Altering wall positions to find good wheelchair spaces has been employed experimentally using 50mm wall-position changes [34].) Given that small changes to the wall 'A' position may have a substantial effect on turning difficulties, and that it has been noted that for the Figure 8 layout type lateral drift effectively brings wall 'A' closer, the results for lateral drift magnitude may have practical importance. For if the assistants who would benefit from a small movement of wall 'A' had instead had the wheelchair wheels replaced (as a matter of imagination to illustrate the argument rather than assuming their existence) with wheels producing negligible lateral drift, what was lateral drift would be clearance between the right rear-wheel and wall 'A'. Thus, as such clearance might reduce collision risks, reversing the argument, lateral drift may be contributing collision risks.

It should also be noted that as the referenced lift has a 280kg total mass capacity and the example occupant was 100kg, so if the initial consideration was repeated for some real-world wheelchair occupants, the lateral drift could be much larger than 33 mm.

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² A mathematical analysis could establish and define the set of all theoretically successful turn paths but as that is beyond the scope of this article interpretation is based on a looser intuitive sense suggested by visual inspection of Figure 8.

³ It has also been assumed that theoretically possible turn paths does not include those paths which require the assistant to have to lift the wheelchair handles to swivel on the casters or to make non-continuous turns, having to stop, adjust curvature radius or make straight motions and restart. The former causes manual handling risks. The latter is assumed contrary to the spirit of wheelchair accessibility as it incurs the larger forces associated with both discrete changes in caster wheels curvature centres [21] and start-up [32].

Additionally, the need to tighten the turn may also add to assistant difficulties when there is a heavier occupant or a weaker assistant. This follows since, layouts which require tighter turns incur greater motion resistance [30] and increase manual handling risks [33] whereas assistants found it easier to turn with a co-rolling turn rather than single or contra-rolling turn [22].

For some real-world Figure 8 type layouts, it is expected that the combination of the wall 'A' - lift dimension and lateral drift being directed into the pinch point, particularly for heavier occupants, requires assistants to make tighter more difficult turns. As some assistants may not have the ability to make these more difficult turns, it is reasonable to expect that lateral drift may contribute to collision risks.

CONCLUSION

Some wheelchair users have to manoeuvre in buildings with unavoidable pinch points. Whether they avoid or experience collisions hinges on just how small the clearances are. Given that assisted wheelchair users can be subject to up to 40mm lateral drift after a quarter-turn — even more for occupants over 100kg — clearances at pinch points must not be evaluated with a basic wheelchair turning kinematic, i.e. assuming zero lateral drift: doing so may indicate a clearance whereas in reality there is no clearance whatsoever. In which case, assistants would then have to make tighter, more difficult turns or adopt unsafe practices, such as turning by lifting the wheelchair by its handles to avoid collision, and also have the frustration of collisions, near collisions and corrective movements. When clearance is small, lateral drift should be considered, especially for heavier wheelchair users. A subsequent article will present details of further layouts where lateral drift contributes to collision risks and solutions.

APPENDIX

Assumptions and Notation

Motions are planar, on level ground. As per Figure 3, orthonormal frame $[B]$, $\hat{v}, \hat{u}, \hat{S}$ is attached at fixed wheel roll-axis midpoint B, with $\hat{v}$ co-directional with roll axis at angle θ to global direction ${}^G\hat{x}$, rotating with angular velocity $\dot{\theta}$ about $\hat{S}$ which is perpendicular to ground-plane with positive upward.

Transformation from global frame $[G]$ to $[B]$ is denoted ${}^B_G R$ where

$${}^B_G R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

B velocity is ${}^G\dot{\vec{b}}$ with $[B]$ components of lateral velocity $\dot{b}_v = {}^B_G R {}^G\dot{\vec{b}} \cdot \hat{v}$ and $\dot{b}_u = {}^B_G R {}^G\dot{\vec{b}} \cdot \hat{u}$. For $\dot{b}_v = b_v(t)$, where t is continuous time, lateral drift is given by $b_v = \int b_v(t)dt$.

Where upright subscript S denotes right (R) or left (L) for all symbols, fixed-wheel equator centres have $\hat{u}$ velocity components $\dot{w}_S$, and global positions ${}^G\vec{w}_S$, and are displaced from B by $\pm w_f$ along $\hat{v}$.

Forks, have orthonormal frame $[F_S]$, $\hat{n}_S, \hat{t}_S, \hat{S}$ attached at fork spindle centres S_S , with: one revolute degree of freedom rotating about $\hat{S}$, $\hat{n}_S$ at angle ${}^B\theta_S$ to $\hat{u}$, S_S displaced from B by $\pm w_f$ along $\hat{v}$,

caster wheel geometric centres denoted C_S displaced f along $\overline{S_S C_S}$ perpendicular to $\hat{S}$. The caster fork trail orientation position occurs when $\hat{n}_S = \hat{u}$.

Wheelchair base length is $\overline{S_S B} \cdot \hat{u} = b$.

Fixed wheels and caster wheels have one revolute degree of freedom about roll-axes directed as $\hat{v}$ and $\hat{t}_S$, respectively.

Centre of curvature, hereinafter, "curvature centre" is located at Z and curvature radius is derived from $|\overline{BZ}| \cdot \hat{v} = z_v$, $|\overline{BZ}| \cdot \hat{u} = z_u$.

The $\hat{v}$ directed handle force summation for both handles is I_v .

When curvature radius (z_v) is non-zero, lateral drift (b_v) is meaningfully described as away from or towards curvature centre. However, where $z_v = 0$, as both directions are away from curvature centre, lateral drift direction is termed left and right (assistant view).

Determining Curvature radius from Fixed Wheel Velocities

Components $\dot{w}_R$ and $\dot{w}_L$, $\dot{\theta}$ and z_v are related as follows (Figure 3) :

$$\dot{\theta}(z_v + w_r) = \dot{w}_R \quad (1),$$

$$\dot{\theta}(z_v - w_r) = \dot{w}_L \quad (2)$$

and using Equations 1, 2 and solving for z_v ,

$$z_v = \left[\frac{dW_R + dW_L}{2d\theta} \right]_t^{t+1} \quad (3).$$

Where upright t denotes a discrete time index of measurement, as will be the case for the experimental method, Equation (3) may be written as

$$z_v \approx \left[\frac{\Delta W_R + \Delta W_L}{2\Delta\theta} \right]_t^{t+1} \quad (4)$$

providing a useful approximation of curvature radius when time step and displacements are sufficiently small.

Lateral Velocity and Curvature centre Located Off the Roll-Axis

For $\dot{b}_v \neq 0$, as Figure 3 illustrates for $\dot{\theta} > 0$ and $\dot{b}_v > 0$ (dotted line from Z to B), $z_u \neq 0$, that is the curvature centre is not located on the roll axis. Determination of z_u will prove useful for considering the caster fork steady state and is as follows

$$\dot{\theta} \hat{S} \times z_u \hat{u} = -\dot{b}_v \hat{v}$$

$$\Rightarrow z_u = -\frac{\dot{b}_v}{\dot{\theta}} = -\frac{db_v}{d\theta} \quad (5).$$

Figure 9 assists an analysis of errors related to determining lateral drift from discrete displacements with possible concurrent curvature radius changes with: $s_1 \rightarrow s_2$, $s_3 \rightarrow s_4$ and $s_6 \rightarrow s_7$ are global start and end positions of the path of ${}^G\bar{w}_S$ (or any point on the fixed wheel roll axis), identical $\Delta\theta$ – assumed small, but magnified to facilitate visual inspection of the geometry, constant z_v , $z_v + \Delta z_v$ and $z_v - \Delta z_v$, respectively, occurring at some θ (only shown for s_1), necessary geometry to assist mathematical formulation, and localised vector $\bar{z}_v$ at s_8 .

Lateral drift measurement error

As the investigative method determines b_v (lateral drift) from ${}^G\bar{w}_S$ at discrete time indices t , errors e_i arise as follows. The true value of b_v where ${}^B_R t$ is ${}^B_R$ constructed at t , and ${}^G\bar{w}_{t+1}$ is ${}^G\bar{w}_S$ constructed at $t + 1$ is

$$b_v = {}^B_R t {}^G\bar{w}_{t+1} \cdot \hat{v} - e_1 - e_2 - e_3 \quad (6).$$

Errors e_i are illustrated (Figure 9) for three measurements where $\dot{b}_v = 0$, and therefore $z_u = 0$ (Equation 5) and e_i are wholly responsible for b_v . A first error type for all e_i arises because ${}^B_R t$ is constructed at s_1, s_3, s_6 to determine b_v at $t + 1$, that is s_2, s_4, s_7 . Thus, for example, at $s_1 \rightarrow s_2$, inspection shows that a vector, directed as the ${}^B_R t$ unit vector orthogonal to $\hat{v}$ with origin at s_1 has its tip laterally displaced from s_2 and thus erroneously $b_v = e_1$ and not zero. A second error type, applicable to e_2, e_3 is that Z displacement may occur during $t \rightarrow t + 1$ as illustrated by $\pm \Delta z_v$.

Theoretical maximum e_i are now determined. If these are magnitudes of order smaller than experimental measurements the usefulness of results is confirmed. The following observation shows that the first error type is maximised when $\dot{b}_v = 0$. Defining $\bar{z}_v$ at s_8 as a *localised* vector, lateral drift occurs for Δz_v when the vector origin is displaced along $\hat{v}$ but a curvature centre location change occurs when the vector tip is displaced along $\hat{v}$. Therefore, e_i is maximised when $\dot{b}_v = 0$ is assumed since all apparent lateral drift is due to Δz . The second error, for $|e_2|$ and $|e_3|$, is maximised when $+\Delta z_v$ and $-\Delta z_v$ are instantaneous changes at s_3 for $Z_1 \rightarrow Z_3$ and at s_6 for $Z_1 \rightarrow Z_6$, respectively.

Maximum error sizes are determined as follows. By inspection angles constructed from midpoint of chord $|\overline{s_3 s_4}|$, Z_3 and s_3 , and s_5 , s_3 , and s_4 are $\Delta\theta/2$ and using small angle approximation, the chord is $(z + \Delta z)\Delta\theta$ and therefore,

$$e_2 = (z + \Delta z) \frac{\Delta\theta^2}{2} \quad (7)$$

It also follows that $e_1 = (z)\Delta\theta^2/2$ and $e_3 = (z - \Delta z)\Delta\theta^2/2$ and therefore for identical $\Delta\theta$ and Δz , $|e_3| < |e_1| < |e_2|$. Therefore, as $|e_i|$ are maximised for all uncertainties, where $|e_2|$ is magnitudes of order smaller than $|b_v|$, ${}^B R_t {}^G \bar{w}_{t+1} \cdot \hat{v}$ usefully approximates lateral drift.

Figure 10 shows (left) kinematics for fork steady state and an example graph of fork orientation against wheelchair orientation (right fork for 459mm target for a push turn) for zero (solid line) and non-zero (dot line) lateral drift with wheelchair steady state (θ_{Wstcd}) θ position indicated by a thick vertical dash line and fork steady state (${}^B \theta_{FstcdR}$) location on ${}^B \theta_s$ indicated by a thin horizontal dash line.

Steady State

The investigative method requires the numerical θ , denoted θ_{Wstcd} , at which ${}^B \dot{\theta}_s = 0$, that is caster fork steady state denoted ${}^B \theta_{FstcdS}$, where ${}^B \dot{\theta}_s = d{}^B \theta_s / dt$. Where (Figure 10 left) ${}^G \dot{\hat{x}}_S$ is the S_s global velocity, ${}^G \dot{\hat{x}}_S = \dot{\theta} \hat{S} \times \bar{Z}_S + \dot{b}_v \hat{v}$, and where $\dot{\hat{x}}_{CS}$ is the $\hat{t}_S$ component of the global C_s velocity,

1 then, using the principle of relative motion

$$2 \quad \dot{x}_{CS} = \left((\dot{\theta} \hat{S} \times \bar{Z}_S + \dot{b}_v \hat{v}) + \dot{\theta} \hat{S} \times f \hat{n}_S \right) \cdot \hat{t}_S \quad (8).$$

3 For the general case neither z_v nor z_u are constants but assuming $z_v = f(\theta)$ and given Equation 5
4 using $z_u(\theta)$, and where $\bar{Z}_{S0}$ is the initial $\bar{Z}_S$, then $|\bar{Z}_S(\theta)| =$

$$5 \quad \sqrt{(|\bar{Z}_{S0}| \cdot \hat{v} - z_v(\theta))^2 + (|\bar{Z}_{S0}| \cdot \hat{u} - z_u(\theta))^2} \text{ and}$$

$$6 \quad {}^B\theta_{ZS}(\theta) = \arctan2(|\bar{Z}_{S0}| \cdot \hat{v} - z_v(\theta), |\bar{Z}_{S0}| \cdot \hat{u} - z_u(\theta)), \text{ and where}$$

$$7 \quad \dot{b}_v = \frac{db_v}{dt} \frac{d\theta}{d\theta} = \dot{\theta} \frac{db_v}{d\theta},$$

8 rewriting Equation 8

$$9 \quad \dot{x}_{CS} = \dot{\theta} \left(|\bar{Z}_S(\theta)| \cos({}^B\theta_S - {}^B\theta_{ZS}(\theta)) - f + \frac{db_v}{d\theta} \cos({}^B\theta_S) \right) - {}^B\dot{\theta}_S f \quad (9).$$

10 Assuming a non-holonomic caster wheel constraint and setting $\dot{x}_{CS} = 0$, and rewriting Equation 9

$$11 \quad {}^B\dot{\theta}_S = \frac{\dot{\theta} \left(|\bar{Z}_S(\theta)| \cos({}^B\theta_S - {}^B\theta_{ZS}(\theta)) - f + \frac{db_v}{d\theta} \cos({}^B\theta_S) \right)}{f} \quad (10)$$

12 Equation 10 is neither in standard differential form nor easily solved. Furthermore, a numerical
13 solution for Equation 10 for ${}^B\theta_S = f(\theta)$ (Figure 10 right, dotted line) does not necessarily have a
14 ${}^B\dot{\theta}_S = 0$ result, i.e. only an approximately steady state is reached. However, a useful θ_{Wsted}
15 approximation is as follows.

16 Assuming, that $\dot{b}_v = 0$ and therefore $z_u = 0$ (Equation 5), variables $\bar{Z}_S$ and ${}^B\theta_{ZS}$ are then replaced
17 with constants, $\bar{Z}_S$ and ${}^B\theta_{ZS}$, respectively and therefore

$$18 \quad {}^B\dot{\theta}_S = \frac{\dot{\theta} (|\bar{Z}_S| \cos({}^B\theta_S - {}^B\theta_{ZS}) - f)}{f} \quad (11).$$

19

20

z_v mm	0.95 ${}^B\theta_{FstedL}$ rad		0.95 ${}^B\theta_{FstedR}$ rad		θ_{Wsted} rad	
	$\dot{\theta} > 0$	$\dot{\theta} < 0$	$\dot{\theta} > 0$	$\dot{\theta} < 0$	$\dot{\theta} > 0$	$\dot{\theta} < 0$
0	1.8	-1.01	1.01	-1.80	0.33	-0.29
153	1.54	-1.25	0.81	-2.02	0.34	-0.30
306	1.27	-1.53	0.67	-2.18	0.33	-0.27
358	1.18	-1.62	0.63	-2.23	0.32	-0.26
459	1.02	-1.79	0.56	-2.31	0.30	-0.24
613	0.82	-2.01	0.48	-2.41	0.26	-0.22

Table 3 shows the model caster fork orientation steady state angles and the wheelchair orientation at which these occur for six target curvature radii: caster fork orientation at 95% steady state for left caster fork (columns 2-3) and right caster fork (columns 4-5) caster forks, and wheelchair angular orientation (columns 6-7), each for push $\dot{\theta} > 0$ and pull turns $\dot{\theta} < 0$.

21

22 Equation 11 provides, by standard integration methods, ${}^B\theta_S = f_1(\theta)$ as shown (Figure 10 right, solid
23 line) from which $\theta = f_2({}^B\theta_S)$ can first be determined, see [21] for details. As $f_1(\theta)$ reaches ${}^B\theta_{FstedS}$

very slowly, ${}^B\theta_{\text{FstedS}}$ can be determined for the experimental parameters with ${}^B\theta_{\text{FstedS}} = f_1(2\pi)$. An approximate θ_{Wsted} is then provided by

$$\theta_{\text{Wsted}} \approx \max(|f_2(0.95 {}^B\theta_{\text{FstedR}})|, |f_2(0.95 {}^B\theta_{\text{FstedL}})|) \quad (12)$$

with the 0.95 coefficient is chosen to exclude most of the non-steady state as apparent from Figure 10 (right). Table 3 details $f_2(0.95 {}^B\theta_{\text{FstedS}})$ for Figure 3 wheelchair parameters and Table 1 curvature radii and it can be seen (rightward two columns) that the steady state occurs after a relatively short wheelchair angular displacement. The θ_{Wsted} is a useful approximation where experimental caster fork orientations follow the distinctive shape of Figure 10 right.

Keywords wheelchair; kinematics

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