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Galactic dynamos supported by magnetic helicity fluxes

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ABSTRACT
We present a simple semi-analytical model of nonlinear, mean-field galactic dynamos and use it to study the effects of various magnetic helicity fluxes. The dynamo equations are reduced using the 'no-z' approximation to a nonlinear system of ordinary differential equations in time; we demonstrate that the model reproduces accurately earlier results, including those where nonlinear behaviour is driven by a magnetic helicity flux. We discuss the implications and interplay of two types of magnetic helicity flux, one produced by advection (e.g., due to the galactic fountain or wind) and the other, arising from anisotropy of turbulence as suggested by Vishniac & Cho (2001). We argue that the latter is significant if the galactic differential rotation is strong enough: in our model, for $R_\omega \lesssim -10$ in terms of the corresponding turbulent magnetic Reynolds number. We confirm that the intensity of gas outflow from the galactic disc optimal for the dynamo action is close to that expected for normal spiral galaxies. The steady-state strength of the large-scale magnetic field supported by the helicity advection is still weaker than that corresponding to equipartition with the turbulent energy. However, the Vishniac-Cho helicity flux can boost magnetic field further to achieve energy equipartition with turbulence. For stronger outflows that may occur in starburst galaxies, the Vishniac-Cho flux can be essential for the dynamo action. However, this mechanism requires a large-scale magnetic field of at least $\simeq 1\,\mu\mathrm{G}$ to be launched, so that it has to be preceded by a conventional dynamo assisted by the advection of magnetic helicity by the fountain or wind.

Key words: magnetic fields – turbulence – ISM: magnetic fields – galaxies: magnetic fields – galaxies: ISM

1 INTRODUCTION

Conservation of magnetic helicity has recently been recognized as a key constraint on the evolution of cosmic magnetic fields, especially those produced by the large-scale dynamo action. Any large-scale magnetic field (if not supported by external currents) must have both poloidal and toroidal parts in order to mitigate diffusive losses; therefore, any self-sustained large-scale magnetic field must have non-zero magnetic helicity. One of consequences of the helicity conservation is the suppression of the α-effect in a medium of high electrical conductivity. Since the large-scale magnetic field has a non-zero helicity in each hemisphere through the mutual linkage of poloidal and toroidal fields, the dynamo must be producing small-scale helical magnetic fields with the opposite sign of the magnetic helicity. In several closure models (Pouquet, Frisch & Léorat 1976; Kleeorin & Ruzmaikin 1982; Gruzinov & Diamond 1994; Blackman & Field 2002), this leads to a suppression of the dynamo action due to the current helicity of the growing small-scale magnetic field. The suppression can be catastrophic in the sense that the steady-state energy density of the mean magnetic field is as small as R_m^{-1} times the kinetic energy density, where $R_\mathrm{m} \gg 1$ is the magnetic Reynolds number. Thus, efficient action of the mean-field dynamo requires that the magnetic helicity due to the small-scale magnetic field is removed from the dynamo active region (Blackman & Field 2000, 2001; Kleeorin et al. 2000).

Several mechanisms have been suggested to produce the required helicity flux, including the anisotropy of the turbulence produced by differential rotation (Vishniac & Cho 2001; Subramanian & Brandenburg 2004), and the non-uniformity of the α-effect (Kleeorin et al. 2000, 2002). More recently, Shukurov et al. (2006) suggested a simpler mechanism involving advection of small-scale magnetic fields (together with their helicity) by an outflow from the dynamo-active region, e.g., the galactic fountain or wind in the case of spiral galaxies. We explore these effects using a simple model where dynamo equations are reduced to a system of ordinary differential equations, which captures the salient features of the nonlinear dynamo action and helicity evolution. The simplicity of the model allows us to gain deeper insight into the role and interaction of various mechanisms, and to more extensively explore the parameter space. We consider both an advective helicity flux and that arising from the combined action of velocity shear due to differential rotation and anisotropy of the tur-

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bulence produced by it. We demonstrate the efficiency of the galactic dynamo action provided it is facilitated by the helicity fluxes.

2 HELICITY BALANCE AND MEAN-FIELD DYNAMOS

The generation of a large-scale magnetic field by small-scale random motions is described by the mean-field electrodynamics (Krause & Rädler 1980), where relevant physical quantities are split into mean and fluctuating parts, e.g., $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$ for magnetic field and $\boldsymbol{A} = \overline{\boldsymbol{A}} + \boldsymbol{a}$ for the vector potential, where overbar denotes relevant averaging and $\overline{\boldsymbol{b}} = 0$, $\overline{\boldsymbol{a}} = 0$. This results in the mean-field dynamo equation

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \nabla \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \boldsymbol{\mathcal{E}} - \eta \overline{\boldsymbol{J}}) , \tag{1}$$

where $\overline{\boldsymbol{U}}$ is the mean velocity field, η is the Ohmic magnetic diffusivity, and $\boldsymbol{J} = \nabla \times \boldsymbol{B}/\mu_0$ (for the vacuum magnetic permeability we assume $\mu_0 = 1$ hereafter). Furthermore,

$$\boldsymbol{\mathcal{E}} \equiv \overline{\boldsymbol{u} \times \boldsymbol{b}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \overline{\boldsymbol{J}}$$

is the mean turbulent electromotive force (emf) (assuming isotropic turbulence), with α and $\eta_{\rm t}$ the turbulent transport coefficients responsible for the α-effect and turbulent magnetic diffusion, respectively, and $\boldsymbol{u}$ is the small-scale velocity field. The dependence of α on $\overline{\boldsymbol{B}}$ and $\boldsymbol{b}$ is a subject of ongoing research, and our knowledge of this is based on limited closure models and numerical simulations. In some closure models, such as EDQNM (Pouquet, Frisch & Léorat 1976) and the τ-approximation (Blackman & Field 2002; Rädler, Kleeorin & Rogachevskii 2003; Brandenburg & Subramanian 2005b), the effect of the small-scale magnetic field on the α-effect is described by (Brandenburg & Subramanian 2005a)

$$\alpha = \alpha_{\rm K} + \alpha_{\rm m} , \tag{2}$$

where $\alpha_{\rm K}$ represents the kinetic α-effect, $\alpha_{\rm K} = -\frac{1}{3}\overline{\tau \boldsymbol{u} \cdot \nabla \times \boldsymbol{u}}$, and $\alpha_{\rm m} = \frac{1}{3}\rho^{-1}\overline{\tau \boldsymbol{j} \cdot \boldsymbol{b}}$ is the magnetic contribution to the α-effect, with ρ the fluid density and τ the correlation time of the turbulent velocity field $\boldsymbol{u}$ (assumed to be short). Further, $\eta_{\rm t} = \frac{1}{3}\overline{\tau \boldsymbol{u}^2}$. It is not clear how general is the form (2). Using a low Reynolds number approximation, where unambiguous analysis is feasible, Sur, Subramanian & Brandenburg (2007) argue that Eq. (2) is acceptable, with $\alpha_{\rm K}$ modestly affected by magnetic field.

To constrain $\alpha_{\rm m}$, we use the helicity conservation equation written in terms of the helicity density χ of the small-scale magnetic field, defined in a gauge-invariant form in terms of the number density of the links of $\boldsymbol{b}$. Since the small-scale magnetic field has finite correlation length, one can meaningfully introduce its *helicity density* and then derive its transport equation in the form (Subramanian & Brandenburg 2006)

$$\frac{\partial \chi}{\partial t} + \nabla \cdot \boldsymbol{F} = -2\boldsymbol{\mathcal{E}} \cdot \overline{\boldsymbol{B}} - 2\eta \overline{\boldsymbol{j} \cdot \boldsymbol{b}} , \tag{3}$$

where $\boldsymbol{F}$ is the helicity flux density (specified below) and $\boldsymbol{j} = \nabla \times \boldsymbol{b}$. For practical purposes, χ is approximately equal to $\overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ given that $\boldsymbol{a}$ is defined in the Coulomb gauge.

In the steady state, Eq. (3) yields $\boldsymbol{\mathcal{E}} \cdot \overline{\boldsymbol{B}} = -\frac{1}{2}\nabla \cdot \boldsymbol{F} - \eta \overline{\boldsymbol{j} \cdot \boldsymbol{b}}$. For $\boldsymbol{F} = 0$, it follows that $\boldsymbol{\mathcal{E}} \cdot \overline{\boldsymbol{B}} \to 0$ as $\eta \to 0$ under reasonable assumptions about the spectrum of current helicity $\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$ (specifically that it peaks at a scale independent of η or at least at such a scale that $\eta \overline{\boldsymbol{j} \cdot \boldsymbol{b}} \to 0$ as $\eta \to 0$). Hence the component of the emf parallel to the mean magnetic field vanishes, and so the dynamo becomes inefficient. This catastrophic quenching of the α-effect in highly conducting medium is, however, avoided if $\boldsymbol{F} \neq 0$. The simplest contribution to the flux,

$$\boldsymbol{F} = \chi \overline{\boldsymbol{U}} , \tag{4}$$

arises from the net effect of advection of magnetic fields by a velocity field $\overline{\boldsymbol{U}}$ directed away from the dynamo active region, as suggested by Shukurov et al. (2006). We also include another helicity flux proposed by Vishniac & Cho (2001).

In order to relate $\alpha_{\rm m}$ to χ, we argue that the main contribution to $\alpha_{\rm m}$ comes from the integral scale of the turbulence $l_0 = 2\pi/k_0$ (Shukurov et al. 2006), so that $\overline{\boldsymbol{j} \cdot \boldsymbol{b}} \simeq l_0^{-2}\overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ and then

$$\alpha_{\rm m} \simeq \tfrac{1}{3}\tau \frac{1}{l_0^2} \frac{\chi}{\rho} . \tag{5}$$

To justify this relation, we note that the spectral density of $\alpha_{\rm m} \propto \overline{\tau \boldsymbol{j} \cdot \boldsymbol{b}}$ differs from that of $\chi \simeq \overline{\boldsymbol{a} \cdot \boldsymbol{b}}$ by a factor $\tau_k k^2$. When the spectral density of χ decreases with k fast enough, the spectrum of $\alpha_{\rm m}$ decreases with k as well, and then both χ and $\alpha_{\rm m}$ are dominated by the integral scale, $2\pi/k_0$, which is the meaning of Eq. (5). For example, for the Kolmogorov spectrum $M_k = k^{-1}b_k^2 \propto k^{-5/3}$, we have $b_k \propto k^{-1/3}$ and $\tau_k \propto k^{-2/3}$, so that the spectrum of χ is $a_k b_k \propto k^{-5/3}$, that of the current helicity is $j_k b_k \propto k^{1/3}$, and $\tau_k j_k b_k \propto k^{-1/3}$ for $\alpha_{\rm m}$. Then both $\alpha_{\rm m}$ and χ are dominated by the integral scale. These arguments are similar to those presented in Sect. 10.IV of Zeldovich, Ruzmaikin & Sokoloff (1983). Analytical (Blackman 2003) and numerical results (Brandenburg & Subramanian 2005b) suggest that the spectrum of the current helicity in fact decreases as $k^{-2/3}$; this lends further support to Eq. (5).

Introducing a reference magnetic field strength $B_{\rm eq}^2 \equiv \rho\overline{u^2}$ and defining the magnetic Reynolds number as $R_{\rm m} = \eta_{\rm t}/\eta$, we rewrite Eq. (3), using Eqs (4) and (5), as

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -\frac{2\eta_{\rm t}}{l_0^2}\left(\frac{\boldsymbol{\mathcal{E}} \cdot \overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_{\rm m}}\right) - \nabla \cdot \left(\alpha_{\rm m}\overline{\boldsymbol{U}}\right) . \tag{6}$$

This equation should supplement the mean-field dynamo equations to provide our model of nonlinear dynamo action. Since galactic discs are thin, it suffices to consider a one-dimensional model, retaining only the z-derivatives of the variables (Ruzmaikin, Shukurov & Sokoloff 1988). In terms of cylindrical coordinates (r, ϕ, z), we consider a mean flow consisting of differential rotation and vertical advection, with $\overline{\boldsymbol{U}} = (0, \overline{U}_\phi, \overline{U}_z)$.

Altogether, the system of nonlinear mean-field dynamo equations has the dimensionless form

$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}\left(R_U \overline{U}_z \overline{B}_r + R_\alpha \alpha \overline{B}_\phi\right) + \frac{\partial^2 \overline{B}_r}{\partial z^2} , \tag{7}$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = R_\omega \overline{B}_r - R_U \frac{\partial}{\partial z}\left(\overline{U}_z \overline{B}_\phi\right) + \frac{\partial^2 \overline{B}_\phi}{\partial z^2} , \tag{8}$$

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -C\left(\alpha \overline{B}^2 - R_\alpha^{-1}\overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} + \frac{\alpha_{\rm m}}{R_{\rm m}}\right) - R_U \frac{\partial}{\partial z}\left(\alpha_{\rm m}\overline{U}_z\right) , \tag{9}$$

where, retaining only the z-derivatives,

$$\overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} = \overline{B}_\phi \frac{\partial \overline{B}_r}{\partial z} - \overline{B}_r \frac{\partial \overline{B}_\phi}{\partial z} , \tag{10}$$

and we have neglected the term proportional to R_α in Eq. (8) thus adopting the $\alpha\omega$-dynamo approximation for convenience; this term can easily be restored. Here the time and length units are $h^2/\eta_{\rm t}$ and h, respectively, with h the semi-thickness of the disc; the unit of α is denoted α_0, and that of $\overline{U}_z$ is U_0; magnetic field is measured in the units of $B_{\rm eq}$. We have introduced dimensionless parameters defined as

$$R_U = \frac{U_0 h}{\eta_{\rm t}}\,, \quad R_\omega = \frac{Gh^2}{\eta_{\rm t}}\,, \quad R_\alpha = \frac{\alpha_0 h}{\eta_{\rm t}}\,, \quad C = 2\left(\frac{h}{l_0}\right)^2, \quad (11)$$

where $G = r\,d\Omega/dr$ and we have neglected $\overline{B}_z$ in comparison with the other two components of the mean magnetic field, which is appropriate in a thin disc (Ruzmaikin et al. 1988). We shall also use the dynamo number defined as $D = R_\alpha R_\omega$.

3 REDUCED DYNAMO EQUATIONS

We reduce the dynamo equations to a dynamical system, where all the variables are functions of time alone, by using the 'no-z' approximation (Subramanian and Mestel 1993; Moss 1995; Phillips 2001). Specifically, the z-derivatives of all the variables are replaced by appropriate division by the disc semi-thickness, $\partial/\partial z \to 1/h$ and $\partial^2/\partial z^2 \to -1/h^2$. The field components $\overline{B}_r, \overline{B}_\phi$ appearing in the resulting ordinary differential equations can be thought of as representing either the corresponding mid-plane values or vertical averages. By its nature, the no-z approximation captures the quadrupole modes of the dynamo which are dominant in a thin disc.

The current helicity density of the large-scale field $\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}$ vanishes in the no-z approximation – see Eq. (10). Therefore, we calculated it using a perturbation solution of one-dimensional, kinematic mean-field dynamo equations as described in Appendix A:

$$\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}} \approx -\tfrac{3}{8}(-\pi D)^{1/2}\overline{B}_r\overline{B}_\phi\,. \quad (12)$$

When applying this estimate to study nonlinear solutions, we recall that $D \propto \alpha = \alpha_{\rm K} + \alpha_{\rm m}$. Furthermore, we follow Phillips (2001) who suggested that the accuracy of the no-z approximation can be improved by using

$$\alpha \to \frac{2}{\pi}\alpha, \quad \frac{\partial^2}{\partial z^2} \to -\frac{\pi^2}{4}\,\frac{1}{h^2}\,. \quad (13)$$

These rescalings are consistent with the z-dependence of $\overline{\boldsymbol{B}}$ obtained in Appendix A. Derivatives similar to $\partial(\alpha\overline{B}_\phi)/\partial z$ have to be treated cautiously in the no-z approximation since this term is positive near $z = 0$ and negative near $z = 1$ (note that $\alpha = 0$ at $z = 0$ and $\overline{B}_\phi = 0$ at $z = 1$). The main contribution to the dynamo action comes from the vicinity of $z = 0$ where $|\overline{B}_\phi|$ is maximum, so that the part of the above term important for the dynamo action is $\overline{B}_\phi\partial\alpha/\partial z$ (Sokoloff 1996), where $\partial\alpha/\partial z > 0$ near $z = 0$ in galactic discs. This suggests that the no-z approximation of this term is $\alpha\overline{B}_\phi/h$ rather than $-\alpha\overline{B}_\phi/h$.

The resulting reduced Eqs (7)–(9) are given by

$$\frac{d\overline{B}_r}{dt} = -\frac{2}{\pi}R_\alpha\,(1+\alpha_{\rm m})\,\overline{B}_\phi - \left(R_U + \frac{\pi^2}{4}\right)\overline{B}_r\,, \quad (14)$$

$$\frac{d\overline{B}_\phi}{dt} = R_\omega\overline{B}_r - \left(R_U + \frac{\pi^2}{4}\right)\overline{B}_\phi\,, \quad (15)$$

$$\frac{d\alpha_{\rm m}}{dt} = -R_U\alpha_{\rm m} - C\left[(1+\alpha_{\rm m})(\overline{B}_r^2+\overline{B}_\phi^2) + (1+\alpha_{\rm m})^{1/2}\,\frac{3(-\pi D)^{1/2}}{8R_\alpha}\overline{B}_r\overline{B}_\phi + \frac{\alpha_{\rm m}}{R_{\rm m}}\right], \quad (16)$$

where $\alpha_{\rm K} = \alpha_0$ is absorbed into R_α, so that $\alpha = 1 + \alpha_{\rm m}$ in terms of dimensionless variables, and $\overline{U}_z$ can now be put equal to unity. Note that D must be replaced by $D(1+\alpha_{\rm m})$ when Eq. (12) is used in the helicity evolution equation because D in that equation has the meaning of the instantaneous dynamo number.

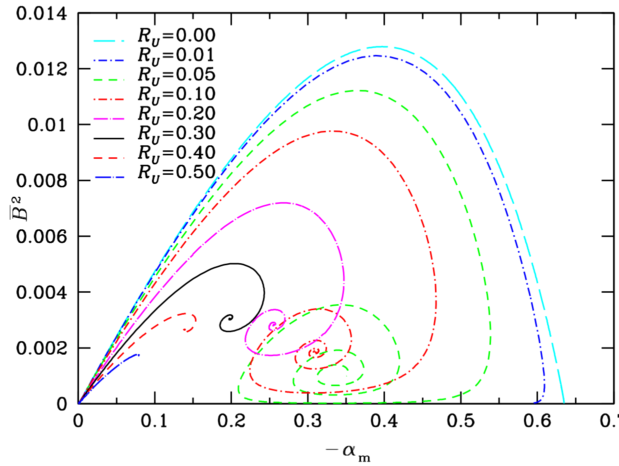

Figure 1. The phase portrait of the dynamical system (14)–(17) in the $(\overline{B}^2, -\alpha_{\rm m})$-plane with and without vertical advection, $0 \leqslant R_U \leqslant 0.5$. Parameter values used here are $R_\alpha = 1$, $R_\omega = -15$, $C = 50$, $R_{\rm m} = 10^5$, the values of R_U are shown in the legend, and the time span is $t \leqslant 100h^2/\eta_{\rm t}$.

Parameter values typical of the Solar neighbourhood of the Milky Way are as follows. The scale height of the ionized gas layer is $h \simeq 500\,{\rm pc}$, the integral scale of interstellar turbulence is $l_0 \simeq 100\,{\rm pc}$, which yields $C = 2(h/l_0)^2 \simeq 50$. With the turbulent magnetic diffusivity $\eta_{\rm t} \simeq 10^{26}\,{\rm cm^2\,s^{-1}}$ and $U_0 = 1\,{\rm km\,s^{-1}}$ (Shukurov et al. 2006), we obtain $R_U \approx 1.5$; we consider a range $R_U = 0$–2. Furthermore, we use standard values of the dynamo parameters, $R_\alpha = 1$ and $R_\omega = -15$. These estimates are often adopted as typical of spiral galaxies in general. However, many galaxies rotate faster than the Milky Way, and even within the Milky Way the angular velocity of rotation rapidly increases towards the centre. Likewise, the other parameters vary broadly between galaxies and across a given galaxy. Hence, any identification of the above parameter ranges as generally applicable to spiral galaxies should be treated with caution.

To illustrate the accuracy of the no-z approximation, we calculate the critical dynamo number (corresponding to $\partial\overline{\boldsymbol{B}}/\partial t = 0$) from Eqs (14) and (15) with $\alpha_{\rm m} = 0$ and $R_U = 0$ as $D_{\rm c} \approx -(\pi/2)^5 \approx -9.6$, as compared to the more accurate values (obtained from a numerical solution of the boundary value problem) $D_{\rm c} \approx -8$ and -11 for $\alpha_{\rm K} = \sin\pi z$ and z, respectively (Ruzmaikin et al. 1988). In the next section we demonstrate that relevant features of the numerical solutions of the nonlinear equations (7)–(9), obtained by Shukurov et al. (2006), are accurately reproduced as well.

4 THE EFFECTS OF THE ADVECTIVE FLUX

We solved numerically Eqs (14)–(16) firstly in order to verify the accuracy of the no-z approximation, and then to study the nonlinear evolution of the dynamo. The initial conditions used are

$$\overline{B}_r = 0\,, \quad \overline{B}_\phi = 10^{-3}\,, \quad \alpha_{\rm m} = -10^{-3} \quad \text{at } t = 0. \quad (17)$$

The choice $\alpha_{\rm m}|_{t=0} = 0$ does not affect the results.

In Fig. 1 we show the phase portrait of the system in the $(\overline{B}^2, -\alpha_{\rm m})$-plane, obtained for various values of R_U. For weak

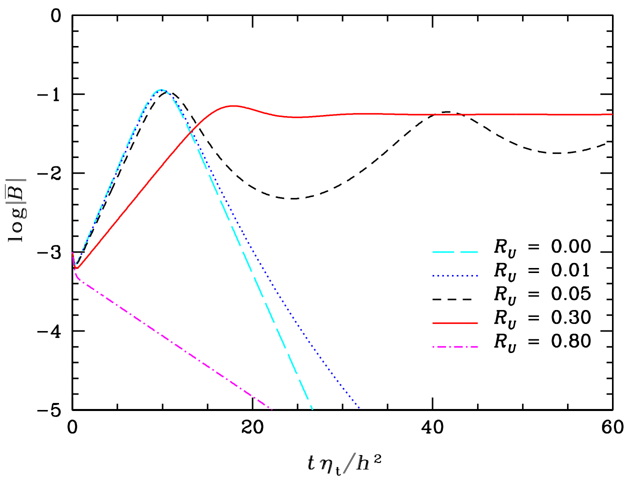

Figure 2. The evolution of the magnetic field strength for various magnitudes of the vertical advective flux shows that magnetic field eventually decays for both small and large values of R_U. Parameter values are the same as in Fig. 1. Time is given in the units of the magnetic diffusion time across the gas layer, about 7.5×10^8 yr.

advection ($R_U \lesssim 0.01$), $-\alpha_\mathrm{m}$ monotonically increases with time to reduce the net α-effect, so that the dynamo eventually becomes subcritical and magnetic energy reduces to negligible values. The dynamo action can resume again as soon as α_m will have decayed together with the large-scale magnetic field and $\alpha = \alpha_\mathrm{K} + \alpha_\mathrm{m}$ becomes supercritical again. Without the advective flux of helicity, the time scale for such a recovery of the dynamo would be of the order of the Ohmic diffusion time, as controlled by the term $\alpha_\mathrm{m}/R_\mathrm{m}$ on the right-hand side of Eq. (16). However, if R_U is of order unity, the relaxation time of the magnetic helicity becomes significantly shorter because of the advection represented by the first term on the right-hand side of Eq. (16), and magnetic field grows again to exhibit nonlinear oscillations before it approaches a steady state. The time scale of the oscillations is long at about 2×10^{10} yr for $R_U = 0.05$. The magnitude of $\overline{B}^2$ in the steady state first grows with R_U, but then reaches a maximum at $R_U \approx 0.3$. Stronger advection affects the dynamo adversely by removing $\overline{\boldsymbol{B}}$ too fast.

The variation of magnetic field strength with time is also shown in Fig. 2. The initial exponential growth of the magnetic field is catastrophically quenched in the absence of the advective flux, and the field decays at about the same rate as it grew (long-dashed). However, even a moderate advective flux ($R_U = 0.3$, solid) compensates the catastrophic quenching allowing the magnetic field to reach a steady-state value of about $0.1B_\mathrm{eq}$. For stronger advection ($R_U \geqslant 0.8$, see the dash-doted curve), the dynamo action is suppressed again since the mean-field is removed too rapidly from the dynamo-active region.

The above results agree quite accurately with those obtained by Shukurov et al. (2006) who solved the differential equations (7)–(9). This confirms the applicability of the no-z approximation to nonlinear dynamo equations. In addition, this indicates that our estimate of $\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}$ in Eq. (12) is appropriate.

To clarify further the effect of the advection on the dynamo, we consider the steady-state solution, $d/dt = 0$ in Eqs (14)–(16), where we introduce the critical dynamo number D_c such that, in the steady state,

$$[D(1+\alpha_\mathrm{m})]_\text{steady state} = D_\mathrm{c}\,. \tag{18}$$

Then Eqs (14) and (15) yield

$$D_\mathrm{c} = -\frac{\pi}{2}\left(R_U + \frac{\pi^2}{4}\right)^2, \quad \overline{B}_r = -\left(\frac{2D_\mathrm{c}}{\pi}\right)^{1/2}\frac{\overline{B}_\phi}{R_\omega}, \tag{19}$$

the first of which shows that the critical dynamo number is affected by the advection; in this sense, the outflow hinders the dynamo action, as might be expected.The second of this relations shows that $|\overline{B}_r| \ll |\overline{B}_\phi|$ if $|R_\omega| \gg 1$. Then Eq. (16) yields, neglecting $\overline{B}_r^2$ in comparison with $\overline{B}_\phi^2$,

$$\overline{B}^2 \approx \frac{\xi}{C}\left(\frac{D}{D_\mathrm{c}} - 1\right)\left(R_U + \frac{C}{R_\mathrm{m}}\right), \tag{20}$$

where

$$\xi = \frac{1}{1-\frac{3}{8}\sqrt{2}} \approx 2\,. \tag{21}$$

It is clear that $\overline{B} \propto R_\mathrm{m}^{-1/2}$ for $R_U = 0$, which is the case of catastrophic α-quenching (with $\overline{B}$ perhaps exhibiting long-term variations around this level – Brandenburg & Subramanian 2005c). However, the magnetic field strength is approximately proportional to $R_U^{1/2}$ for $R_\mathrm{m} \gg 1$ and small R_U. On the other hand, $|D_\mathrm{c}|$ increases with R_U, so that there is an optimal value of R_U providing maximum field strength. For $D = -15$, the steady state is nontrivial ($\overline{B}^2 > 0$) for $R_U \lesssim 0.6$, with $\overline{B}^2$ being maximum for $R_U \approx 0.3$. In the optimal case, $R_U = 0.3$, we obtain $\overline{B} \approx 0.1$, where we recall that the unit magnetic field corresponds to equipartition between magnetic and turbulent energy densities. We note that Eq. (20) is similar to Eq. (10) of Shukurov et al. (2006), but here we use an arguably better estimate of $\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}$.

The steady-state magnetic field remains of order $C^{-1/2}B_\mathrm{eq} \simeq 0.1B_\mathrm{eq}$, with $C = 2(h/l_0)^2 \simeq 50$, even in the presence of the advective flux. Therefore, we consider additional helicity fluxes to examine if they can lead to stronger magnetic fields such that $\overline{B} \simeq B_\mathrm{eq}$.

5 THE VISHNIAC-CHO FLUX

A flux of magnetic helicity discovered by Vishniac & Cho (2001) relies on the anisotropy of turbulence which is naturally produced by velocity shear due to differential rotation (Subramanian & Brandenburg 2004; Brandenburg & Subramanian 2005c). Indeed, the regular velocity shear $\overline{U}_y(x)$ produces an additional y-component of the turbulent velocity via $\partial u_y/\partial t \simeq (\boldsymbol{u}\cdot\nabla)\overline{U}_y$, so that the resulting anisotropy, produced during one correlation time τ, is $u_y/u_x \simeq 1 + \tau\partial\overline{U}_y/\partial x$. The x and y directions can be identified with the radial and azimuthal ones in the galactic disc, so that $\partial\overline{U}_y/\partial x$ is replaced by $G = r\,d\Omega/dr$. The anisotropic part of the turbulent velocity correlation tensor can be estimated as $\overline{u_x u_y} \simeq \tau G\overline{u^2}$, where $\boldsymbol{u}$ is the background isotropic turbulent velocity field. A convenient expression for the flux of Vishniac & Cho has been obtained by Brandenburg & Subramanian (2005c) and Subramanian & Brandenburg (2006). Using Eq. (12) of Brandenburg & Subramanian (2005c), with their C_VC calculated using the results of Subramanian & Brandenburg (2006), we can represent the vertical flux of the magnetic helicity of the small-scale magnetic field in the following dimensional form:

$$F_z \simeq \tfrac{1}{2}(u\tau)^2 G(\overline{B}_r^2 - \overline{B}_\phi^2)\,. \tag{22}$$

This flux will add the following term to the right-hand side of Eq. (16) written in the dimensionless form (and using the no-z approximation):

$$\frac{\partial \alpha_{\mathrm{m}}}{\partial t} = \cdots - \frac{\partial F_z}{\partial z}, \qquad \frac{\partial F_z}{\partial z} \approx -\frac{R_\omega}{R_\alpha}\left(\overline{B}_r^2 - \overline{B}_\phi^2\right), \tag{23}$$

where dots denote the terms already included in Eq. (16). Concerning the steady state, Eqs. (18) and (19) still apply and the steady-state strength of the mean field is given by Eq. (20), but now with

$$\xi = \frac{1}{1 - \frac{3}{8}\sqrt{2} + R_\omega^2/(CD_{\mathrm{c}})} \tag{24}$$

instead of (21). Since $D_{\mathrm{c}} < 0$, the additional flux results in a stronger steady-state magnetic field than that with advective flux alone – cf. Eq. (21).

If $|R_\omega|$ is large enough, the additional term can formally lead to a singularity in $\overline{B}^2$ where the denominator in Eq. (24) vanishes (and $\overline{B}^2 < 0$ for larger values of $|R_\omega|$). The reason is that the Vishniac-Cho flux produces $\alpha_{\mathrm{m}} > 0$ enhancing the hydrodynamic α-effect. If R_ω^2 is large enough in comparison with $C|D_{\mathrm{c}}|$, the flux of Vishniac-Cho can lead to a dynamo action of its own, independently of the kinetic α-effect (Vishniac & Cho 2001). As shown in Sect. 5.1, equations governing this regime are in fact linear in $\overline{\boldsymbol{B}}$, hence $\overline{B}^2$ grows exponentially as $t \to \infty$, which corresponds to the singularity in Eqs (20) and (24). Despite the formally linear governing equations, this type of dynamo is essentially nonlinear and relies on either a strong seed magnetic field or an earlier conventional dynamo action to produce a strong magnetic field required to build up α_{m} at a sufficiently short time scale (Sect. 5.1).

For the parameter values typical of the Solar neighbourhood, $C = 50$ and $D_{\mathrm{c}} = -10$, the effect of the additional helicity flux becomes significant as R_ω approaches about -15. For $|R_\omega| \lesssim 15$, this effect does not lead to independent dynamo action (see Sect. 5.1) but rather increases the steady-state strength of the large-scale magnetic field to values close to B_{eq}.

With the addition of the Vishniac-Cho flux, we plot in Fig. 3 the variation of the magnetic field strength driven by both helicity fluxes for a particular value of R_α and varying R_ω, as obtained from a numerical solution of Eqs (14)–(16) with the modification (23). The useful contribution of the Vishniac-Cho flux at larger values of R_U is now evident. With the advective helicity flux alone, magnetic field decays if R_U is large enough; this is true with the Vishniac-Cho flux added, but now for larger values of R_U. For example, $\overline{B}$ decays for $R_U > 0.8$ if $R_\omega = -15$ (either with the flux of Vishniac & Cho – Fig. 3a, or without it – Fig. 2), but only for $R_U > 1$ if $R_\omega = -17$ (Fig. 3c). This is a result of the joint action of the two mechanisms, the $\alpha\omega$-dynamo and that driven by the Vishniac-Cho flux: both become stronger as $|R_\omega|$ increases. For $R_\omega \leqslant -15.5$, the solutions for $R_U = 0$ do not decay because the Vishniac-Cho flux drives a dynamo of its own (the dash-dotted curves in Fig. 3b,c).

Thus, the advective helicity flux facilitates the dynamo action by alleviating the magnetic helicity conservation constraint. On the other hand, it ensures that the dynamo action is eventually saturated. On the contrary, the helicity flux of Vishniac & Cho, also mitigating the helicity constraint, can lead to an unbounded growth of magnetic field and must be quenched by further physical processes. The advective flux can ensure such a quenching for moderate values of $|R_\omega|$ (e.g., for $R_U = 0.3$–0.8 in Fig. 3).

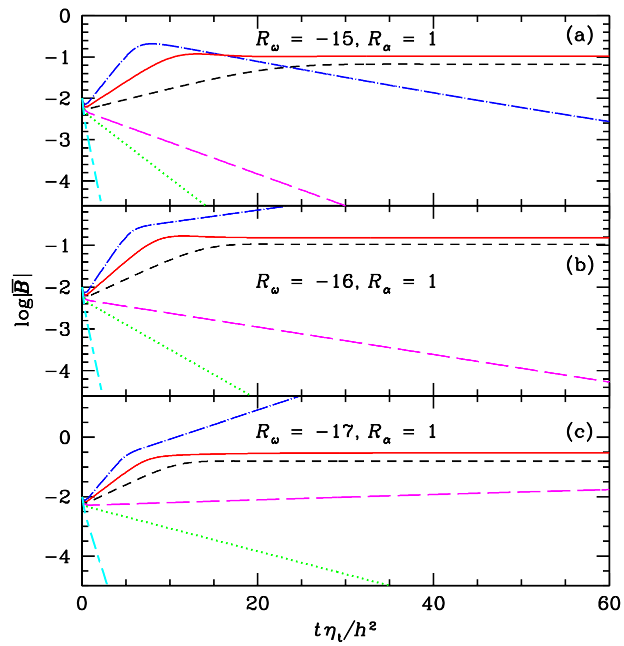

Figure 3. Evolution of the magnetic field strength with both the advective helicity flux and that of Vishniac & Cho, flux for various magnitudes of the velocity shear: (a)$R_\omega = -15$, (b) $R_\omega = -16$ and (c) $R_\omega = -17$. The values of R_U presented are: 0 (dash-dotted), 0.3 (solid), 0.5 (dashed), 0.8 (long-dashed), 1.0 (dotted) and 3.0 (short-long-dashed). The Vishniac-Cho helicity flux supports the dynamo action for larger values of R_U: note decay at $R_U = 0.8$ in (a) but not in (c). The dynamo action is similarly facilitated for small R_U, where solution with $R_U = 0$ decays in (a) but not in (b) and (c). The parameters used here are $C = 50$, $R_{\mathrm{m}} = 10^5$, $\overline{B}_\phi|_{t=0} = 10^{-2}$ and $\overline{B}_r|_{t=0} = 0$, with the values of R_α and R_ω indicated in the legends.

5.1 Dynamo action due to the Vishniac-Cho flux

In this section we present an illustrative model of dynamo action driven by the helicity flux of Vishniac & Cho (2001) alone. For this purpose, we use Eq. (9) with additional term (23) but now we do not employ the no-z approximation, take $R_U = 0$ and neglect any hydrodynamic α-effect, $\alpha_{\mathrm{K}} = 0$. To make the system easily tractable, we make two key assumptions which can be justified a posteriori using the solution obtained: we neglect $\overline{B}_r^2$ in comparison with $\overline{B}_\phi^2$ in Eq. (9) and assume that $\overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}}$ is negligible. Neglecting also $\alpha_{\mathrm{m}}/R_{\mathrm{m}}$, this yields

$$\frac{\partial \alpha_{\mathrm{m}}}{\partial t} = -\alpha_{\mathrm{m}} C \overline{B}_\phi^2 - \frac{\partial F_z}{\partial z},$$

where the dimensional form of F_z is given in Eq. (22), and we use dimensionless variables as defined in Sect. 2. Then Eqs (7)–(9) reduce to

$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}(\alpha_{\mathrm{m}}\overline{B}_\phi) + \frac{\partial^2 \overline{B}_r}{\partial z^2}, \tag{25}$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = R_\omega \overline{B}_r + \frac{\partial^2 \overline{B}_\phi}{\partial z^2}, \tag{26}$$

$$\frac{\partial \alpha_{\mathrm{m}}}{\partial t} = -\alpha_{\mathrm{m}} C \overline{B}_\phi^2 + R_\omega \frac{\partial \overline{B}_\phi^2}{\partial z}, \tag{27}$$

where we have put $R_\alpha = 1$, which is equivalent to choosing the unit of α_{m} to be η_{t}/h. The characteristic time of the variation of α_{m}, of the order of the turbulent time scale, is much shorter than that of the magnetic field (see Sect. 6). Therefore, we can take $\partial\alpha_\mathrm{m}/\partial t = 0$, which yields

$$\alpha_\mathrm{m} \simeq 2\frac{R_\omega}{C}\frac{\partial \ln \overline{B}_\phi}{\partial z}\,.$$

Perhaps unexpectedly, Eqs. (25) and (26) now become linear,

$$\frac{\partial \overline{B}_r}{\partial t} = -2\frac{R_\omega}{C}\frac{\partial^2 \overline{B}_\phi}{\partial z^2} + \frac{\partial^2 \overline{B}_r}{\partial z^2}\,, \qquad (28)$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = R_\omega \overline{B}_r + \frac{\partial^2 \overline{B}_\phi}{\partial z^2}\,, \qquad (29)$$

and have the following exact quadrupolar solution satisfying the vacuum boundary conditions (A3)

$$\begin{pmatrix} \overline{B}_r \\ \overline{B}_\phi \end{pmatrix} = K \begin{pmatrix} -\pi\kappa\sqrt{2/C} \\ 1 \end{pmatrix} \cos(\pi\kappa z)\exp(\gamma t)\,,$$

where growing magnetic fields have

$$\gamma = \pi\kappa\left(|R_\omega|\sqrt{2/C} - \pi\kappa\right), \quad \kappa = \tfrac{1}{2} + n\,, \quad n = 0, 1, \ldots\,,$$

and K is an arbitrary constant. Given that $R_\omega < 0$ in galactic discs, the dynamo produces a growing magnetic field if $R_\omega < R_{\omega\mathrm{c}}$ with

$$R_{\omega\mathrm{c}} = -\pi\sqrt{C/2} \approx -8\,. \qquad (30)$$

As we argue below, this is a necessary but not sufficient condition for the dynamo action of this type.

The assumptions made to derive this solution can now be verified: $|\overline{B}_r/\overline{B}_\phi| = \pi/\sqrt{2C} \ll 1$ for the lowest mode, and $\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}} \equiv 0$ since $\overline{B}_r/\overline{B}_\phi$ is independent of z in this approximation.

An essential feature of this mechanism is that it cannot be launched unless the large-scale magnetic field is strong enough: for $\overline{B}^2 \ll 1$, we have $|\alpha_\mathrm{m}| \ll 1$ and the field must decay. For α_m to grow, its variation rate $C\overline{B}^2$ should be larger than the magnetic diffusion rate across the disc, $\pi^2/4$ of Eq. (13). This yields $\overline{B} \gtrsim \pi/(2\sqrt{C}) \approx 0.2$, where we recall that magnetic field is measured in the units of B_eq. For galactic parameters, the minimum magnetic field is of order $1\,\mu$G. The initial magnetic field needs to be even stronger if we take into account that it first decays until α_m has grown enough. These arguments equally apply to the helicity flux discussed by Kleeorin et al. (2000, 2002).

Therefore, conditions for the dynamo action driven by the Vishniac-Cho alone are, firstly, the inequality (30) and, secondly, the initial large-scale magnetic field must be strong enough, say $\overline{B}^2|_{t=0} > B_0^2$, where B_0 strongly depends on R_ω. As a result, the dynamo threshold $R_{\omega\mathrm{c}}$ is a function of the initial magnetic field; in this sense, Eq. (30) is just a necessary condition. Numerical solution of the dynamo equations in the no-z approximation with $\alpha_\mathrm{K} = 0$ shows that magnetic field decays for $R_\omega > -22$ for any initial magnetic field and grows for $R_\omega < -23$ if $B_0 > 0.4$ and for $R_\omega < -40$ if $B_0 > 0.1$. This dynamo mechanism seems to be suitable for an additional amplification of the large-scale magnetic field produced by the conventional mean-field dynamo (assisted by the advective helicity flux as described in Sect. 4) closer to equipartition with turbulent energy.

The Vishniac-Cho dynamo can be saturated by quenching the corresponding helicity flux, with $\partial F_z/\partial z$ in Eq. (23) multiplied by $1/(1+\overline{B}^2)$. We have confirmed, using the no-z approximation, that this indeed produces a steady state with $\overline{B} = O(1)$. Curiously, the 'standard' α-quenching, with $\alpha_\mathrm{m} \to \alpha_\mathrm{m}/(1+\overline{B}^2)$ in both Eqs (25) and (27), cannot lead to a saturated state because of the trivial cancellation of the quenching factor.

6 CONCLUSIONS AND DISCUSSION

The simple model suggested above reproduces a feature of the mean-field dynamo which has become well known: if the helicity of the small-scale magnetic field cannot escape from the dynamo active region, the mean magnetic field eventually decays to negligible levels being constrained by the conservation of magnetic helicity. Correspondingly, Eq. (20) yields $\overline{B} \simeq R_\mathrm{m}^{-1/2} \ll 1$ for $R_U = 0$. An outflow from the dynamo region can prevent this catastrophic suppression of the dynamo as it carries away small-scale magnetic fields together with their contribution to the total magnetic helicity, giving breathing space to the large-scale magnetic field (Shukurov et al. 2006). Thus, $\overline{B} \propto R_U^{1/2}(l_0/h)B_\mathrm{eq}$ in Eq. (20) with $R_\mathrm{m} \gg 1$, where R_U is the turbulent magnetic Reynolds number of the outflow and B_eq is the magnetic field strength corresponding to equipartition with the turbulent energy; $B_\mathrm{eq} \simeq 5\,\mu$G in the Solar neighbourhood of the Milky Way. Thus, the outflow must be strong enough to support the dynamo action, $R_U \gtrsim 0.1$ according to our results (Figs 1 and 2). However, any outflow is removing the large-scale magnetic field as well, and thus adversely affects the dynamo action. Hence, the large-scale magnetic field decays when $R_U \gtrsim 0.8$. The strength of the outflow optimal for the dynamo action is $R_U \approx 0.3$, and this is consistent with the plausible range of $\overline{U}_z$ in spiral galaxies estimated by Shukurov et al. (2006) as $\overline{U}_z = 1$–$2\,\mathrm{km\,s^{-1}}$.

For $R_U = 0.3$, the dynamo achieves a steady state in about $10^{10}\,\mathrm{yr}$ for parameter values typical of the Solar neighbourhood of the Milky Way, with the amplification factor of about 10^4 in terms of magnetic energy. This imposes significant restrictions on the strength of the seed magnetic field required for the dynamo (Beck et al. 1996). However, this estimate, often taken as representative of spiral galaxies as a whole, only applies to the Solar vicinity of the Milky Way and, in addition, relies on various poorly known factors as well as on the approximations of this paper. Closer to the Galactic centre, the angular velocity of rotation $\Omega \propto r^{-1}$ is larger together with the dynamo number, $D \propto \alpha\Omega \propto \Omega^2$. For example, at a galactocentric distance equal to half the Solar orbit radius, the dynamo number is four times larger than near the Sun, and the dynamo growth time $\tau \propto (\sqrt{D} - \sqrt{D_\mathrm{c}})^{-1}$ is more than five times shorter than near the Sun. This applies to other galaxies as well: for example, the growth rate of the large-scale magnetic field in the nearby galaxy M51 is estimated to be *ten* times larger than that in the Solar neighbourhood (§VII.9 in Ruzmaikin et al. 1988).

These features of the mean-field dynamo facilitated by the advective flux of magnetic helicity appear to be quite satisfactory. However, the steady-state strength of the large-scale magnetic field, about $0.1B_\mathrm{eq} \simeq 0.5\,\mu$G for $D = 2D_\mathrm{c}$ and $R_U = 0.3$ – see Eq. (20) and Fig. 2, is near the lower end of the range observed in spiral galaxies, 1–5 μG (Beck 2000). The main factor which makes the magnetic field strength low is $\overline{B} \propto C^{-1/2} \propto l_0/h \simeq 0.2$ in Eq. (20). The origin of this factor is the fact that the magnetic helicity evolves over the time scale $l_0^2/\eta_\mathrm{t} \simeq l_0/v_0$ – see Eq. (6) – which is shorter than the evolution time scale of the mean magnetic field, h^2/η_t. Thus, even in the presence of the advective helicity flux magnetic helicity can partially cancel the kinetic one before the large-scale magnetic field has grown enough. (Without any helicity flux, this cancellation is more complete and the mean magnetic field is catastrophically quenched.) The advection term in Eq. (6) opposes the growth of $|\alpha_\mathrm{m}|$, and so the steady-state strength of the mean field increases with R_U as $(R_U/C)^{1/2}$. We note in this connection that it is important that Eq. (5) involves a scale (l_0) independent of the magnetic Reynolds number; otherwise, the strength of the mean magnetic field would be catastrophically small for large $R_{\rm m}$.

Although the agreement with observations within an order of magnitude may be sufficient for a crude model explored, we discuss in the Sect. 5 an additional effect that can make the magnetic field stronger and accelerate its growth. We have considered an additional flux of magnetic helicity, suggested by Vishniac & Cho (2001), which arises because of the symmetry breaking and anisotropy introduced in the turbulent flow by differential rotation. This attractive mechanism is essentially nonlinear and can only be efficient if the large-scale magnetic field is strong enough – in fact, almost exactly as produced owing to the advective helicity flux (Sect. 5.1). Therefore, we believe that the primary role of the Vishniac-Cho helicity flux in galactic dynamos is to complement the action of the advective flux of magnetic helicity. We have not explicitly included another type of magnetic helicity flux suggested by Kleeorin et al. (2000) and Kleeorin et al. (2002), which also relies on anisotropy of turbulence. However, the main elements of the functional form of that flux are, at least in the approximation used here, similar to that of the Vishniac-Cho flux, and we believe that the results would remain qualitatively unchanged.

The additional helicity flux due to anisotropy of turbulence can be essential for supporting large-scale magnetic fields in starburst galaxies or in galaxies with a relatively strong wind, where the strong outflow could otherwise suppress the dynamo action.

Even for $R_U = 0$, the mean magnetic field achieves strength of order $0.1B_{\rm eq}$ before it is catastrophically quenched. As follows from Fig. 2, the field strength is maximum at a time of order $10^{10}\,{\rm yr}$, which is comparable to the galactic lifetime, and the subsequent decay by a factor of ten takes another $10^{10}\,{\rm yr}$. This implies that some galaxies can have a significant mean magnetic field even if their fountain flow is too weak to alleviate the catastrophic quenching of the mean-field dynamo.

A feature of both the earlier models of dynamical quenching and the present work is that the turbulent diffusion is assumed to be unaffected by magnetic field. Due to this reason, in the absence of a helicity flux, as α goes below a critical value (due to increase in $|\alpha_{\rm m}|$), turbulent diffusion leads to a decay of the mean field faster than at the resistive time-scale, resulting in the rapid decay of the mean field for $R_U = 0$ mentioned in the previous paragraph. However this is not accompanied in the models by an equally rapid decay of $\alpha_{\rm m}$. This is consistent with the fact that the steady-state solution of Eq. (6), $\alpha_{\rm m} = R_{\rm m}\boldsymbol{\mathcal{E}}\cdot\overline{\boldsymbol{B}}/B_{\rm eq}^2$ for $R_U = 0$, can be of order unity if $\boldsymbol{\mathcal{E}}\cdot\overline{\boldsymbol{B}} \sim R_{\rm m}^{-1}$. One possibility is that current models are limited in how accurately they incorporate strict total helicity conservation. It is also possible that one may have a "turbulent" diffusive contribution to the small-scale helicity flux, when one goes to the next order in large-scale derivatives in Eq. 3 and the above problem might have resulted from the neglect of this flux.

Alternatively, a mean-field decay without the correspondingly strong decay in $\alpha_{\rm m}$, can still be consistent with strict total helicity conservation if there is a preferential loss of the mean-field helicity through the boundary (without the corresponding loss of the small-scale helicity). Indeed such preferential loss of the large-scale field and its helicity (apparently due to the turbulent magnetic diffusion of the mean field) are seen in the direct simulations of Brandenburg & Dobler (2001) where open boundary conditions were used. In these simulations helical turbulence was driven in a slab between open boundaries, with the field was taken to be purely vertical on the boundaries. The resulting α^2-dynamo led to a mean-field, but with a steady-state magnetic energy decreasing with $R_{\rm m}$. This presumably resulted from the fact that turbulent diffusion of the mean-field (and its helicity) through the boundary was more efficient than similar turbulent diffusive losses of the small-scale helicity. Such a behaviour is similar to the catastrophic quenching of the mean field that obtains in one-dimensional mean field dynamo models (including α^2 dynamo models) with open boundaries, such as this work or those of Brandenburg & Subramanian (2005c) and Shukurov et al. (2006), in the limit of zero small-scale helicity flux. Remarkably, in a domain with closed or periodic boundaries, where both large- and small- scale helicity losses are zero, a different type of $R_{\rm m}$-dependent evolution obtains. Direct simulations of Brandenburg (2001) show that in this case the magnitude of the large-scale magnetic field can eventually reach larger, super-equipartition values, but the time scale to reach this value increases with $R_{\rm m}$. Clearly, the case of open boundaries is more relevant for galaxies, and both the direct simulations and mean field dynamo models indicate that efficient removal of the small-scale magnetic helicity from the dynamo region is necessary for a healthy mean-field dynamo action.

On the technical side, we have confirmed the applicability of the no-z approximation to the fairly complicated nonlinear dynamo system discussed here. In particular, it is reassuring that, the time evolution of the magnetic field in the no-z model follows closely that obtained in numerical solutions of the corresponding partial differential equations, including the oscillatory behaviour (Shukurov et al. 2006). A nontrivial issue in this term approach is how to approximate $\overline{\boldsymbol{J}}\cdot\overline{\boldsymbol{B}}$, since this vanishes in the no-z approximation. Using a different approximation (Appendix A), we show that this term is proportional to $-|D|^{1/2}\overline{B}_r\overline{B}_\phi$. Kleeorin et al. (2002) assume a similar form but without the factor $-|D|^{1/2}$ which noticeably affects the quantitative results (especially because of the different sign of the term). Shukurov et al. (2006) approximate this term as being proportional to $\alpha_{\rm m}$ in their analytical estimate of the steady-state field, their Eq. (10), which affects the result for $R_U = 0$. With this form, the time evolution of the field (in the no-z approximation) is also not correctly reproduced for $R_U = 0$.

The mechanisms of dynamo action discussed here can have extensive implications for galactic magnetic fields. For example, the modulation of the dynamo action by the outflow may contribute to the formation of magnetic arms, i.e., spiral-shaped regions of strong large-scale magnetic field located between the gaseous spiral arms, as in the galaxy NGC 6946 (e.g., Sect. 5 in Beck 2000). Magnetic arms occur not in all galaxies, and in some cases (e.g., M51) the large-scale magnetic field is stronger in the gaseous arms at some radii and between them elsewhere. It is reasonable to expect that the intensity of the galactic fountain, quantified by R_U, is higher in the arms where star formation is more intense. If then the average value of R_U is less then 0.3, the enhancement of R_U in the gaseous arms will lead to a stronger magnetic field there. Otherwise, for stronger overall outflow, enhancement of R_U will suppress the magnetic field in the arms, which can produce magnetic arms interlaced with the gaseous spiral arms.

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APPENDIX A: PERTURBATION SOLUTION FOR THE $\alpha\omega$-DYNAMO

The kinematic $\alpha\omega$-dynamo in a thin disc is governed by the following equations written in dimensionless form (e.g., Ruzmaikin et al. 1988):

$$\gamma\overline{B}_r = -R_\alpha \frac{\partial}{\partial z}(\alpha\overline{B}_\phi) + \frac{\partial^2 \overline{B}_r}{\partial z^2}\,, \tag{A1}$$

$$\gamma\overline{B}_\phi = R_\omega \overline{B}_r + \frac{\partial^2 \overline{B}_\phi}{\partial z^2}\,, \tag{A2}$$

with the vacuum boundary conditions at the disc surface,

$$\overline{B}_r|_{z=1} = \overline{B}_\phi|_{z=1} = 0\,. \tag{A3}$$

where γ is the growth rate of the magnetic field, $\partial\overline{\boldsymbol{B}}/\partial t = \gamma\overline{\boldsymbol{B}}$, and $\overline{B}_z$ can be recovered from the solenoidality condition. Here we derive an approximate solution of this eigenvalue problem for $|D| \equiv |R_\alpha R_\omega| \ll 1$. As it often happens with such asymptotic methods, the solution can be applied even for a relatively large $|D|$. For the sake of definiteness, we assume $D < 0$, $\alpha(z) = \sin \pi z$ and consider quadrupolar modes that are dominant in a thin disc.

Using new variables $\overline{B}_r' = R_\alpha^{-1}\overline{B}_r$ and $\overline{B}_\phi' = |D|^{-1/2}\overline{B}_\phi$, we rewrite Eqs (A1) and (A2) in terms of the dynamo number and then represented them in a symmetric form

$$\gamma\overline{\boldsymbol{B}} = (\widehat{W} + \epsilon\widehat{V})\overline{\boldsymbol{B}}\,, \tag{A4}$$

where $\epsilon = |D|^{1/2}$ is a small parameter and for $D < 0$,

$$\widehat{W} = \begin{pmatrix} \partial^2/\partial z^2 & 0 \\ 0 & \partial^2/\partial z^2 \end{pmatrix},$$

$$\widehat{V}\overline{\boldsymbol{B}} = \begin{pmatrix} 0 & -\partial(\alpha\overline{B}_\phi)/\partial z \\ -\overline{B}_r & 0 \end{pmatrix},$$

and we have dropped dash at the newly introduced variables.

The eigensolutions of the unperturbed (free-decay) system $\lambda_n \boldsymbol{b}_n = \widehat{W}\boldsymbol{b}_n$ (with the above boundary conditions) are doubly degenerate and given by

$$\lambda_n = -\pi^2\left(n + \tfrac{1}{2}\right)^2, \qquad n = 0, 1, 2, \ldots, \tag{A5}$$

$$\boldsymbol{b}_n = \begin{pmatrix} \sqrt{2}\cos[\pi(n+\frac{1}{2})z] \\ 0 \end{pmatrix},$$

$$\boldsymbol{b}_n' = \begin{pmatrix} 0 \\ \sqrt{2}\cos[\pi(n+\frac{1}{2})z] \end{pmatrix},$$

where we have normalized them to $\int_0^1 \boldsymbol{b}_n^2\,dz = \int_0^1 \boldsymbol{b}_n'^2\,dz = 1$; the eigenfunctions should not be confused with the small-scale magnetic field denoted $\boldsymbol{b}$ in the main text.

The expansions

$$\gamma = \gamma_0 + \epsilon\gamma_1 + \epsilon^2\gamma_2 + \ldots\,, \tag{A6}$$

$$\overline{\boldsymbol{B}} = C_0\boldsymbol{b}_0 + C_0'\boldsymbol{b}_0' + \epsilon C_1\boldsymbol{b}_1 + \epsilon C_1'\boldsymbol{b}_1' + \ldots \tag{A7}$$

are substituted into Eqs (A1) and (A2), terms of like order in ϵ collected, the dot product of the resulting equations taken first with $\boldsymbol{b}_n$ and then with $\boldsymbol{b}_n'$, and results are integrated over $0 \leqslant z \leqslant 1$. To the lowest order, this yields $\gamma_0 = \lambda_0$. A homogeneous system of algebraic equations for C_0 and C_0' follows from terms of order ϵ, whose solvability condition yields $\gamma_1 = (V_{0'0}V_{00'})^{1/2} = \frac{1}{2}\sqrt{\pi}$, and $C_0' = C_0\gamma_1/V_{00'} = -2C_0/\sqrt{\pi}$, where $V_{nm} = \int_0^1 \boldsymbol{b}_n \cdot \widehat{V}\boldsymbol{b}_m\,dz$ are the perturbation matrix elements, whose direct calculation yields $V_{00} = V_{0'0'} = V_{10} = V_{1'0} = V_{1'0'} = 0$, $V_{00'} = -\pi/4$, $V_{0'0} = -1$ and $V_{10'} = -3\pi/4$. To the same order in ϵ, we similarly obtain inhomogeneous algebraic equations for C_1 and C_1', which yield $C_1' = 0$ and $C_1 = C_0'V_{10'}/(\lambda_0 - \lambda_1) = 3C_0/(4\pi^{3/2})$.

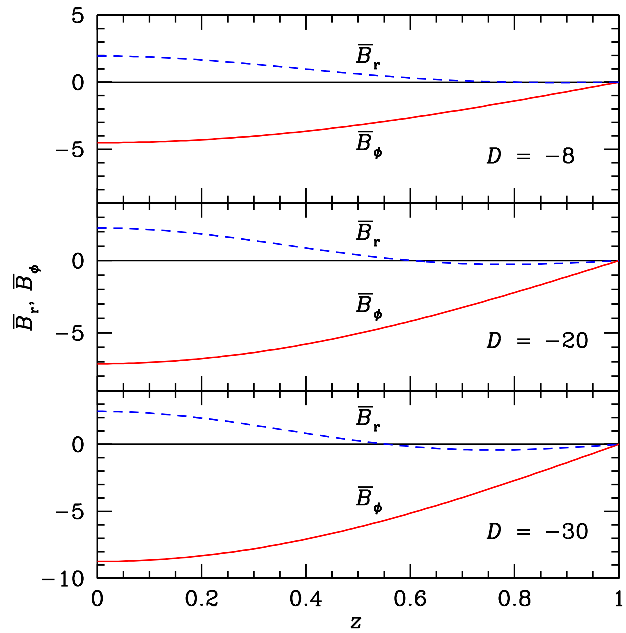

Figure A1. The approximate eigenfunctions $\overline{B}_r$ (dashed) and $\overline{B}_\phi$ (solid) from Eqs (A8) and (A9) for $D = -8, -20, -30$ (with $D_c \approx -8$), $C_0 = 1$ and $R_\alpha = 1$.

In terms of the original variables $\overline{B}_{r,\phi}$, the resulting approximate solution is given by

$$\overline{B}_r \approx R_\alpha C_0 \left[\cos\left(\frac{\pi z}{2}\right) + \frac{3}{4\pi^{3/2}}\sqrt{|D|}\cos\left(\frac{3\pi z}{2}\right)\right], \tag{A8}$$

$$\overline{B}_\phi \approx -2C_0\sqrt{\frac{|D|}{\pi}}\cos\left(\frac{\pi z}{2}\right), \tag{A9}$$

$$\gamma \approx -\tfrac{1}{4}\pi^2 + \tfrac{1}{2}\sqrt{\pi|D|}\,, \tag{A10}$$

where C_0 is an arbitrary constant. Thus, $\gamma > 0$ for $D < D_c$ with $D_c \approx -\pi^3/4 \approx -8$. This estimate of D_c is impressively close to that obtained numerically (Ruzmaikin et al. 1988). We plot the components of the magnetic field for various values of the dynamo number in Fig. A1; the result reproduces very closely the more accurate numerical solutions presented, e.g., in Fig. VII.1 of Ruzmaikin et al. (1988), including such a subtle detail as a zero of $\overline{B}_r$ at $0 < z < 1$ for $D < D_c$ which shifts to smaller z as $|D - D_c|$ increases.

The current helicity density of this magnetic field follows as

$$\overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} \approx -\frac{3\pi^{1/2}}{8}|D|^{1/2}\overline{B}_r\overline{B}_\phi \tag{A11}$$

to the lowest order in D. This estimate is used in Sect. 3.

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